**Chapter Fouir** 

# **Discrete Probability Distributions**

Introduction to Probability Distributions Geometric and Binomial Probabilities Binomial Distributions

## **Probability Distributions**

A **probability distribution** states each possible outcome or range of outcomes of an event and how likely it is.

A probability distribution can be displayed in many ways, such as a sentence, table, or graph. When the variable is numerical, a histogram is commonly used.

Flip two coins.	Sentence	Table	Graph
Probability distribution for number of heads	There is a 25% chance of getting 0 heads, a 50% chance of getting exactly 1 head, and a 25% chance of getting 2 heads.	Probability of exactly <i>r</i> heads out of 2 coin flips $\frac{r \mid P(r)}{0 \mid 25\%}$ 1 \ 50% 2 \ 25%	Probability of exactly r heads out of 2 coin flips 50% 25% 0% 0 1 2 # of heads

### **Discrete and Continuous Variables**

Variables for which specific values are counted are **discrete**. Variables for which values are sorted into ranges are **continuous**.

Variable type	Definition	In a histogram	Examples
Discrete	There exist values	There is a bar for	• iPhone capacity (128GB, 256GB, 512GB, etc.
	between which	each possible	but not 200GB or other values in between)
	no other values of	value. Each bar is	• number of people in a kitchen (0, 1, 2, 3,,
	the variable are	labeled.	but not 2¾ or other values in between)
	possible.		
Continuous	There are infinitely	Each bar	<ul> <li>book weight (0 – 100 grams, 100 – 200</li> </ul>
	many possible	represents a range	grams,)
	values of the	of values. The	<ul> <li>tree height (0 – 2 meters, 2 – 4 meters,)</li> </ul>
	variable between	boundaries of the	
	any two values of	bars are labeled	
	it.	instead of the bars	
		themselves.	

Usually for distributions with more than a handful of possible values, the values are grouped into ranges and treated as continuous even if they are technically discrete, such as number of people at a school.

### **Mean and Standard Deviation of Probability Distributions**

The mean of a probability distribution is its expected value, which is the same as its weighted average.

The standard deviation is calculated the same as it is for grouped data, except that the probabilities P(x) are used instead of the frequencies f. Since the sum of the probabilities must be 1, the mean is  $\sum xP(x) \div 1$ , that is  $\mu = \sum xP(x)$ .

In the example below, a deck has six white cards worth 0 points, three green cards worth 10 points, and one red card worth 25 points.

Event	X	<b>P(x)</b>	<i>xP</i> ( <i>x</i> )	x – µ	$(x - \mu)^2$	$P(x)(x-\mu)^2$
white card	0	<u>6</u> 10	0.0	-5.5	30.25	18.15
green card	10	<u>3</u> 10	3.0	4.5	20.25	6.08
red card	25	<u>1</u> 10	2.5	19.5	380.25	38.03
TOTAL		1.00	5.5			$\sigma^2 = 62.26$

 $\sigma = \sqrt{62.26} \approx 7.89$ 

#### **Binomial Probabilities**

A **binomial experiment** is a scenario in which a specific independent event is attempted multiple times so see how many successes there are.

Value	Meaning	Example: 3 correct predictions in ten 6-sided die rolls
n	number of trials	10 rolls were made.
r	number of successes	3 rolls were correctly predicted.
р	probability of success on each individual trial	Each roll had a $\frac{1}{6}$ chance of being correctly predicted.
q	probability of failure on each Each roll had a $\frac{5}{6}$ chance of being incorrectly predicted. individual trial ( $q = p'$ )	
<b>p</b> <sup>r</sup>	probability of <i>r</i> successes out of <i>r</i> trials	There is a $(\frac{1}{6})^3 = \frac{1}{216}$ chance of three out of three rolls being correctly predicted.
<b>q</b> <sup>n-r</sup>	probability of <i>n</i> — <i>r</i> failures out of <i>n</i> trials	There is a $\binom{5}{6}^7 = \frac{78125}{279936}$ chance of seven out of seven rolls being incorrectly predicted.
( <i>n</i> / <i>r</i> )	number of possible orders of <i>r</i> successes out of <i>n</i> total trials	There are $\binom{10}{3} = 120$ different ways to choose which 3 of the 10 rolls were correctly predicted.
$\binom{n}{r}p^{r}q^{n-r}$	probability of exactly <i>r</i> The probability of correctly predicting	
	successes (and <i>n</i> – <i>r</i> failures)	exactly 3 out of 10 rolls on a 6-sided die is
	out of <i>n</i> trials	$\binom{10}{3}\binom{1}{6}^{3}\binom{5}{6}^{7} = 120\binom{1}{216}\binom{78125}{279936} = \frac{9375000}{60466176} \approx 15.5\%$

### **Binomial Calculator Functions**

The probability of exactly *r* successes in a binomial experiment,  $\binom{n}{r}p^{r}q^{n-r}$ , can be found on the calculator with the binompaf function.

The probability of at most *r* successes in a binomial experiment can be found on the calculator with the binomedf function.

The probability of at least *r* successes in a binomial experiment can be found on the calculator by taking the complement of at most r - 1 successes.

The binomial functions can be selected from the DISTR menu.

Function	Probability of	Example: Roll a die 8 times
<pre>binompdf(n,p,r)</pre>	exactly <i>r</i> successes	Exactly 2 sixes:
		binompdf(8,1/6,2) $\approx$ 26.0%
<pre>binomcdf(n,p,r)</pre>	at most <i>r</i> successes	At most 2 sixes:
		binomcdf(8,1/6,2) $\approx$ 86.5%
1-binomcdf(n,p,r-1)	at least <i>r</i> successes	At least 2 sixes:
		$1-binomcdf(8, 1/6, 1) \approx 39.5\%$

## Mean and Standard Deviation of Binomial Distributions

For binomial probability distributions, the calculations for mean and standard deviation are greatly simplified. In addition, the mean is also the most likely outcome, or between the two most likely outcomes if the mean itself is not a whole number.

Statistic	Formula	Example for number of 6's out of 12 dice
Mean	$\mu = np$	$\mu = 12(\frac{1}{6}) = 2$
Standard Deviation	$\sigma = \sqrt{npq}$	$\sigma = \sqrt{12(\frac{1}{6})(\frac{5}{6})} \approx 1.29$