Chapter Four

Factoring

Monomial Factors

Binomial Factors

Solving

Common Monomials

Many polynomials can be written as the product of a monomial and another polynomial. A monomial that divides into every term of a polynomial like this is called a **common monomial**, as in the examples below.

Polynomial	Common Monomial	Factorization
$4x - 6$	2	$2(2x - 3)$
$4x^8 - 6x^7 + 6x^3$	$2x^3$	$2x^3(2x^5 - 3x^4 + 3)$
$4x^8y^4 + 12x^5y^6$	$4x^5y^4$	$4x^5y^4(x^3 + 3y^2)$

Factoring $ax^2 + bx + c$

Some quadratic trinomials can be factored as the product of two binomials. This can be done by splitting the linear term into two so that the expression can be written as the sum of two binomials.

In order for this method to work, the linear coefficient must be split so that the two new b values multiplied together equal the same as a and c multiplied together. This will result in each binomial having the same factor.

In the example below, $8 \times 15 = 120$, so 26 must be split up into two numbers that also equal 120 when multiplied. When this is done, both binomials have the same factor, $4x + 3$.

Step	Explanation
$8x^2 + 26x + 15$	original polynomial
$8x^2 + 6x + 20x + 15$	$6x + 20x$ is still $26x$, and $6(20) = 120$, the same as $8(15)$
$(8x^2 + 6x) + (20x + 15)$	Group $8x^2 + 6x$ into one binomial and $20x + 15$ into another.
$2x(4x + 3) + 5(4x + 3)$	Factor $2x$ out of the first binomial and 5 out of the other.
$(2x + 5)(4x + 3)$	There are $2x$ of the $(4x + 3)$ factor, plus another 5 of them.

It can be tricky to determine how to split b up in order to factor, and in some cases the polynomial is not factorable. There are different strategies for what numbers to try, as discussed in class.

Factoring $x^2 + bx + c$

$x^2 + bx + c$ is factored as $(x + b_1)(x + b_2)$ if it is factorable, where b_1 and b_2 add together to equal b and multiply together to equal c.

Polynomial Example	$b_1 + b_2 = b$	$b_1 \times b_2 = c$	Factors
$x^2 + 10x + 21$	$3 + 7 = 10$	$3 \times 7 = 21$	$(x + 3)(x + 7)$
$x^2 + 7x - 18$	$-2 + 9 = 7$	$-2 \times 9 = -18$	$(x - 2)(x + 9)$
$x^2 - 12x + 20$	$-2 - 10 = -12$	$-2 \times -10 = 20$	$(x - 2)(x - 10)$
$x^2 - 6x - 55$	$5 - 11 = -6$	$5 \times -11 = -55$	$(x + 5)(x - 11)$

$x^2 + bx + c$ can also be factored using the methed for factoring $ax^2 + bx + c$, with $a = 1$.

Special Factoring Patterns

A binomial times its conjugate, $(a+b)(a-b)=a^2-b^2$, is a **difference of squares**.

For a binomial of the form a^2-b^2, the factors are $(a+b)(a-b)$.

Example	a	b	Factors
x^2-9	x	3	$(x+3)(x-3)$
x^2+9	x	3	not factorable
$4x^2-9$	$2x$	3	$(2x+3)(2x-3)$
$4x^{10}-9y^2z^{16}$	$2x^5$	$3yz^8$	$(2x^5+3yz^8)(2x^5-3yz^8)$

A binomial times itself, $(a+b)^2=a^2+2ab+b^2$, is a **perfect square** trinomial.

For a trinomial of the form $a^2+2ab+b^2$, the factors are $(a+b)^2$.

Example	a	b	$2ab$	Factors
x^2+6x+9	x	3	$2(x)(3)=6x$	$(x+3)^2$
x^2-6x+9	x	-3	$2(x)(-3)=-6x$	$(x-3)^2$
$4x^2-6x+9$	$2x$	-3	$2(2x)(-3)=-12x\neq -6x$	not factorable
$4x^2-12x+9$	$2x$	-3	$2(2x)(-3)=-12x$	$(2x-3)^2$

Solving by Factoring

If a product is zero, then one of the factors must be zero. Therefore, an equation in which one side is factored and the other side is zero can be solved by setting each factor equal to zero, as in the three examples below.

Step	$(x-8)(2x+10)=0$	$x^2+9x+20=0$	$x^2+9x+20=2$
Set one side equal to zero.			$x^2+9x+18=0$
Factor the other side.		$(x+4)(x+5)=0$	$(x+3)(x+6)=0$
Set each factor equal to zero.	$x-8=0$ $2x+10=0$	$x+4=0$ $x+5=0$	$x+3=0$ $x+6=0$
Solve each equation.	$x=8$ $x=-5$	$x=-4$ $x=-5$	$x=-3$ $x=-6$