## CHAPTER ONE: FUNDAMENTALS OF ALGEBRA

**Review September 13 ⚘ Test September 25**

*Thorough understanding and fluency of the concepts and methods in this chapter is a cornerstone to success in the rest of this course and most future math courses as well.*

### 1-A Fractions — Monday • 8/19

❶ Convert a whole number to a fraction.
❷ Multiply by a fraction.
❸ Divide by a fraction.
❹ Reduce a fraction.
❺ Add or subtract fractions with the same denominator.
❻ Add or subtract fractions with different denominators.
❼ Convert a percentage to a decimal or fraction.

### 1-B Expressions — Friday • 8/23

term • expression • argument • polynomial • monomial • binomial • trinomial • degree • constant • linear • quadratic • cubic • standard form • coefficient • leading coefficient • scientific notation

❶ Identify terms of an expression.
❷ Identify the argument of a function.
❸ Multiply a rational expression by an integer.
❹ Simplify a fraction with multiple terms in the numerator or denominator.
❺ Classify a polynomial in one variable.
❻ Convert calculator notation to scientific notation and to standard notation.
❼ Convert standard notation to scientific notation.

### 1-C Solving Equations — Wednesday • 8/28

inverse • equation

❶ Apply order of operations.
❷ Identify the inverse of an operation by definition.
❸ Solve a linear equation.
❹ Show proper notation in solving an equation.
❺ Express an answer.

### 1-D Properties of Exponents — Wednesday • 9/4

❶ Simplify an expression using properties of exponents.

## 1-E Addition, Subtraction, and Multiplication of Polynomials

**Wednesday • 9/11**

conjugate

1. Add or subtract polynomials.
2. Multiply two polynomials.
3. Multiply more than two polynomials.
4. Multiply a binomial by its conjugate.
5. Square a binomial.

## 1-A Fractions

❶ Convert a whole number to a fraction.

1. The numerator is the original number, and the denominator is 1.

❶ 10 can be written as the fraction $\frac{10}{1}$.

❷ Multiply by a fraction.

1. Multiply the numerators together.
2. Multiply the denominators together.

❷ a) $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$ b) $\frac{2}{5} \times 3 = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$

❸ Divide by a fraction.

1. Multiply by the reciprocal of the fraction (see ❷).

❸ a) $\frac{2}{5} \div \frac{4}{3} = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$ b) $\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times 3 = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$

❹ Reduce a fraction.

1. Divide the numerator and denominator by a number that divides evenly into both.
2. Repeat step 1 until no number divides evenly into both the numerator and the denominator.

❹ Reduce $\frac{140}{350}$.

1. 10 divides evenly into 140 and into 350. $140 \div 10 = 14$, and $350 \div 10 = 35$. $\frac{140}{350} = \frac{14}{35}$
2. 7 divides evenly into 14 and into 35. $14 \div 7 = 2$, and $35 \div 7 = 5$. $\frac{14}{35} = \frac{2}{5}$

It also would have worked to divide 140 and 350 both by 70 in step 1.

❺ Add or subtract fractions with the same denominator.

1. Add or subtract the numerators.
2. Keep the same denominator.

❺ $\frac{12}{20} - \frac{5}{20} = \frac{7}{20}$

❻ Add or subtract fractions with different denominators.

1. Multiply the first fraction by $\frac{b}{b}$, where $b$ is the denominator of the second fraction.
2. Multiply the second fraction by $\frac{a}{a}$, where $a$ is the denominator of the first fraction.

❻ a) $\frac{3}{5} - \frac{1}{4}$

1. $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$
   $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$
2. $\frac{12}{20} - \frac{5}{20} = \frac{7}{20}$

$x$% means $\frac{x}{100}$. 100% is the same as $\frac{100}{100}$ or 1.

❼ Convert a percentage to a decimal or fraction.

1. To convert to a fraction, multiply by $\frac{1}{100\%}$, canceling the % symbols. If there is a decimal in the numerator, multiply the fraction by $\frac{10}{10}$ until there is not.
2. To convert to a decimal, move the decimal point two places to the left, and remove the % symbol.

