CHAPTER THREE: GEOMETRY

Test: Tuesday, October 31

This chapter sets up basic concepts for describing, sketching, and solving geometric scenarios.

3-A Lines and Line Segments — Tuesday • 10/10

point • line • ray • line segment • parallel • perpendicular • congruent • circle • radius • chord • diameter • secant • tangent • point of tangency • perpendicular bisector

❶ Sketch and label geometric diagrams.
❷ Identify types of lines and line segments intersecting circles.
❸ Find the slope of a line.
❹ Write the equation of a line, given two points on the line.
❺ Find the midpoint of a line segment.
❻ Write the equation of the perpendicular bisector of a line segment.

3-B Angles — Thursday • 10/12

angle • vertex • complementary • supplementary • transversal • linear angles • vertical angles • corresponding angles • alternate interior angles • alternate exterior angles • arc • semicircle • minor arc • major arc • central angle • inscribed angle • circumscribed angle • polygon • regular polygon • interior angle • exterior angle

❶ Use properties of angles to determine angle measures.
❷ Calculate the measure of an angle that intercepts two points on a circle.
❸ Find the sum of the measures of the interior angles in a polygon.
❹ Find the measure of an angle of a polygon based on the other angles.
❺ Find the number of sides of a polygon based on its angles.

3-C Quadrilaterals — Monday • 10/16

quadrilateral • trapezoid • parallelogram • rhombus • rectangle • kite • midsegment

❶ Identify types of quadrilaterals.
❷ Calculate values based on angles within a rectangle.
❸ Calculate values based on lengths within a rectangle.
❹ Calculate values based on angles within a rhombus.
❺ Calculate the length of the midsegment or a base of a trapezoid.
❻ Calculate values based on angles within a parallelogram.
❼ Calculate values based on lengths within a parallelogram.
❽ Find the point of intersection of the diagonals of a parallelogram.
❾ Identify the coordinates of a vertex of a parallelogram.

3-D Transformations — Thursday • 10/19

transformation • mapping • pre-image • image • translation • reflection • dilation

❶ Translate a figure.
❷ Reflect a figure.
❸ Dilate a figure.
❹ Apply multiple transformations.

3-E Scale Factors — Monday • 10/23

scale factor • similar

❶ Determine a scale factor that maps a figure onto a similar figure.
❷ Calculate lengths in similar figures.
❸ Find the perimeter of a similar figure.
❹ Find the area or volume of a similar figure.
❺ Use lengths to determine whether or not two figures are similar.
❻ Determine whether or not two triangles are similar.

3-A Lines and Line Segments

A POINT is a geomtric object with a location, represented by coordinates such as (4, 9), but no size. Points are typically labeled with a capital letter.

The geometric object that connects two points is called a LINE if it extends infinitely in both directions, a RAY if it extends infinitely in one direction and has one endpoint (like an arrow), and a LINE SEGMENT if it has two endpoints. When sketched, lines have an arrowhead in each direction and rays have an arrowhead in one direction.

The notation for a line, ray, or line segment is a bar over two letters representing two points. For lines, the bar has arrows on each side, such as $\overleftrightarrow{AB}$. For rays, the bar has an arrow on the side that extends infinitely, such as $\overrightarrow{AB}$. For line segments, the bar has no arrows, such as $\overline{AB}$.

A pair of points without a bar, such as AB, represents the length of the line segment. Other variations of this notation include $|AB|$ and mAB.

Lines that never intersect within a plane are PARALLEL, written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and indicated graphically with arrowheads within the lines.

Lines that intersect at a 90° angle are PERPENDICULAR, written $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and indicated graphically with a box at the intersection.

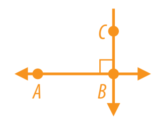
Line segments that are the same length are CONGRUENT, written $\overline{AB} \cong \overline{AB}$ and indicated graphically with a tic mark on each line segment.

If typed, letters are italicized when used to represent points, lines, and other geometric objects, just like variables.

❶ Sketch and label geometric diagrams.

1. Sketch each item. Make sure perpendicular lines appear perpendicular, congruent segments appear congruent, etc.
2. Label each point with a capital letter.
3. Avoid mistakes such as a single point labeled in two different places or a ray pointing in the wrong direction.

❶ Sketch $\overleftrightarrow{AB} \perp \overleftrightarrow{CB}$.



