CHAPTER SIX: GRAPHS OF QUADRATIC FUNCTIONS — Test: Tuesday, March 3

The graph of a quadratic function is a parabola. The most basic quadratic function is $y = x^2$, which has a vertex of (0, 0) and an x-intercept of (0, 0). If the vertex or x-intercepts are different, or if the parabola is wider, skinnier, or up-side down, the equation changes, and it can be expressed in vertex form, intercept form, or standard form.

6-A Parabolas — Thursday • 2/5

quadratic function • parabola • axis of symmetry • vertex • minimum • maximum

❶ Identify the vertex, minimum or maximum value, axis of symmetry, domain, and range of a graphed quadratic function.

❷ Graph $f(x) = ax^2$.

❸ Sketch $y = ax^2$ for different values of a.

6-B Vertex Form — Monday • 2/9

vertex form

❶ Identify the vertex of a parabola in vertex form.

❷ Sketch $f(x) = a(x - h)^2 + k$.

❸ Identify the y-intercept of a parabola.

❹ Identify the x-intercepts of a parabola in vertex form.

6-C Intercept Form — Thursday • 2/12

intercept form

❶ Identify the vertex of a parabola in intercept form.

❷ Sketch $f(x) = a(x - p)(x - q)$.

6-D Standard Form — Tuesday • 2/17

standard form

❶ Identify the vertex of a parabola in standard form.

❷ Convert standard form to vertex form.

❸ Convert vertex form or intercept form to standard form.

❹ Write, in standard form, the equation of a parabola from a graph.

6-A Parabolas

A PARABOLA is the U-shaped curve created by a quadratic equation such as $y = ax^2$.

The tip of a parabola is its VERTEX, labeled (h, k).

If the leading coefficient a is positive, the parabola opens upward and the vertex is at the bottom, making its y value the MINIMUM Value of the parabola. The reverse is true if the leading coefficient a is negative.

The shape of the parabola is also determined by a: The closer a is to zero, the flatter the parabola is.

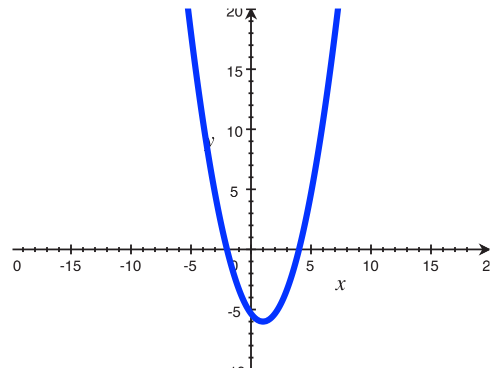
The domain of a quadratic function is all real numbers, because any number can be squared.

The range of a parabola is all values at or above the minimum value if the parabola opens upward, or all values at or below the maximum value if the parabola opens downward.

The vertical line $x = h$ going through a parabola's vertex divides it exactly in half. This line is the parabola's AXIS OF SYMMETRY.

❶ Identify the vertex, minimum or maximum value, axis of symmetry, intercepts, domain, and range of a graphed quadratic function.

1. The vertex is the tip of the parabola. Label it (h, k).
2. The maximum or minimum value is k.
3. The axis of symmetry is the vertical line $x = h$.
4. The y-intercept is where the parabola crosses the y-axis (where $x = 0$).
5. The x-intercepts, if any, are where the parabola crosses the x-axis (where $y = 0$).
6. The domain is all real numbers.
7. The range is $y \geq k$ if the parabola opens upward or $y \leq k$ if the parabola opens downward.

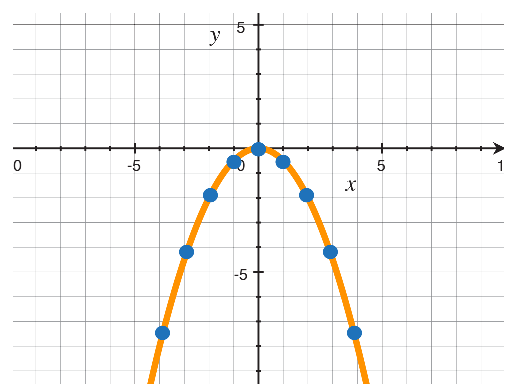


❶

1. The vertex is the point (1, -6).
2. The minimum value is -6.
3. The axis of symmetry is the line $x = 1$.
4. The y-intercept is approximately (0, -5.5).
5. The x-intercepts are (-2, 0) and (4, 0).
6. The domain is all real numbers.
7. The range is $y \geq -6$.

❷ Graph $f(x) = ax^2$.

1. Start at the vertex (0, 0).
2. Make a table of values using $x = 1, 2, 3 \ldots$.
3. Plot these points.
4. Make a mirror image by plotting the same y-values but negative x-values.
5. Connect the dots in a parabola.