❼ a) 9%

1. $9\% \times \frac{1}{100\%} = \frac{9\%}{100\%} = \frac{9}{100}$

2. $9\% = .09$

b) .09%

1. $.09\% \times \frac{1}{100\%} = \frac{.09\%}{100\%} = \frac{.09}{100}$

$\frac{.09}{100} \times \frac{10}{10} = \frac{.9}{1000}$

$\frac{.9}{1000} \times \frac{10}{10} = \frac{9}{10{,}000}$

2. $.09\% = .0009$

## 1-B Expressions

A TERM is the product of any number of constants, variables, and function values. For example, 5, $5x^3$, and $5x^3yz^2 \sin x$ are all terms.

The COEFFICIENT of a term is the factor of the term that does not vary, and is usually written at the beginning of the term. The coefficient of each the terms above is 5.

An Algebraic EXPRESSION is one or more terms added together. For example, 5, $5 + 5x^3$, and $5 + 5x^3 - 5x^3yz^2 \sin x$ are all expressions.

❶ Identify terms and their coefficients.

1. Each individual term includes no addition or subtraction, but when the terms of an expression are added the result is the whole expression.
2. Divide out the variable factors from each term to find the coefficients. Keep in mind that an “invisible” coefficient equals 1 (or -1), not 0.

❶ Identify the terms of the expression $5x^6 + x^3 + \frac{8x}{7} - 9 - 10\sqrt{x - \cos 3x}$.

1. $5x^6$, $x^3$, $\frac{8x}{7}$, -9, and $-10\sqrt{x - \cos 3x}$ are the terms of the expression, because when added the result is the whole expression $5x^2 - \frac{8x}{7} + 9 + 10\sqrt{x - \cos 3x}$. Note that -10, $x$, $3x$, and -cos $3x$ are not terms of this expression.
2. The coefficients are 5, 1, $\frac{8}{7}$, -9, and -10.

An ARGUMENT of a Function is an expression input into the function.

❷ Identify the argument of a function.

1. An argument is a value input into a function.

❷ Identify all arguments in the expression $5x^2 - \frac{8x}{7} + 9 + 10\sqrt{x - \cos 3x}$.

$3x$ is the argument of the cosine function.

$x - \cos 3x$ is the argument of the square root function.

A function and its argument is a single term. An operation on such a term affects the term as a whole, not the argument separately.

❸ Multiply a rational expression by an integer.

1. Multiply each term in the numerator by the integer, making sure to avoid the following:
   - multiplying a term more than once
   - multiplying an argument
   - multiplying a term in the denominator

❸ Identify the error, if any, in each of the following attempts to multiply $\frac{11(4x) - \sqrt{10x}}{5}$ by 2.

a) $\frac{22(8x) - 2\sqrt{10x}}{5}$ The term $11(3x)$ should be multiplied by 2 one time, but it was multiplied by 2 once on the 11 and again on the $3x$.

b) $\frac{22(4x) - \sqrt{20x}}{5}$ $10x$ is an argument and should not be multiplied.

c) $\frac{22(4x) - 2\sqrt{10x}}{10}$ The denominator should not be multiplied.

d) $\frac{22(4x) - 2\sqrt{10x}}{5}$ This is correct, because each term in the numerator was multiplied by 2 one time.

❹ Simplify a fraction with multiple terms in the numerator or denominator.

1. Identify the terms of the numerator and the denominator.
2. Identify a factor that divides evenly into all of the terms.
3. Rewrite the fraction with the factor divided out of each term. If it is variable, specify that it cannot be zero.