A CIRCLE is formally defined as the set of all points that are a set distance from a center point.

A RADIUS is a line segment or distance from the center of a circle to the circle itself. $\overline{ED}$ and $\overline{FD}$ at right are radii.

A CHORD is a line segment from any point on a circle to any other point on the circle. $\overline{BC}$ and $\overline{FC}$ at right are chords.

A DIAMETER is a chord that passes through the center of the circle, making it equal to two radii. $\overline{FC}$ at right is a diameter.

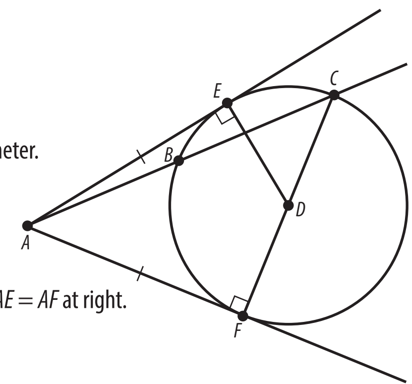
A SECANT is a line that contains a chord. $\overleftrightarrow{BC}$ at right is a secant.

A TANGENT is line, ray, or line segment that intersects a circle at exactly one point. $\overline{AE}$ and $\overline{AF}$ at right are tangents.

The point at which a tangent intersects a circle is a POINT OF TANGENCY. E and F at right are points of tangency.

Every point outside a circle is on two tangent lines to the circle and is equidistant from the two points of tangency. $AE = AF$ at right.

A tangent is perpendicular to the radius at the point of tangency. $\overline{AE} \perp \overline{DE}$ and $\overline{AF} \perp \overline{DF}$ at right.

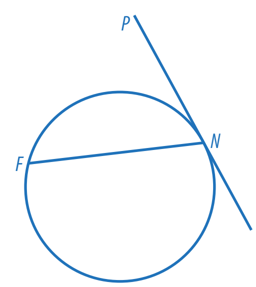


❷ Identify types of lines and line segments intersecting circles.

1. If it goes from the center to the circle, it is a radius.
2. If it touches the circle at two points, it is a chord if it does not leave the circle, or a secant if it does continue infinitely outside the circle. If it is a chord passing through the center, it is a diameter.
3. If it touches the circle but goes past it without entering it, like a road and wheel, then it is a tangent.

❷ What type of line segments are $\overline{FN}$ and $\overline{PN}$ in the diagram at right?

2. $\overline{FN}$ touches the circle twice, but does not go outside the circle, so it is a chord.
3. $\overline{PN}$ touches the circle but keeps going past it without entering it, so it is a tangent.



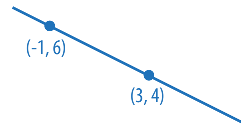
$y = mx + b$ is the equation of a line, where m is the slope, b is the y-intercept, and x and y are variables.

The slope between two points can be found by dividing the vertical distance between them by the horizontal distance between them: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

❸ Find the slope of a line.

1. Identify one point on the line as (x_1, y_1) and another point as (x_2, y_2).
2. Calculate $m = \frac{y_2 - y_1}{x_2 - x_1}$.

❸ What is the slope of the line at right?



1 $(x_1, y_1) = (-1, 6)$
$(x_2, y_2) = (3, 4)$
2. $m = \frac{4 - 6}{3 - (-1)} = \frac{-2}{4} = -\frac{1}{2}$

Once the slope is known, the y-intercept of a line can be calculated by using the x value and the y value of any point on the line and solving for b.

❹ Write the equation of a line, given two points on the line.

1. Calculate m (see ❸).
2. In the equation $y = mx + b$, plug in m and the given values of x and y from one of the two points.
3. Solve for b.
4. Write the equation $y = mx + b$, replacing m and b with their calculated values.

❹ Write an equation of the line passing through the points (-1, 6) and (3, 4).

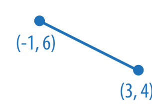
1. $m = -\frac{1}{2}$ (see ❸)
2. $4 = -\frac{1}{2}(3) + b$
3. $\frac{11}{2} = b$
4. $y = -\frac{1}{2}x + \frac{11}{2}$

The MIDPOINT of a line segment is the point that averages the x-coordinates and averages the y-coordinates of the endpoints of the line segment: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

❺ Find the midpoint of a line segment.

