CHAPTER FOUR: FACTORING

Review January 14 ~ Test January 26

Factoring a number is breaking it down to smaller numbers that, when multiplied together equal the original number, such as $30 = 2 \times 3 \times 5$. Likewise, factoring a polynomial is breaking it down to smaller polynomials that, when multiplied together, equal the original polynomial, such as $x^3 + 8x^2 + 15x = x(x + 3)(x + 5)$. If the product of factors is zero, then one of the factors must be zero. This property is frequently used to solve polynomial equations.

4-A Monomial Factors

Monday • 1/4

1. Factor a common monomial out of each term of a polynomial.
2. Factor a polynomial by grouping.

4-B Binomial Factors

Thursday • 1/7

1. Factor $ax^2 + bx + c$.
2. Factor $x^2 + bc + c$.
3. Identify and factor a perfect square trinomial.
4. Factor a difference of squares.
5. Factor a polynomial.

4-C Solving

Thursday • 1/14

1. Solve an equation in factored form.
2. Solve an equation by factoring.

4-A Monomial Factors

Just as a factor of a whole number is a whole number that divides evenly into it, a FACTOR of a Polynomial is a polynomial that divides evenly into it. For example, 3 and 4 are factors of 12, and $2x$ and $x - 5$ are factors of $2x^2 - 10x$. Factoring a polynomial is writing it as a product of other polynomials.

A COMMON Monomial is a term that divides evenly into every term in a polynomial.

❶ Factor a common monomial out of each term of a polynomial.

1. Find a monomial that divides evenly into every term in the polynomial.
2. Divide every term by the common monomial.
3. Write the common monomial, followed by the new polynomial in parentheses.

❶ $40x^5 - 8x^3 + 20x^2$

1. $4x^2$ divides into all three terms.
2. $(40x^5 - 8x^3 + 20x^2) \div 4x^2 = 10x^3 - 2x + 5$
3. $4x^2\,(10x^3 - 2x + 5)$

Sometimes one monomial is common to some of the terms in a polynomial, and another monomial is common to the other terms. If factoring these separate monomials out of their respective terms leaves the same quotient each time, this grouping can help in factoring the polynomial.

❷ Factor a polynomial by grouping.

1. Sort the terms of the polynomial into groups such that each group has its own common monomial.
2. Factor a common monomial out of each group.
3. If the quotient of each group is the same, the factors of the polynomial are this quotient and the sum of the common monomials.

❷ $2x^3 - 8x^2 + 5x - 20$

1. $(2x^3 - 8x^2) + (5x - 20)$
2. $2x^2(x - 4) + 5(x - 4)$
3. $(2x^2 + 5)(x - 4)$

4-B Binomial Factors

A factorable trinomial $ax^2 + bx + c$ can be factored by grouping after splitting up bx into $b_1x + b_2x$, where $b_1b_2 = ac$.

❶ Factor $ax^2 + bx + c$.

1. Calculate ac.
2. Identify a pair of factors of ac whose sum is b.
3. Using this pair of factors, rewrite bx as the sum of two separate terms.
4. Rewrite the polynomial as the sum of two binomials.
5. Factor a common monomial out of each of the binomials. The remaining factor will be the same for both binomials.
6. One factor is the remaining factor from both binomials. The other is the sum of the monomials that were factored out.

❶ $18x^2 + 27x + 4$

1. $ac = 18(4) = 72$ — The first coefficient times the last coefficient is 72.
2. $3(24) = 72$, $3 + 24 = 27$ — Find another pair of numbers that has a product of 72. This pair must add up to b, which is 24.
3. $18x^2 + 3x + 24x + 4$ — Split the middle term $27x$ into $3x + 24x$.
4. $(18x^2 + 3x) + (24x + 4)$ — Group $18x^2 + 3x$ into one binomial and $24x + 4$ into another.
5. $3x(6x + 1) + 4(6x + 1)$ — Factor $3x$ out of the first binomial and 4 out of the second binomial. The remaining factor is $(6x + 1)$ both times.
6. $(3x + 4)(6x + 1)$ — There are $3x$ of the $(6x + 1)$ factor, plus another 4 of them, for a total of $(3x + 4)$ of them.