❹ $\frac{6x^2 - 9x\sqrt{30x}}{6x^2 - 9x}$

1. The terms are $6x^2$ and $-9x\sqrt{30x}$ in the numerator, and $6x^2$ and $-9x$ in the denominator. $3x$ is an argument of the square root function, not a term of the numerator.
2. $6x^2 \div 3x = 2x$
   $-9x\sqrt{30x} \div 3x = -3\sqrt{30x}$ (not $-3\sqrt{10}$)
   $6x^2 \div 3x = 2x$
   $-9x \div 3x = -3$
3. $\frac{2x - 3\sqrt{30x}}{2x - 3}$, $x \neq 0$ (We divided by $3x$, so our answer doesn't work if $3x$ is zero.)

Note that $6x^2$ in the numerator and $6x^2$ in the denominator do not cancel each other out because there are other terms.

A MONOMIAL is a nonzero term with a whole number exponent. The exponent is the DEGREE.

| Degree | Name | Example |
|---|---|---|
| 0 | CONSTANT | 5 (that is, $5x^0$) |
| 1 | LINEAR | $5x$ (that is, $5x^1$) |
| 2 | QUADRATIC | $5x^2$ |
| 3 | CUBIC | $5x^3$ |
| $n$ | $n^{\text{th}}$ degree | $5x^{12}$ |

A POLYNOMIAL is an expression of the sum of one or more monomials, after like terms are combined. The degree of a polynomial in one variable is the highest degree of its terms.

A BINOMIAL is a polynomial with two terms.

A TRINOMIAL is a polynomial with three terms.

A polynomial in one variable with its terms written in order from highest degree to lowest is in STANDARD Form.

The LEADING Coefficient of a Polynomial is the coefficient of the term with the highest degree.

❺ Classify a polynomial in one variable.

1. A single term is a monomial, two terms is a binomial, and three terms is a trinomial.
2. Consider the term with the highest exponent:
   If there is no variable at all, the polynomial is a constant.
   If there is a variable but no exponent (that is, there is an unwritten exponent of 1), the polynomial is linear.
   If the variable is to the second power, the polynomial is quadratic.
   If the variable is to the third power, the polynomial is cubic.
   If the variable is to the fourth power or higher, look up what it is called, or simply refer to the polynomial as "$n^{\text{th}}$ degree", where $n$ is the exponent.

❺ Write the following polynomials in standard form, classify them, and identify the leading coefficient.

| | a) $x + 4x^3$ | b) $-15x$ | c) $8x^2 - 2x^9 + 3$ | d) $2 - \frac{7x^3}{5} + 6x^2 + x$ |
|---|---|---|---|---|
| standard form: | $4x^3 + x$ | $-15x$ | $-2x^9 + 8x^2 + 3$ | $-\frac{7x^3}{5} + 6x^2 + x + 2$ |
| classification: | cubic binomial | linear monomial | $9^{\text{th}}$ degree trinomial | cubic polynomial |
| leading coefficient: | 4 | -15 | -2 | $-\frac{7}{5}$ |

SCIENTIFIC NOTATION is $a \times 10^b$, where $1 \le a < 10$ and $b$ is an integer.

Many calculators use the notation `aEb` instead of $a \times 10^b$ to display scientific notation. Do not write numbers in this notation.

❻ Convert calculator notation to scientific notation and to standard notation.

1. To convert `aEb` to scientific notation, change "`E`" into "$\times$ 10", and make $b$ an exponent.
2. To convert $a \times 10^b$ to standard notation, move the decimal point right $b$ spaces (which will be left if $b$ is negative). Fill in 0's as needed.

| ❻ | a) `2.57E3` | b) `2.57E-3` |
|---|---|---|
| 1. | $2.57 \times 10^3$ | $2.57 \times 10^{-3}$ |
| 2. | 2570 | .00257 |

❼ Convert standard notation to scientific notation.