1. Identify the coordinates of the two endpoints (x_1, y_1) and (x_2, y_2) of the line segment.
2. Find the average of the x coordinates: $x = \frac{x_1 + x_2}{2}$.
3. Find the average of the y coordinates: $y = \frac{y_1 + y_2}{2}$.
4. The coordinates of the midpoint are these averages (x, y).

❺ Find the midpoint of the line segment shown at right.



1 $(x_1, y_1) = (-1, 6)$
$(x_2, y_2) = (3, 4)$
2. $x = \frac{-1 + 3}{2} = 1$
3. $y = \frac{6 + 4}{2} = 5$
4. The midpoint is (1, 5).

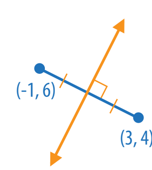
The slope of a perpendicular is the negative reciprocal of the original slope m_1: $m = -\frac{1}{m_1}$.

A PERPENDICULAR BISECTOR intersects with a line segment at a right angle and cuts the line segment exactly in half.

❻ Write the equation of the perpendicular bisector of a line segment.

1. Find the midpoint (see ❺).
2. Find the slope m_1 of the line segment (see ❸).
3. Find the perpendicular slope: $m = -\frac{1}{m_1}$.
4. In the equation $y = mx + b$, plug in m and the x and y values of the midpoint.
5. Solve for b.
6. Write the equation $y = mx + b$, replacing m and b with their calculated values.

❻ Write the equation of the perpendicular bisector of the line segment shown at right.



1. The midpoint is (1, 5) (see ❺).
2. The slope of the original line segment is $-\frac{1}{2}$ (see ❸).
3. $m = -\frac{1}{-1/2} = 2$
4. $5 = 2(1) + b$
5. $3 = b$
6. $y = 2x + 3$

3-B Angles

An ANGLE is formed by a pair of rays sharing the same endpoint, called the VERTEX.

An angle can be written as the $\angle$ sign followed by three points, such as $\angle ABC$, where the middle point is the vertex.

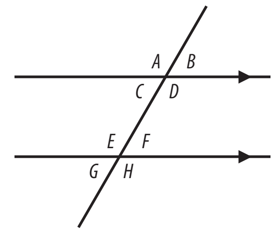
An angle can also be written as the $\angle$ sign followed by the name of the angle written within, either a number or a capital letter, such as $\angle B$.

The measure of an angle can be notated by putting m before the angle, such as $m\angle ABC$.

Two angles are COMPLEMENTARY if they total 90°.

Two angles are SUPPLEMENTARY if they total 180°.

A TRANSVERSAL is a line that intersects two other lines at two different points, creating eight angles. If the two lines are parallel, each of the eight angles are supplementary with or congruent to each of the other angles.



There are four pairs of LINEAR Angles, such as $\angle A$ and $\angle B$, which are supplementary.

There are four pairs of VERTICAL Angles, such as $\angle A$ and $\angle D$, which are congruent.

There are four pairs of CORRESPONDING Angles, such as $\angle A$ and $\angle E$, which are congruent.

There are two pairs of CONSECUTIVE Angles, such as $\angle C$ and $\angle E$, which are supplementary.

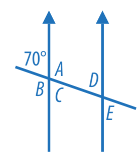
There are four pairs of ALTERNATE EXTERIOR Angles, such as $\angle A$ and $\angle H$, which are congruent.

There are four pairs of ALTERNATE INTERIOR Angles, such as $\angle C$ and $\angle F$, which are congruent.

❶ Use properties of angles to determine angle measures.

1. Identify any pair of angles that are linear (together they make a line). The measure of one can be found by subtracting the other from 180°.
2. Identify any pair of angles that are vertical (they are opposite each other but made from the same lines). These angles are congruent.
3. Identify any pair of angles made by a transversal through parallel lines. The angles are congruent if they are corresponding (positioned the same on each of the two parallel lines), alternate interior (opposite each other and inside the parallel lines), or alternate exterior (opposite each other and outside the parallel lines).

❶ State the relationship between the 70° angle and each of the other labeled angles, and use this to determine the measure of each labeled angle.



1. $\angle A$ and the 70° angle are linear, so they are supplementary and $\angle A = 180° - 70° = 110°$. $\angle B = 110°$ for the same reason.
2. $\angle C$ and the 70° angle are vertical, so they are congruent and $\angle C = 70°$.
3. $\angle D$ and the 70° angle are corresponding, so they are congruent and $\angle D = 70°$.
   $\angle E$ and the 70° angle are alternate exterior angles, so they are congruent and $\angle E = 70°$.

