Chapter Three

Geometry

Lines and Line Segments

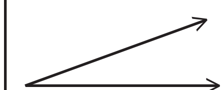
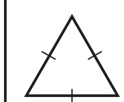
Angles

Quadrilaterals

Transformations

Scale Factors

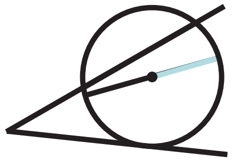
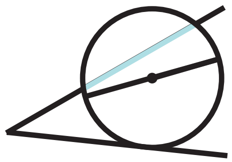
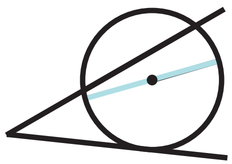
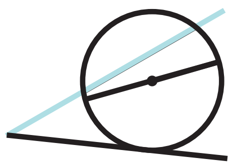
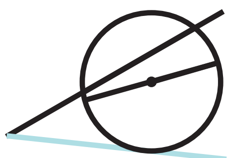
Basic Geometric Terminology and Notation

Term	Notation	Definition	Sketch
Point	A	zero-dimensional object	
Line	$\overleftrightarrow{AB}$	one-dimensional object extending infinitely in both directions	
Ray	$\overrightarrow{AB}$	one-dimensional object extending infinitely in one direction	
Line Segment	$\overline{AB}$	finite one-dimensional object	
Length	AB	distance from one point to another	
Angle	$\angle A$	two rays that share a common endpoint, called the vertex	
Congruent	$\cong$	equal in size and shape	
Parallel	$\parallel$	extending forever in a plane but never intersecting	
Perpendicular	$\perp$	intersecting at a 90° angle	

Components of Circles

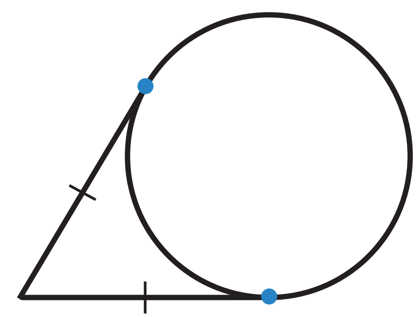
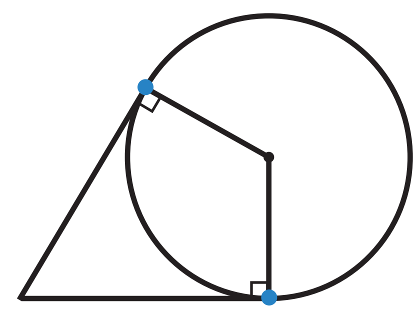
A circle can be defined as the set of points that are a set distance from a given point.

There are special names for each of the types of lines and line segments that intersect a circle.

Line or Line Segment	Definition	Example
Radius	line segment from the center to a point on the circle	
Chord	line segment from a point on the circle to another point on the circle	
Diameter	chord passing through the center	
Secant	line that contains a chord	
Tangent	line, ray, or line segment that intersects the circle at exactly one point without entering it	

Tangency Theorems

Every point outside a circle is on two lines tangent to the circle. Each of these lines intersect the circle at a **point of tangency**.

Theorem	Description	Sketch
External tangent congruence	The two line segments connecting a given point outside the circle to a point of tangency are congruent.	
Tangent line to circle	A tangent is perpendicular to the radius it intersects.	