1. Move the decimal place left so that it follows the first nonzero digit, and count how many spaces $b$ it was moved left.
2. Drop any zeros on the end, unless you know they were actually measured, not rounded. If the number is a decimal, no zeros were rounded.
3. Multiply this number by $10^b$.

| ❼ | a) 2700 | b) 2700 exactly | c) .002700 |
|---|---|---|---|
| 1. | 2.700 ($b = 3$) | 2.700 ($b = 3$) | 2.700 ($b = -3$) |
| 2. | 2.7 | 2.700 | 2.700 |
| 3. | $2.7 \times 10^3$ | $2.700 \times 10^3$ | $2.700 \times 10^{-3}$ |

## 1-C Solving Equations

When evaluating an expression with no parentheses, exponents are done first, followed by multiplication and division, and then addition and subtraction.

When evaluating an expression with parentheses in it, each expression within parentheses is evaluated on its own first, following the order above, before the overall expression is evaluated.

Numerators and denominators always have parentheses around them. For example, $\frac{1+3}{3+5}$ is actually $\frac{(1+3)}{(3+5)}$. These parentheses only have to be shown if the numerator is not written above the denominator. For example, on a calculator, $\frac{1+3}{3+5}$ can be typed as $(1+3)\ /\ (3+5)$, but not as $1 + 3 / 3 + 5$.

Negative signs mean "multiply by negative 1." Since they represent multiplication, they are ignored until after exponents are calculated.

❶ Apply order of operations.

1. Apply the steps below to everything within parentheses, including hidden parentheses.
2. Calculate exponenets.
3. Calculate multiplication and division, including multiplying by -1 for negative signs.
4. Calculate addition and subtraction.

❶ a) $4 + 5 \times 2$
$4 + 10$
$14$

b) $4 + 5(2)^3$
$4 + 5(8)$
$4 + 40$
$44$

c) $-2^4$
$-(16)$
$-16$

d) $(-2)^4$
$16$

e) $\frac{5+6}{5-3}$
$\frac{11}{3}$

The INVERSE of an operation is the operation that cancels the original operation.

❷ Identify the inverse of an operation by definition.

1. The inverse of addition is subtraction.
2. The inverse of multiplication is division.
3. The inverse of a power is a root.

❷ Identify the inverse of each operation in the equation $175x + 900 = 1600$.

The inverse of adding 900 is subtracting 900.

The inverse of multiplying by 175 is dividing by 175.

An Algebraic EQUATION is one expression set equal to another.

An equation is solved by applying one or more inverse operations to each side of the equation. These are done in reverse order of operations.

❸ Solve a linear equation.

1. If there are parentheses, distribute to eliminate them. (This step is not needed if the variable is only on one side of the equation.)
2. If needed, subtract a value or expression to each side of the equation to cancel addition, or vice versa.
3. If needed, divide a value on each of the equation to cancel multiplication, or vice versa.
4. If needed, take the root of each side of the equation to cancel a power, or vice versa. If the root is even (e.g., square root), put ±.
5. If there are parentheses around one side of the equation, remove them now and redo steps 1-5.

❸ a) $3x + 1 = 13$

$3x = 12$

$x = 4$

b) $3x^2 + 1 = 13$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

c) $\frac{3}{5}(3x + 1) + 1 = 13$

$\frac{3}{5}(3x + 1) = 12$

$\frac{5}{3}\left(\frac{3}{5}(3x + 1)\right) = \frac{5}{3}\left(\frac{12}{1}\right)$

$3x + 1 = 20$

$3x = 19$

$x = \frac{19}{3}$

d) $5(3x + 1) + 1 = 4x$

$15x + 5 + 1 = 4x$

$6 = 4x - 15x$

$6 = -11x$

$x = -\frac{6}{11}$

It is not incorrect to write basic algebraic operations like subtraction or division, such as subtracting 900 or dividing by 175 in the problem below, but there is no need, and it is never done in advanced math. If you do choose to show these, they should be shown as part of each expression, not below them, next to them, or part of only one of them.

Do not use the symbol "×" for multiplication in an equation in which $x$ is a variable.

If $t$ is a variable, make sure it does not look like "+".

❹ Show proper notation in solving an equation.