For $x^2 + bx + c$ (that is, when $a = 1$), ac is equal to c, and not all the steps above are needed. The two factors of c are the two numbers in the final answer.

❷ Factor $x^2 + bc + c$.

1. Find two numbers p and q such that $b_1 + b_2 = b$ and $b_1b_2 = c$.
2. The factorization is $(x + b_1)(x + b_2)$.

❷ $x^2 - 13x + 22$

1. $-2 + -11 = -13$
   $-2(-11) = 22$
2. $(x - 2)(x - 11)$

$(a+b)^2 = a^2 + 2ab + b^2$ is called a PERFECT SQUARE Trinomial.

❸ Identify and factor a perfect square trinomial.

1. Find the square root, a, of the first term, and find the square root, b, of the last term. Make b negative if the middle term is negative.
2. If the middle term equals 2 times ab, then the trinomial factors as $(a+b)^2$. Otherwise, the trinomial is not a perfect square.

❸

a) $16x^2 - 24x + 9$

1. $a = \sqrt{16x^2} = 4x$
   $b = -\sqrt{9} = -3$
2. $2(4x)(-3) = -24x$, so $16x^2 - 24x + 9$ is the perfect square $(4x-3)^2$.

b) $x^2 + 10x + 100$

$a = \sqrt{x^2} = x$
$b = \sqrt{100} = 10$
$2(x)(10) = 20x \neq 10x$, so $x^2 + 10x + 100$ is not a perfect square.

$(a+b)(a-b) = a^2 - b^2$ is called a DIFFERENCE OF SQUARES because it is one squared expression minus another squared expression.

❹ Factor a difference of squares.

1. Find the square root a of the first term, and find the square root b of the last term (ignoring the negative).
2. The factors are $(a+b)(a-b)$.

❹ $16x^2 - 9$

1. $a = \sqrt{16x^2} = 4x$
   $b = \sqrt{9} = 3$
2. $16x^2 - 9 = (4x+3)(4x-3)$

❺ Factor a polynomial.

1. If possible, factor out a common monomial from every term (see 4-A ❶).
2. If possible, factor a perfect square or a difference of squares (see ❸ or ❹).
3. If possible, continue factoring, such as by the methods in ❶ or ❷.

❺ $80x^7 + 300x + 180x^5$

1. $20x^5(4x^2 + 15x + 9)$
3. $20x^5(4x^2 + 12x + 3x + 9)$
   $20x^5(4x(x+3) + 3(x+3))$
   $20x^5(4x+3)(x+3)$

4-C Solving

If $ab = 0$, then a or b must be zero.

❶ Solve an equation in factored form.

1. Set each factor equal to zero.
2. Solve each equation in step 1.

❶ $3x^2(x+6)(2x-9)(x^2-16)(x^2+7x+10) = 0$

1. $3x^2 = 0$	$x+6=0$	$2x-9=0$	$x^2-16=0$	$x^2+7x+10=0$
2. $x^2 = 0$		$2x=9$	$x^2=16$	$(x+2)(x+5)=0$
$x = 0$	$x = -6$	$x = \frac{9}{2}$	$x = \pm 4$	$x = -2, x = -5$

❷ Solve an equation by factoring.

1. Isolate zero on one side of the equation. If this cannot be done, the equation cannot be solved by this method.
2. Factor the expression.
3. Set each factor equal to zero.
4. Solve each equation in step 3.

❷ $18x^3 = 50x$

1. $18x^3 - 50x = 0$
2. $2x(9x^2 - 25) = 0$
   $2x(3x+5)(3x-5) = 0$

3. $2x = 0$	$3x+5=0$	$3x-5=0$
4. $x = 0$	$x = -\frac{5}{3}$	$x = \frac{5}{3}$