Equations of Lines

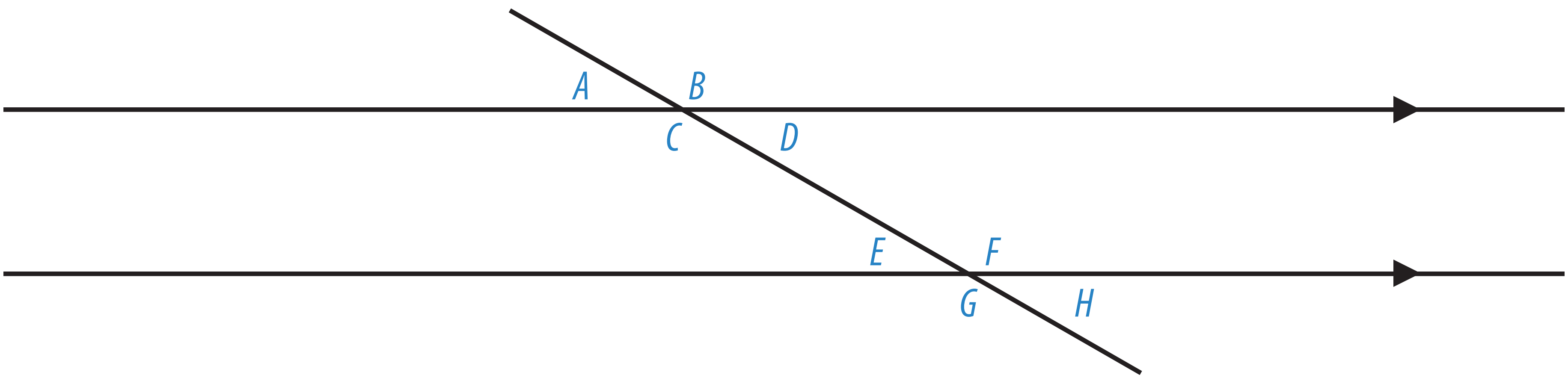
The equation of a line can be written $y = mx + b$, where m is the slope and b is the y-intercept. The y-intercept can be found by using m and an (x, y) point in the equation.

Item	Equation	Example for (4, -5) and (8, 1)
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	$m = \frac{1-(-5)}{8-4} = \frac{6}{4} = \frac{3}{2}$
y-intercept	Solve the line equation for b, using the value of m and one of the known points for x and y.	$1 = \frac{3}{2}(8) + b$ $1 = 12 + b$ $b = -11$
Equation of Line	$y = mx + b$	$y = \frac{3}{2}x - 11$
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	$M = \left(\frac{4+8}{2}, \frac{-5+1}{2}\right) = (6, -2)$
Equation of Perpendicular Bisector	$y = -\frac{1}{m}x + b$ (Solve for b using x and y from the midpoint.)	$-2 = -\frac{2}{3}(6) + b$ $-2 = -4 + b$ $b = 2, y = -\frac{2}{3}x + 2$

Angles from Transversals and Parallel Lines

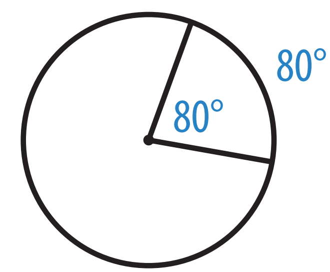
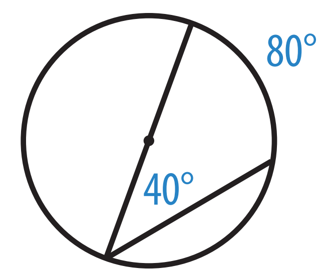
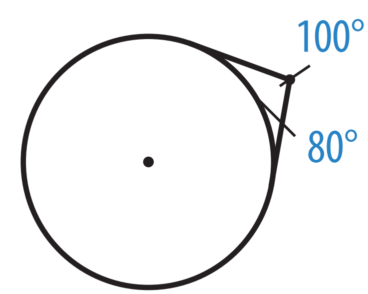
A **transversal** is a line that intersects two other lines at two different points. When these two other lines are parallel, the following types of angles are made with the transversal.