1. If it is in a word problem, make sure you know what the variable represents.
2. Apply the inverse operations to each side of the equation, not to the equation as a whole or to only one side. Be sure each symbol is written in an appropriate place. Show every step except those that are both simple enough to recognize easily and not new to the current chapter.
3. Write the new equation. Be sure the expressions on either side of the equals sign are equivalent.
4. Repeat steps 2 and 3 as needed.

❹ Collin has \$900 this year and plans to save \$175 per year. Solve the equation $900 + 175x = 1600$ to determine when he will have \$1600.

| Incorrect | Reason | Correct |
|---|---|---|
| | | $x$ = years after 2019 |
| $175x + 900 = 1600 - 900$ | The subtraction has to be done on both sides. | $175x + 900 - 900 = 1600 - 900$ |
| $\div 175\ (175x = 700)$ | An equals sign cannot be divided, | $175x = 700$ |
| | and the ÷ symbol can't be before the dividend. | $175x \div 175 = 700 \div 175$ |
| $x = 700 \div 175 = 4 + 2019 = 2023$ | $700 \div 175 \neq 4 + 2017$ | $x = 4$ |
| | $x$ is years after 2019, not the current year | He will have \$1600 in the year 4 + 2018 = 2023. |

❹ $\frac{1}{2}x = 5$

There is no work needed to be shown for this problem. Simply writing $x = 10$ makes it clear that both sides were multiplied by 2.

correct but unnecessary: $2(\frac{1}{2}x) = 2(5)$

| | | |
|---|---|---|
| incorrect: $2(\underset{\times 2}{\frac{1}{2}x} = \underset{\times 2}{5})$ | incorrect: $\times 2\ \frac{1}{2}x = 5 \times 2$ | incorrect: $\frac{1}{2}x = 5$ |

In general, simplifying means writing with smaller numbers if possible, such as by reducing or combining like terms.

Rounding an answer makes it approximate instead of exact. Do not round when asked to simplify.

❺ Express an answer.

1. If the instructions are to round, then round to the indicated place, or choose an appropriate place to round to if none is specified.
2. If the instructions are to simplify, then reduce fractions, simplify square roots, and combine like terms.
3. If the instructions do not specify whether to round or simplify:
   - If the answer is a number or algebraic expression, do not round unless you have a reason to do so.
   - If the answer is a quantity (that is, it has units, such as *centimeters* or *years*), round unless you have a reason not to do so.

❺ Give an appropriate answer for each question.

a) Solve $7x = 9$. $x = \frac{9}{7}$

b) Solve $7x = 9$. Round your answer to the nearest tenth. $x = 1.3$

c) If a pack of seven pens costs \$9, what is the price per pen? Each pen costs \$1.29.

## 1-D Properties of Exponents

The rules below are valid in almost all contexts, including any time $a$ and $b$ are positive or $x$ and $y$ are integers.

| Property | Rule | Example |
|---|---|---|
| Power of a Product | $(ab)^x = a^x b^x$ | $(2x)^3 = 2^3x^3 = 8x^3$ |
| Power of a Quotient | $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$ | $\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3} = \frac{x^3}{8}$ |
| Power of a Power | $(b^x)^y = b^{xy}$ | $(x^5)^3 = x^{15}$ |
| Product of Powers | $b^x b^y = b^{x+y}$ | $x^5x^3 = x^8$ |
| Quotient of Powers | $\frac{b^x}{b^y} = b^{x-y}$ | $\frac{x^5}{x^3} = x^2$ |
| Zero Exponent | $b^0 = 1$ | $2^0 = 1$ |
| Negative Exponent | $b^{-x} = \frac{1}{b^x}$ | $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ |

❶ Simplify an expression using properties of exponents.