An ARC is a portion of a circle. It is a SEMICIRCLE if it is exactly half a circle (180°), a MINOR Arc if smaller, or a MAJOR Arc if larger.

A CENTRAL Angle has its vertex at the center of the circle. It has the same measure as the arc it intercepts.

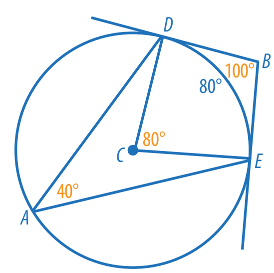
An INSCRIBED Angle has its vertex on the circle. It has half the measure as the arc it intercepts.

A CIRCUMSCRIBED Angle has sides that are tangent to the circle. Its measure is 180° minus the measure of the arc it intercepts.

❷ Calculate the measure of an angle that intercepts two points on a circle.

1. If the vertex is at the center, it is a central angle and it has the same measure as the intercepted arc.
2. If the vertex is on the circle, it is an inscribed angle and its measure is the measure of the intercepted arc divided by 2.
3. If the vertex is outside the circle, it is a circumscribed angle and its measure is 180° minus the measure of the intercepted arc.

❷ Identify the type of angle for each of the following in the diagram at right, and find their measures.



a) $\angle DCE$
   1. central angle
   $m\angle DCE = 80°$

b) $\angle DAE$
   2. inscribed angle
   $m\angle DAE = 80° \div 2 = 40°$

c) $\angle DBE$
   3. circumscribed angle
   $m\angle DBE = 180° - 80° = 100°$

A POLYGON is a figure made up of only straight sides. A REGULAR Polygon is one that has all equal sides and all equal angles.

The INTERIOR Angles of a polygon are the angles inside the shape. The sum of the measure of the interior angles in a polygon with n sides is $S = (n - 2)180°$.

An angle formed between the side of a polygon and another side extended outside the polygon is an EXTERIOR Angle. The sum of the measure of one exterior angle from each side of a polygon is $S = 360°$ for all concave polygons, regardless of the number of sides.

❸ Find the sum of the measures of the interior angles in a polygon.

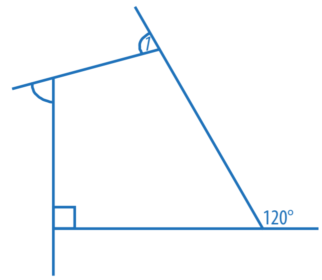
1. Multiply 180° by 2 less than the number of sides.

❸ Find the sum of the interior angles in the pentagon at right.

1. $S = (5 - 2)180° = 540°$

❹ Find the measure of an angle of a polygon based on the other angles.

1. Write an equation showing the sum of the interior angles is $(n - 2)180°$ or an equation showing the sum of one exterior angle per vertex is 360°.
2. Solve.



❹ Find $m\angle 1$ in the diagram at right.

1. $90° + 120° + m\angle 1 + m\angle 1 = 360°$
2. $2(m\angle 1) = 150°$

   $m\angle 1 = 75°$

❺ Find the number of sides of a polygon based on its angles.

1. Identify the sum of the interior angles.
2. Set this sum equal to $(n - 2)180°$.
3. Solve for n.

❺ How many sides does a polygon have if the sum of the interior angles is 1440°?

1. $S = 1440°$
2. $1440° = (n - 2)180°$
3. $1440° = 180n° - 360°$

   $1800° = 180n°$

   $10 = n$

❺ How many sides does a polygon have if each angle is 162°?

1. $S = 162n°$
2. $162n° = (n - 2)180°$
3. $162n° = 180n° - 360°$

   $360° = 18n°$

   $20 = n$

3-C Quadrilaterals

A QUADRILATERAL is a four-sided polygon.

A TRAPEZOID is quadrilateral with a pair of parallel sides. Each of these sides is a BASE of the trapezoid.

A PARALLELOGRAM is a quadrilateral with two pairs of parallel sides.

A RHOMBUS is a parallelogram in which all four sides are congruent.

A RECTANGLE is a parallelogram in which all four angles are congruent.

A KITE is a quadrilateral that has two pairs of congruent sides but no parallel sides.

The diagonals of a rhombuses and of kites are perpendicular.