Type of angle	Example	Relationship
Linear	$\angle A$ and $\angle B$	supplementary
Vertical	$\angle A$ and $\angle D$	congruent
Consecutive	$\angle C$ and $\angle E$	supplementary
Corresponding	$\angle A$ and $\angle E$	congruent
Alternate Interior	$\angle D$ and $\angle E$	congruent
Alternate Exterior	$\angle A$ and $\angle H$	congruent

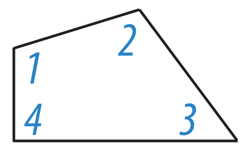
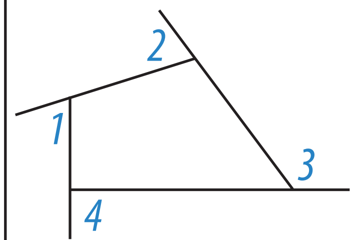


Angles In and Around Circles

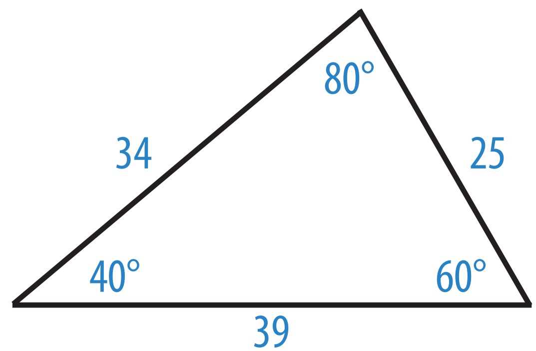
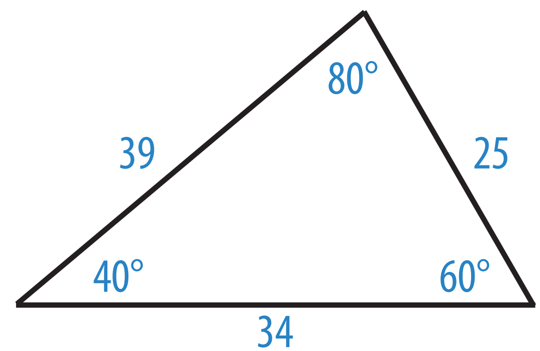
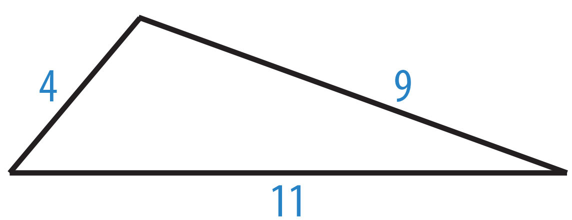
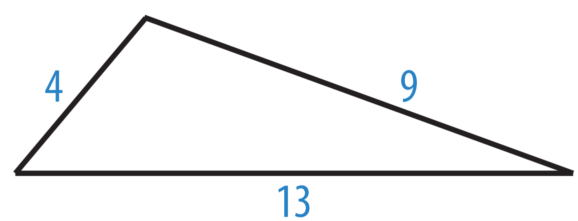
Three types of angles intersecting circles have special names.

Type of angle	Position of vertex	Size	Sketch
Central	center of circle	equal to intercepted arc	
Inscribed	on circle	half of intercepted arc	
Circumscribed	outside circle	180° minus intercepted arc	

Angles of Polygons

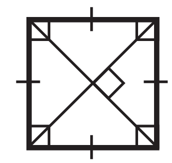
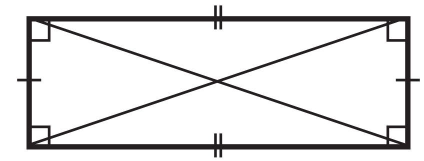
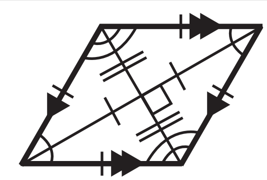
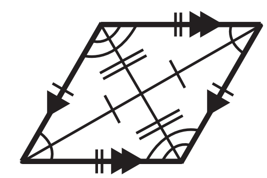
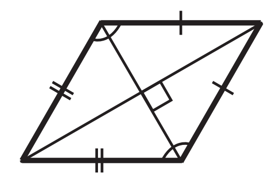
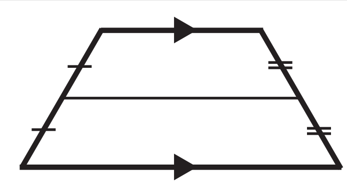
Type of angle	Description	Sketch	Sum of angles
Interior	angle within the polygon		$S = (n - 2)180°$ where n is the number of sides of the polygon
Exterior	angle between one side and the line extending from another side		$S = 360°$ for one angle from each vertex of a convex polygon

Possible Triangles

Restrictions	Possible example	Impossible example
The longest side must be opposite the largest angle. The shortest side must be opposite the smallest angle.		
The longest side must be shorter than the sum of the other two side lengths.		

