CHAPTER THREE: RIGHT TRIANGLE TRIGONOMETRY — Review November 30 ↝ Test December 8

Other than circles, triangles are the most fundamental shape. Many aspects of advanced abstract mathematics and practical applications are based on properties of triangles. In particular, the field of trigonometry is founded on relationships between side lengths of right triangles.

3-A Special Right Triangles — Monday • 11/9

Pythagorean Theorem • radical

❶ Rationalize a denominator.
❷ Calculate the length of the third side of a right triangle.
❸ Find unknown lengths in a 45° right triangle.
❹ Find unknown lengths in a 30° right triangle.

3-B Trigonometric Functions — Monday • 11/16

sine • cosine • tangent

❶ Find the sine, cosine, and tangent of an acute angle in a right triangle with two known sides.
❷ Calculate a side length in a right triangle based on a known angle and known side length.

3-C Inverse Trigonometric Functions — Thursday • 11/19

❶ Calculate an angle measure in a right triangle based on two known side lengths.
❷ Solve a right triangle.

3-A Special Right Triangles

By the PYTHAGOREAN Theorem, the square of the a triangle's hypotenuse c is equal to the sum of the squares of the legs a and b: $a^2 + b^2 = c^2$.

A RADICAL is the square root of a number. The Pythagorean theorem results in radical answers.

Fractions with a radical in the denominator are commonly rewritten by multiplying the numerator and denominator by the radical. This is called rationalizing the denominator.

❶ Rationalize a denominator.

1. Multiply the numerator and denominator by the radical in the denominator to make the denominator a whole number.
2. Simplify.

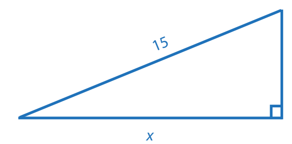
❶ $\frac{20}{\sqrt{2}}$

1. $\frac{20}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{20\sqrt{2}}{2}$
2. $10\sqrt{2}$

❷ Calculate the length of the third side of a right triangle.

1. Fill in the two known values in the equation $a^2 + b^2 = c^2$, with c representing the hypotenuse.
2. Solve for the unknown value.

❷ Find length x in the triangle shown.

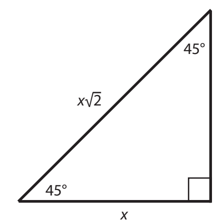


1. $x^2 + 6^2 = 15^2$
2. $x^2 = 225 - 36 = 189$

   $x = \sqrt{189} \approx 13.7$

In an isosceles right triangle, a and b are equal. Therefore, $a^2 + a^2 = c^2$, and $c = \sqrt{2a^2}$. Simplified, this is $c = a\sqrt{2}$.

In other words, in a 45° right triangle, the hypotenuse is $\sqrt{2}$ times as long as the legs.

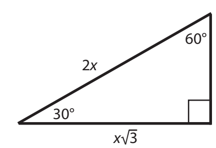


❸ Find unknown lengths in a 45° right triangle.

1. If a leg length is known, multiply it by $\sqrt{2}$ to find the hypotenuse

   If the hypotenuse is known, divide it by $\sqrt{2}$ to find the leg lengths.

If an equilateral triangle with side length c is cut in half by its altitude, making a pair of 30° right triangles, the hypotenuse is twice as long as the shorter leg: $c = 2a$. Therefore, $a^2 + b^2 = (2a)^2$, making the longer leg $b = \sqrt{3a^2}$. Simplified, this is, $b = a\sqrt{3}$.



❹ Find unknown lengths in a 30° right triangle.

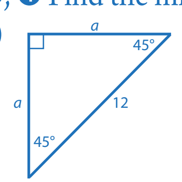
1. If the length of the shorter leg is known, multiply it by $\sqrt{3}$ to find the length of the longer leg, and multiply it by 2 to find the hypotenuse.

   If the length of the longer leg is known, divide it by $\sqrt{3}$ to find the length of the shorter leg, and then multiply this length by 2 to find the hypotenuse.

   If the hypotenuse is known, divide it by 2 to find the length of the shorter leg, and then multiply this length by $\sqrt{3}$ to find the length of the longer leg.

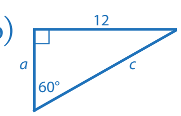
❸, ❹ Find the missing lengths, and simplify.

a)



1. $a = \frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$

b)



1. $a = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

   $c = 2 \bullet 4\sqrt{3} = 8\sqrt{3}$

3-B Trigonometric Functions

The field of trigonometry uses relationships between sides and angles in triangles to solve problems involving unknown lengths and angle measures.

The three primary trigonometric functions are SINE (sin), COSINE (cos), and TANGENT (tan). They show the ratio of the length of one side to another in a right triangle.