The angles of a kite that are between noncongruent sides are congruent.

Parallelograms have many special properties. The following are true for all parallelograms but not for any other quadrilaterals.

- Both pairs of opposite sides are congruent.
- Both pairs of opposite angles are congruent.
- The diagonals bisect each other.
- Consecutive angles are supplementary.

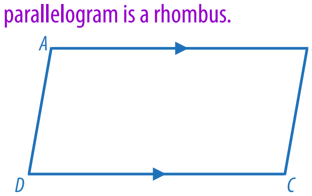
A quadrilateral is a parallelogram if one pair of sides is parallel and congruent.

❶ Identify types of quadrilaterals.

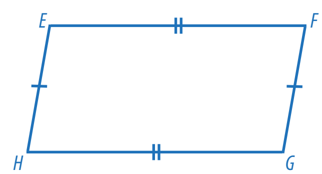
1. If there are two pairs of congruent sides, none of which are parallel, it is a kite.
2. If there is one pair of parallel sides, it is a trapezoid.
3. If there is a pair of congruent parallel sides, two pairs of parallel sides, two pairs of congruent opposite sides, or two pairs of congruent opposite angles, then it is a parallelogram and:
   - If all the angles are congruent (90°), the parallelogram is a rectangle.
   - If all the sides are congruent, if all of the angles are bisected by the diagonals, or if the diagonals are perpendicular, the parallelogram is a rhombus.
   - If it is both a rectangle and a rhombus, it is a square.

Do not assume any information that is not marked.

❶ Identify the quadrilaterals at right.



a) $\overline{AB}$ is parallel to $\overline{DC}$, so *ABCD* is a trapezoid.
   $\overline{AD}$ and $\overline{BC}$ appear parallel, but they are not marked as such so *ABCD* is not necessarily a parallelogram.

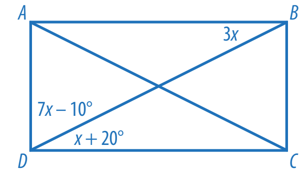


b) There are two pairs of opposite congruent sides, so *EFGH* is a parallelogram.
   Not all four angles are congruent, so *EFGH* is not a rectangle.
   Not all four sides are congruent, so *EFGH* is not a rhombus.

❷ Calculate values based on angles within a rectangle.

1. All four angles of the rectangle are 90°.
2. Alternate interior angles are congruent.

❷ Use each method above to solve for x, given $ABCD$ is a rectangle.



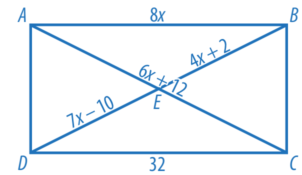
1. $m\angle ADB + m\angle CDB = 90°$
   $7x - 10° + x + 20° = 90°$
   $8x + 10° = 90°$
   $x = 10°$

2. $m\angle ABD = m\angle CDB$
   $3x = x + 20°$
   $2x = 20°$
   $x = 10°$

❸ Calculate values based on lengths within a rectangle.

1. Opposite sides are congruent.
2. The diagonals are congruent.
3. The diagonals bisect each other.

❸ Use each method above to solve for x, given $ABCD$ is a rectangle.



1. $AB = DC$
   $8x = 32$
   $x = 4$

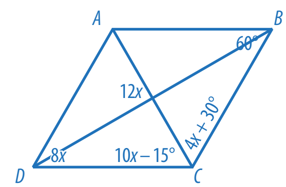
2. $AC = DB$
   $6x + 12 = 7x - 10 + 4x + 2$
   $20 = 5x$
   $x = 4$

3. $DE = BE$
   $7x - 10 = 4x + 2$
   $3x = 12$
   $x = 4$

❹ Calculate values based on angles within a rhombus.

1. Opposite angles are congruent.
2. The diagonals are perpendicular.
3. The angle between a diagonal and a side is congruent to the angle made between that diagonal and any other side.

❹ Use each method above to solve for x, given $ABCD$ is a rhombus.