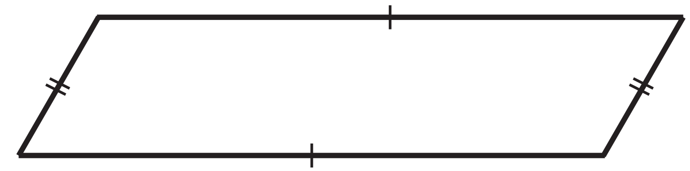
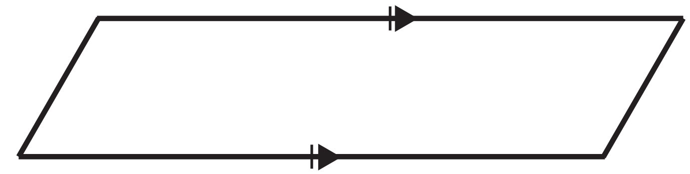
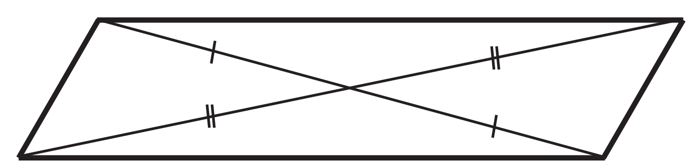
Quadrilaterals

A **quadrilateral** is a four-sided figure that may or may not match one or more of the descriptions below.

Type	Description	Features	Sketch
Square	All four sides are congruent, and all four angles are congruent (90°).	It is a rectangle, rhombus, parallelogram, and regular quadrilateral.	
Rectangle	All four angles are congruent (90°).	It is a parallelogram.	
Rhombus	All four sides are congruent.	It is a parallelogram, and the diagonals are perpendicular.	
Parallelogram	Both pairs of sides are parallel.	The diagonals bisect each other. Opposite sides and angles are congruent.	
Kite	Each side is congruent with exactly one connecting side.	The diagonals are perpendicular. A pair of opposite angles are congruent.	
Trapezoid	Two sides are parallel.	The midsegment length is the average of the lengths of the bases.	

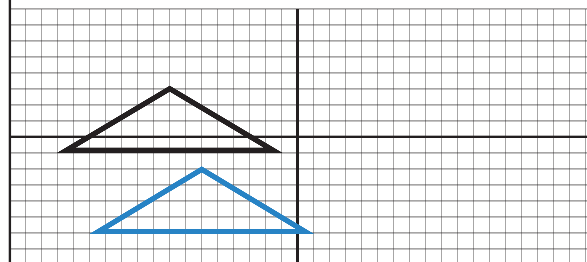
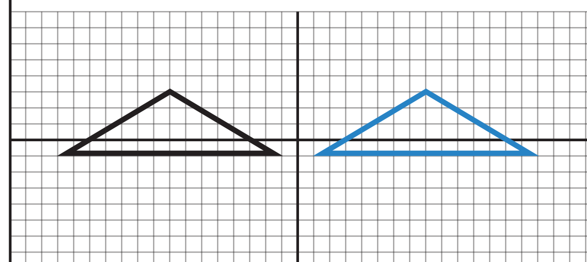
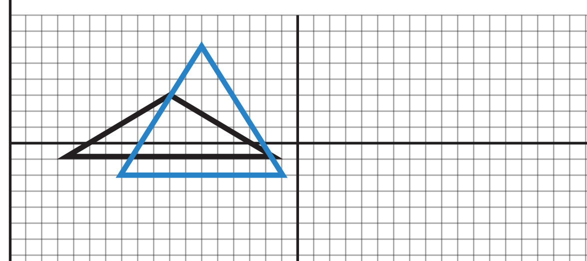
Parallelograms

Each of the following criteria defines a **parallelogram**.