1. Use the properties above as needed.
2. Reduce fractions if needed.

❶ Simplify.

a) $ab^3(a^2b)^5$

$ab^3(a^{10}b^5)$

$a^{11}b^8$

b) $\left(\frac{5a}{2}\right)^3$

$\frac{125a^3}{8}$

c) $\left(\frac{5a}{2}\right)^{-3}$

$\left(\frac{2}{5a}\right)^3$

$\frac{8}{125a^3}$

d) $10\left(\frac{5a}{2}\right)^{-3}$

$10\left(\frac{2}{5a}\right)^3$

$\frac{10(8)}{125a^3}$

$\frac{80}{125a^3}$

$\frac{16}{25a^3}$

e) $\frac{8a^{-3}}{6a^4}$

$\frac{4}{3a^7}$

f) $\frac{6\times10^{-12}}{2\times10^{20}}$

$3\times10^{-32}$

❶ Simplify $\frac{(2a^4b)^3b^6}{a^{12}bc^2}$, and write it without a fraction. State each property of exponents used.

| | | |
|---|---|---|
| $(2a^4b)^3 = 2^3(a^4)^3b^3$ | Power of a Product | $\frac{8(a^4)^3b^3b^6}{a^{12}bc^2}$ |
| $(a^4)^3 = a^{12}$ | Power of a Power | $\frac{8a^{12}b^3b^6}{a^{12}bc^2}$ |
| $b^3b^6 = b^9$ | Product of Powers | $\frac{8a^{12}b^9}{a^{12}bc^2}$ |
| $\frac{a^{12}b^9}{a^{12}b} = a^0b^8$ | Quotient of Powers | $\frac{8a^0b^8}{c^2}$ |
| $a^0 = 1$ | Zero Exponent | $\frac{8(1)b^8}{c^2}$ |
| $\frac{1}{c^2} = c^{-2}$ | Negative Exponent | $8b^8c^{-2}$ |

## 1-E Addition, Subtraction, and Multiplication of Polynomials

❶ Add or subtract polynomials.

1. Distribute any coefficient, including negatives.
2. Combine like terms.

❶ $5(4x^2 + 9x - 3) - (11x - 4)$

1. $(20x^2 + 45x - 15) + (-11x + 4)$
2. $20x^2 + 34x - 11$

❷ Multiply two polynomials.

1. Multiply each term in the first polynomial by each term in the second polynomial.
2. Simplify.
3. Combine like terms.

❷ $(4x^2 - 3x)(x + 5)$

1. $(4x^2 \bullet x) + (4x^2 \bullet 5) + (-3x \bullet x) + (-3x \bullet 5)$
2. $4x^3 + 20x^2 - 3x^2 - 15x$
3. $4x^3 + 17x^2 - 15x$

❸ Multiply more than two polynomials.

1. Multiply two of the polynomials together (see ❷).
2. Multiply the result by the next polynomial.
3. Combine like terms.

❸ $(x + 2)(x + 5)(x - 10)$

1. $(x^2 + 7x + 10)(x - 10)$
2. $(x^3 + 7x^2 + 10x) + (-10x^2 - 70x - 100)$
3. $x^3 - 3x^2 - 60x - 100$

The CONJUGATE of a Binomial is the original binomial except with the sign of one of the terms switched: The conjugate of $a + b$ is $a - b$.
There is a special pattern for multiplying a binomial by itself and for multiplying by its conjugate.
Binomial times itself: $(a + b)^2 = a^2 + 2ab + b^2$
Binomial times its conjugate: $(a + b)(a - b) = a^2 - b^2$

❹ Multiply a binomial by its conjugate.

1. Square each term of the binomial.
2. Subtract the second square from the first.

❹ $(3x - 10)(3x + 10)$

1. $a^2 = (3x)^2 = 9x^2$
   $b^2 = 10^2 = 100$
2. $(3x - 10)(3x + 10) = 9x^2 - 100$

❺ Square a binomial.

1. Square each term of the binomial.
2. Multiply the two terms together, and double this product.
3. Add the three results of the steps above.

❺ $(3x - 10)^2$

1. $a^2 = (3x)^2 = 9x^2$
   $b^2 = (-10)^2 = 100$
2. $2ab = 2(3x)(-10) = -60x$
3. $(3x - 10)^2 = 9x^2 - 60x + 100$