1. $m\angle ADC = m\angle ABC$
   $8x = 60°$
   $x = 7.5°$

2. $\overline{AC} \perp \overline{DB}$
   $12x = 90°$
   $x = 7.5°$

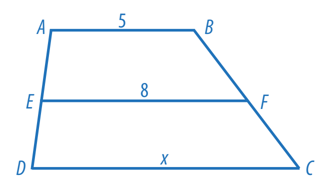
3. $m\angle ACD = m\angle ACB$
   $10x - 15° = 4x + 30°$
   $6x = 45°$
   $x = 7.5°$

A MIDSEGMENT connects the midpoints of two nonparallel sides of a trapezoid or triangle. Its length m is the average of the lengths of the bases b: $b_1 + b_2 = 2m$. (In a triangle, b_2 is zero.)

❺ Calculate the length of the midsegment or a base of a trapezoid.

1. Use two known lengths in the formula $b_1 + b_2 = 2m$.
2. Solve for the unknown length.

❺ Solve for x, given $\overline{EF}$ is a midsegment of trapezoid $ABCD$.

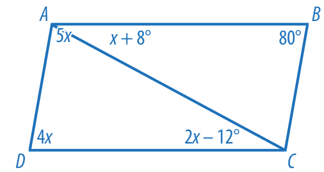


2. $5 + x = 2(8)$
   $5 + x = 16$
   $x = 11$

❻ Calculate values based on angles within a parallelogram.

1. Each angle is congruent to the opposite angle.
2. Consecutive angles are supplementary.
3. Alternate interior angles are congruent.

❻ Use each method above to solve for x, given $ABCD$ is a parallelogram.



1. $m\angle ADC = m\angle CBA$
   $4x = 80°$
   $x = 20°$

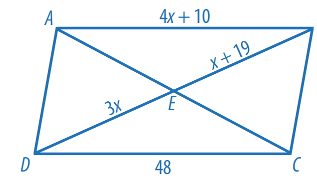
2. $m\angle DAB + m\angle ABC = 180°$
   $5x + 80° = 180°$
   $5x = 100°$
   $x = 20°$

3. $m\angle BAC = m\angle DCA$
   $x + 8° = 2x - 12°$
   $x = 20°$

❼ Calculate values based on lengths within a parallelogram.

1. Each side is congruent to the opposite side.
2. Each diagonal is cut in half by the other diagonal.

❼ Use each method above to solve for x, given $ABCD$ is a parallelogram.



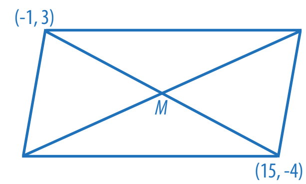
1. $AB = DC$
   $4x + 10 = 48$
   $4x = 38$
   $x = 9.5$

2. $DE = BE$
   $3x = x + 19$
   $2x = 19$
   $x = 9.5$

❽ Find the point of intersection of the diagonals of a parallelogram.

1. Use the midpoint formula to find the midpoint of one of the diagonals.

❽ Find the intersection of the diagonals of the parallelogram at right.

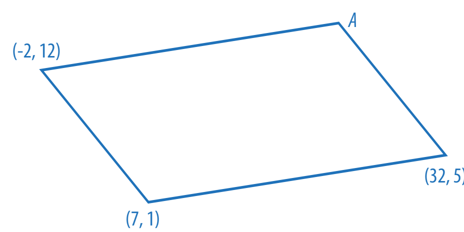


1. $M = \left(\frac{-1+15}{2}, \frac{3-4}{2}\right) = (7, -½)$

❾ Identify the coordinates of a vertex of a parallelogram.

1. Write an equation setting the difference between the x values for one line segment equal to the difference between the x values for the opposite line segment.
2. Solve this equation.
3. Repeat steps 1-2 for y.

❾ Find the coordinates of vertex A in the parallelogram at right.



1. $x - 32 = -2 - 7$ $\quad$ $y - 5 = 12 - 1$
2. $x = 23$ $\quad$ $y = 16$
   $A = (23, 16)$

3-D Transformations

A TRANSFORMATION (or Mapping) takes each point of a figure, called the PRE-IMAGE, and MAPS it onto a new figure, called the IMAGE.

MAPPING Notation uses the symbol ➝ to indicate how each (x, y) point in the pre-image is transformed in the image. For example, $(x, y) \rightarrow (x + 3, y)$ adds 3 to each x value and does not change the y values.

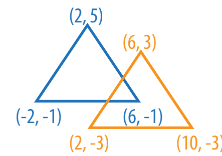
Three main types of transformation are translations, stretches, and reflections.

A TRANSLATION changes the *position* of a figure: $(x, y) \rightarrow (x + h, y + k)$ translates a figure h units right and k units up.