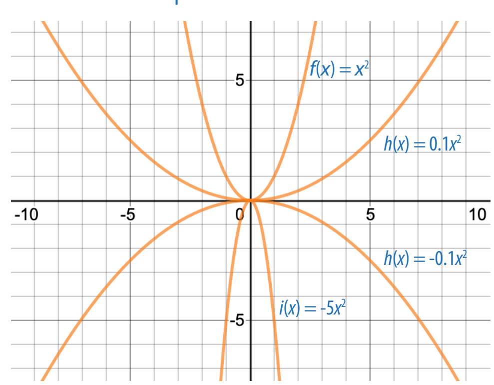
❷ Graph $f(x) = -0.5x^2$.

❸ Sketch $y = ax^2$ for different values of a.

1. The parabola opens upward if a is positive or downward if a is negative.
2. The closer a is to zero, the closer the graph is to a horizontal line.

❸ Sketch the graphs of $f(x) = x^2$, $g(x) = 0.1x^2$, $h(x) = -0.1x^2$, and $i(x) = -5x^2$ on the same set of axes.

1. f and g open upward. h and i open downward.
2. Compared to the others, g and h are fairly flat and close to the x-axis. i is very steep and not close to the x-axis at all except near the vertex.



6-B Vertex Form of a Quadratic

The graph of any equation can be moved to the right by any amount h by subtracting h from each x in the equation (or, equivalently, by adding h to the expression x is set equal to). Likewise, the graph of any equation can be moved up by any amount k by subtracting k from each y in the equation (or, equivalently, by adding k to whatever y is set equal to).

Therefore, the graph of $y = a(x - h)^2 + k$ is the same as the graph of $y = ax^2$ except moved to the right by h units and moved upward by k units. In other words, the shape and direction stay the same and the vertex moves from (0, 0) to (h, k).

A quadratic equation written in the form $y = a(x - h)^2 + k$ is in VERTEX Form.

❶ Identify the vertex of a parabola in vertex form.

1. Identify h, the value subtracted from x. (If a value is being added to x, then the value subtracted from x is negative.)
2. Identify k, the value being added to the whole expression that equals y.
3. The vertex is (h, k).

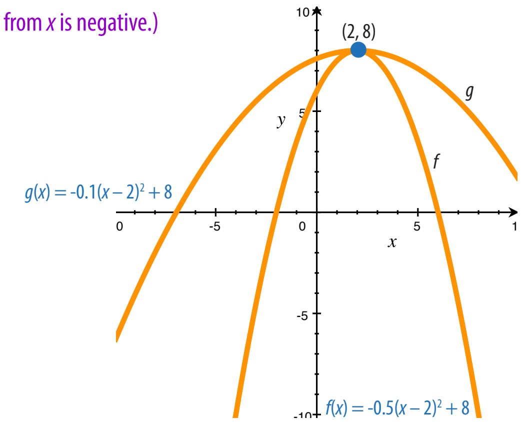
❶ $y = -3(x + 4)^2 + 12$

1. $h = -4$
2. $k = 12$
3. The vertex is (-4, 12).

❷ Sketch $f(x) = a(x - h)^2 + k$.

1. Identify the vertex (h, k) (see ❶), and plot it.
2. Sketch the parabola the same as $y = ax^2$ (see 6-A ❸), except starting at the new vertex.

❷ Sketch $f(x) = -0.5(x - 2)^2 + 8$ and $g(x) = -0.1(x - 2)^2 + 8$ on the same set of axes.



1. The vertex of both parabolas is (2, 8).
2. The parabolas open downward because $a = -0.5$ and $a = -0.1$ are negative. g is flatter than f because -0.1 is closer to zero than -0.5 is.

A y-intercept is a point on a graph where $x = 0$.

An x-intercept is a point on a graph where $y = 0$.

❸ Identify the y-intercept of a parabola.

1. Calculate $f(0)$, where f is the function.
2. The y-intercept is $(0, f(0))$.

❸ Find the y-intercept of $d(x) = -0.5(x - 2)^2 + 8$.

1. $d(0) = -0.5(0 - 2)^2 + 8 = 6$
2. (0, 6)

x-intercepts of parabolas in vertex form can be found by applying inverses to solve an equation the same as in 2-C ❺.

❹ Identify the x-intercepts of a parabola in vertex form.

1. Let $y = 0$ on one side of the equation.
2. Subtract k from each side.
3. Divide each side by a.
4. If one side is a negative number, then there are no x-intercepts. Otherwise, take the square root of each side. Use $\pm$ to indicate both square roots.
5. Split it up into two separate equations, one with plus and one with minus.
6. Solve both equations.
7. The x-intercepts are $(p, 0)$ and $(q, 0)$, where p and q are the solutions in step 5.

❹ Find the x-intercepts of $f(x) = -0.5(x - 2)^2 + 8$

1. $0 = -0.5(x - 2)^2 + 8$
2. $-8 = -0.5(x - 2)^2$
3. $16 = (x - 2)^2$
4. $\pm 4 = x - 2$
5. $4 = x - 2, -4 = x - 2$
6. $x = 6, x = -2$
7. The x-intercepts are (6, 0) and (-2, 0).

6-C Intercept Form of a Quadratic

A quadratic equation written in the form $y = a(x - p)(x - q)$ is in INTERCEPT Form. Its x-intercepts are $(p, 0)$ and $(q, 0)$.