Both of the acute angles in any right triangle are comprised of the hypotenuse and one leg, called the adjacent leg. The other leg is not part of the angle and is called the opposite leg.

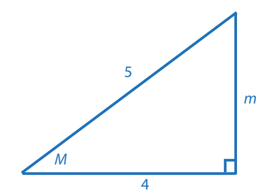
The sine, cosine, and tangent of any acute angle A in a right triangle are defined as the following ratios of side lengths:

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

❶ Find the sine, cosine, and tangent of an acute angle in a right triangle with two known sides.

1. Use the Pythagorean theorem $a^2 + b^2 = c^2$ to find the length of the third side.
2. Identify which length is opposite the given angle, which is adjacent to it, and which is the hypotenuse.
3. For each trig function, use the two appropriate lengths in a fraction based on the definitions above.

❶ Find the sine, cosine, and tangent of M shown at right.



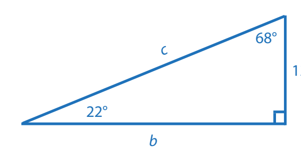
1. $m^2 + 4^2 = 5^2$
   $m = \sqrt{9} = 3$
2. opposite = 3, adjacent = 4, hypotenuse = 5
3. $\sin M = \frac{3}{5}$ $\cos M = \frac{4}{5}$ $\tan M = \frac{3}{4}$

A trigonometric function can be used to calculate an unknown side length in a right triangle if the measure of one side and one acute angle are known.

❷ Calculate a side length in a right triangle based on a known angle and known side length.

1. Write a sine, cosine, or tangent ratio involving the known side, the unknown side, and the known acute angle.
2. Solve for the unknown side.

❷ Solve for c in the triangle at right.



1. $\sin 22° = \frac{12}{c}$
2. $c \sin 22° = 12$
   $c = \frac{12}{\sin 22°} \approx 32.0$

Note that we could have also started with $\cos 68° = \frac{12}{c}$, but $\sin 68° = \frac{b}{c}$ and $\cos 22° = \frac{b}{c}$ cannot be solved because these equations have two unknowns.

3-C Inverse Trigonometric Functions

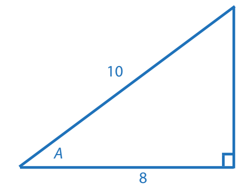
Whereas sin, cos, and tan take an angle and find a ratio for it, the INVERSE Trigonometric Functions SIN^{-1}, COS^{-1}, and TAN^{-1} take a ratio and find an angle for it. For example, $\sin 30° = ½$ and $\sin^{-1} ½ = 30°$.

Given a triangle with hypotenuse c, $\sin^{-1} \sin A = A$, $\cos^{-1} \cos A = A$, and $\tan^{-1} \tan A = A$.

➊ Calculate an angle measure in a right triangle based on two known side lengths.

1. Write a sine, cosine, or tangent ratio involving the angle and the two known sides.
2. Solve for the angle by applying $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ to each side of the equation.

➊ Solve for A in the triangle at right.



1. $\sin A = \frac{6}{10}$
2. $\sin^{-1} \sin A = \sin^{-1} \frac{6}{10}$
3. $A \approx 36.9°$

Note that we could have also started with $\cos A = \frac{8}{10}$ or $\tan A = \frac{6}{8}$.

To solve a triangle is to find every unknown side and angle.

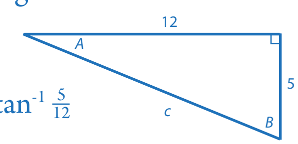
Angles are labeled with capital letters. Each side is labeled with the same letter as the angle opposite it, but lowercase.

➋ Solve a right triangle.

1. If both angles are unknown, solve for one of them using $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ (see ➊).
2. Find the second angle by subtracting the first from 90°.
3. If two sides are unknown, solve for one of them using sin, cos, or tan (see 4-B).
4. Find the last side by using sin, cos, or tan (see 4-B), or by using the Pythagorean theorem.

➋ Solve the triangles shown below.

a)



1. $\tan A = \frac{5}{12}$

   $\tan^{-1} \tan A = \tan^{-1} \frac{5}{12}$

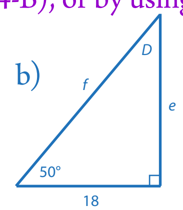
   $A \approx 22.6°$
2. $B \approx 90° - 22.6° = 67.4°$
3.

4. $\sin 22.6° \approx \frac{5}{c}$

   $c \sin 22.6° \approx 5$

   $c \approx \frac{5}{\sin 22.6°} \approx 13.0$

b)



$D = 90° - 50° = 40°$

$\tan 50° = \frac{e}{18}$

$18 \tan 50° = e \approx 21.5$

$\cos 50° = \frac{e}{18}$

$f \cos 50° = 18$

$f = \frac{18}{\cos 50°} \approx 28.0$