Criterion	Sketch
Both pairs of opposite angles are congruent.	
Both pairs of opposite sides are congruent.	
One pair of opposite sides is congruent and parallel.	
The diagonals bisect each other.	

Mappings

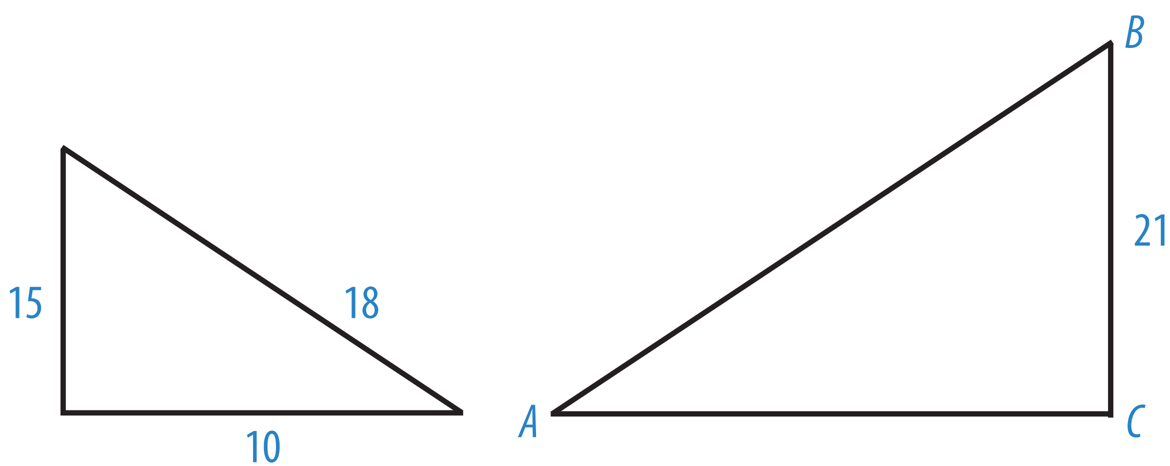
A **mapping**, or transformation, takes an original figure called the **pre-image** and transforms it into a related figure called the **image**.

Mapping	Change	Notation	Example
Translation	position: shifted h units to the right and k units up	$(x, y) \rightarrow (x + h, y + k)$	$h = 2, k = -5$ 
Reflection	direction: reflected horizontally (across the y-axis) or reflected vertically (across the x-axis)	$(x, y) \rightarrow (-x, y)$ or $(x, y) \rightarrow (x, -y)$	horizontal reflection 
Dilation	size: stretched to be b times as wide and a times as tall, centered at the origin	$(x, y) \rightarrow (bx, ay)$	$b = ¾, a = 2$ 

Scale Factors

Two figures that are exactly the same shape and size are **congruent**. Two figures that are exactly the same shape but not necessarily the same size are **similar**.

A **scale factor** is a factor by which a length in the pre-image is multiplied to find the corresponding length in the image. It can be found by dividing a length in the second figure by the corresponding length in the first figure. For similar figures, the scale factor is the same for each length.



Given the triangles above are similar, the scale factor from the small one to the big one is $21 \div 15 = \frac{21}{15}$.