This is because an x-intercept is a point on a graph where $y = 0$, an expression equals zero when one of its factors equals zero, and a factor of the form $(x - p)$ equals zero when x is p.

Since parabolas are symmetrical, the vertex is exactly midway between the two x-intercepts: $h = \frac{p+q}{2}$.

❶ Identify the vertex of a parabola in intercept form.

1. Identify p and q.
2. Calculate $h = (p + q) \div 2$.
3. Plug h into the equation to find k.
4. The vertex is (h, k).

❶ $f(x) = \frac{1}{5}(x - 4)(x + 6)$

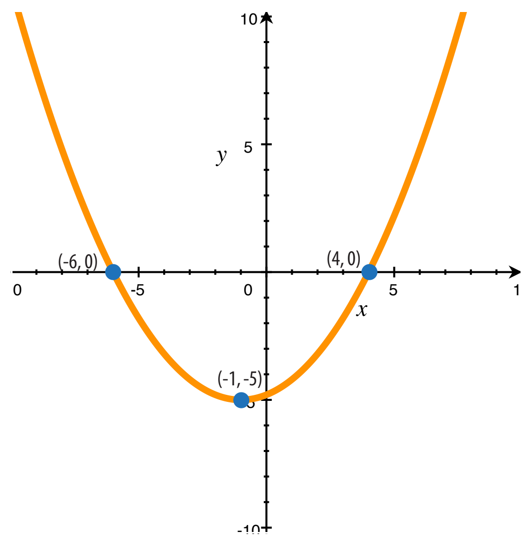
1. $p = 4$, $q = -6$
2. $h = \frac{4-6}{2} = -1$
3. $k = \frac{1}{5}(-1 - 4)(-1 + 6) = -5$
4. The vertex is $(-1, -5)$.

❷ Sketch $f(x) = a(x - p)(x - q)$.

1. Identify the x-intercepts $(p, 0)$ and $(q, 0)$.
2. Identify the vertex (h, k) (see ❶).
3. Connect the vertex and x-intercepts in a parabola.

❷ $f(x) = \frac{1}{5}(x - 4)(x + 6)$

1. The x-intercepts are $(4, 0)$ and $(-6, 0)$.
2. $h = \frac{4-6}{2} = -1$
   $k = \frac{1}{5}(-1 - 4)(-1 + 6) = -5$
   The vertex is $(-1, -5)$.



6-D Standard Form of a Quadratic

A quadratic equation written in the form $y = ax^2 + bx + c$ is in STANDARD Form.

The vertex (h, k) of the graph of a quadratic function f in standard form can be found by calculating $h = \frac{-b}{2a}$ and $k = f(h)$.

❶ Identify the vertex of a parabola in standard form.

1. Identify a and b.
2. Calculate $h = -b \div 2a$.
3. Plug h into the equation to find k.
4. The vertex is (h, k).

❶ $f(x) = 0.25x^2 + x - 3$

1. $a = 0.25$, $b = 1$
2. $h = \frac{-1}{2(0.25)} = -2$
3. $k = f(-2) = 0.25(-2)^2 + (-2) - 3 = -4$
4. The vertex is $(-2, -4)$.

❷ Convert standard form to vertex form.

1. Identify the vertex (h, k) (see ❶).
2. The equation is $f(x) = a(x - h)^2 + k$, using (h, k) from the vertex and a from the original equation.

❷ $f(x) = 0.25x^2 + x - 3$

1. The vertex is $(-2, -4)$ (see ❶).
2. $f(x) = 0.25(x + 2)^2 - 4$

❸ Convert vertex form or intercept form to standard form.

1. Multiply the binomials together.
2. Distribute a.
3. Combine like terms.

❸ $f(x) = 0.25(x + 2)^2 - 4$

1. $f(x) = 0.25(x^2 + 4x + 4) - 4$
2. $f(x) = 0.25x^2 + x + 1 - 4$
3. $f(x) = 0.25x^2 + x - 3$

❹ Write, in standard form, the equation of a parabola from a graph.

1. Identify the vertex (h, k) or the x-intercepts $(p, 0)$ and $(q, 0)$.
2. Identify the coordinates (x, y) of any point on the graph not already used in step 1.
3. Plug h, k, x, and y into the equation $y = a(x - h)^2 + k$, or plug p, q, x, and y into the equation $y = a(x - p)(x - q)$.
4. Solve for a.
5. Write the equation of the parabola, using the values from steps 1 and 4 but leaving x and y as variables.
6. Convert to standard form (see ❸).

❹ Write the equation of the parabola graphed at right.

1. The x-intercepts are $(-1, 0)$ and $(7, 0)$.
2. The point $(3, -32)$ is also on the graph.
3. $-32 = a(3 + 1)(3 - 7)$
4. $-32 = -16a$

   $a = 2$
5. $f(x) = 2(x + 1)(x - 7)$
6. $f(x) = 2(x^2 - 6x - 7)$

   $f(x) = 2x^2 - 12x - 14$