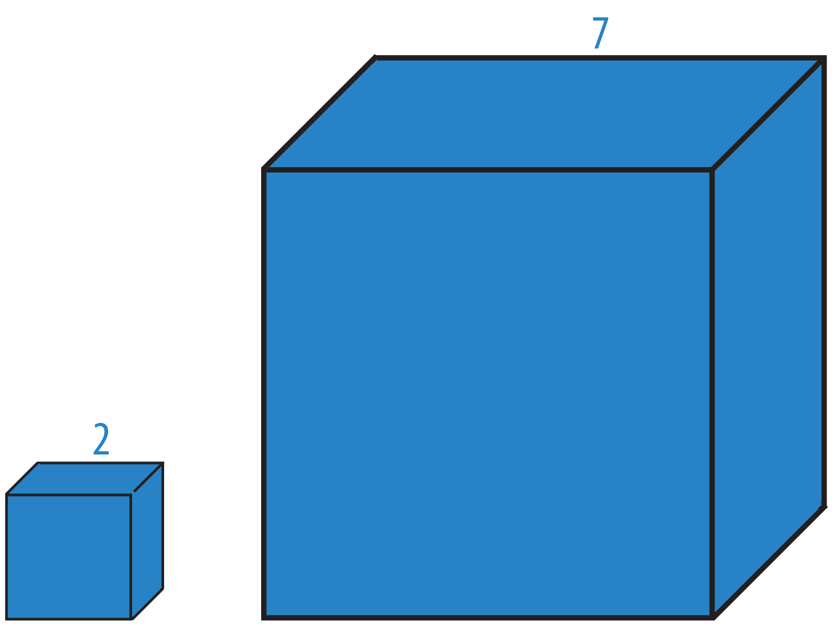
Therefore, $AB = 18(\frac{21}{15}) = 25.2$, and $AC = 10(\frac{21}{15}) = 14$.

The scale factor from the large triangle to the small triangle is the reciprocal of the opposite scale factor: $15 \div 21 = \frac{15}{21}$.

Scale Factors in Multiple Dimensions

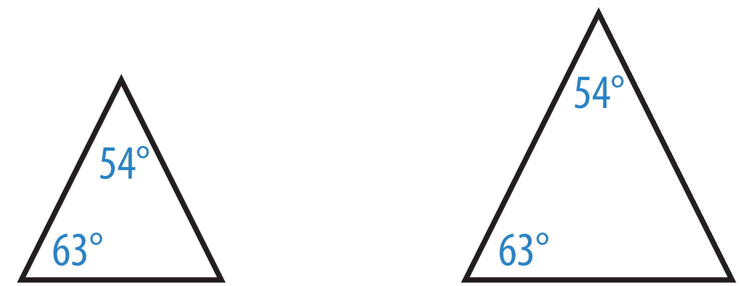
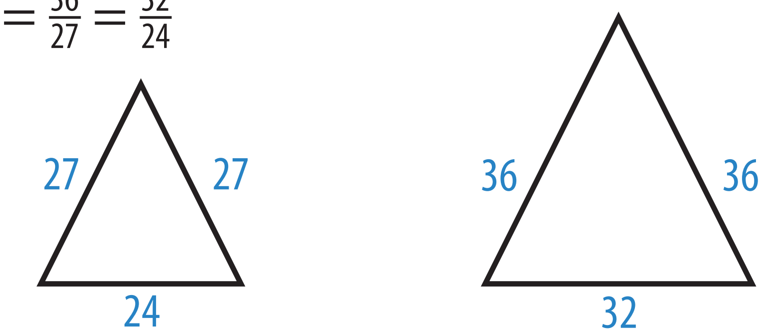
Scale factors can be multiplied multiple times—once per dimension—to find areas (two-dimensional) or volumes (three-dimensional).

Measurement	Example: 2″ cube	Example: 7″ cube (scale factor: $\frac{7}{2}$)
Length per Edge	2	$2(\frac{7}{2}) = 7$
Area per Face	4	$4(\frac{7}{2})(\frac{7}{2}) = 49$
Volume	8	$8(\frac{7}{2})(\frac{7}{2})(\frac{7}{2}) = 343$



Triangle Similarity Theorems

Like for congruence, there are theorems to prove the similarity of two triangles.

Theorem	Description	Example
AA	Two pairs of corresponding angles are congruent.	
SSS	All three pairs of corresponding sides are proportional.	$\frac{36}{27} = \frac{36}{27} = \frac{32}{24}$ 
SAS	Two pairs of corresponding sides are proportional, and the angles between them are congruent.	$\frac{36}{27} = \frac{32}{24}$