❶ Translate a figure.

1. Identify h and k in the mapping $(x, y) \rightarrow (x + h, y + k)$.
2. Add h to each x value in the pre-image.
3. Add k to each y value in the pre-image.

❶ Apply the mapping $(x, y) \rightarrow (x + 4, y - 2)$ to the figure at right.



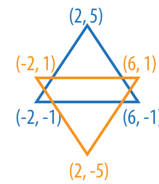
1. $h = 4, k = -2$
2. $x_1 = -2 + 4 = 2$ $\quad x_2 = 2 + 4 = 6$ $\quad x_3 = 6 + 4 = 10$
3. $y_1 = -1 - 2 = -3$ $\quad y_2 = -1 - 2 = -3$ $\quad y_3 = 5 - 2 = 3$

A REFLECTION changes the *direction* of a figure: $(x, y) \rightarrow (-x, y)$ reflects a figure horizontally (across the y-axis), and $(x, y) \rightarrow (x, -y)$ reflects a figure vertically (across the x-axis).

❷ Reflect a figure.

1. For a horizontal reflection, multiply each x value by -1.
2. For a vertical reflection, multiple each y value by -1.

❷ Apply the mapping $(x, y) \rightarrow (x, -y)$ to the figure at right.



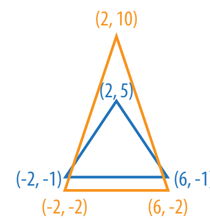
2. $y_1 = -(-1) = 1$ $\quad y_2 = -(-1) = 1$ $\quad y_3 = -(5) = -5$

A DILATION changes the *size* of a figure: $(x, y) \rightarrow (bx, ay)$ dilates a figure by a factor of b horizontally (every point becomes b times as far from the y-axis) and by a factor of a vertically (every point becomes a times as far from the x-axis).

❸ Dilate a figure.

1. Identify b and a in the mapping $(x, y) \rightarrow (bx, ay)$.
2. Multiply each x value by b.
3. Multiply each y value by a.

❸ Apply the mapping $(x, y) \rightarrow (x, 2y)$ to the figure at right.



1. $b = 1, a = 2$
2. x maps onto itself (multiplying by 1), so the x values do not change.
3. $y_1 = 2(-1) = -2$ $\quad y_2 = 2(-1) = -2$ $\quad y_3 = 2(5) = 10$

If more than one type of transformation is applied to a figure in the same direction (horizontally or vertically), the image will be different depending on the order of the transformations.

❹ Apply multiple transformations.

1. Apply ❶, ❷, and ❸, above, in the order specified.

3-E Scale Factors

A SCALE FACTOR is the value of b or a in the mapping $(x, y) \rightarrow (bx, ay)$.

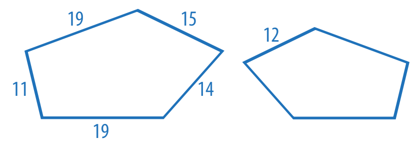
An image is SIMILAR to its pre-image if the horizontal scale factor b is the same as the vertical scale factor a. This means the figures are the same shape but not necessarily the same size.

If an image is similar to its pre-image, the scale factor can be found by dividing any length in the image by the corresponding length in the pre-image.

The symbol to indicate similarity is the same as for congruence but without the equals sign: ~ instead of ≅.

❶ Determine a scale factor that maps a figure onto a similar figure.

1. Identify a length in the original figure and the corresponding length in the similar figure.
2. Divide the second length by the original length.



❶ Determine the scale factor that maps the pentagon on the left onto the similar pentagon on the right.

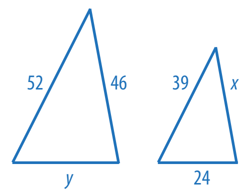
1. The side of 15 in the original pentagon corresponds with the side of 12 in the similar pentagon.
2. $12 \div 15 = \frac{12}{15}$

The scale factor to go from the image back to the preimage is the reciprocal of the original scale factor.

A scale factor can be used to determine lengths in similar figures.

❷ Calculate lengths in similar figures.

1. Determine the scale factor (see ❶).
2. Multiply the corresponding length in the original figure by the scale factor.



❷ Find x and y, given the triangles are similar.

1. a) The scale factor from the big triangle to the small triangle is $\frac{39}{52}$.
2. $x = 46\left(\frac{39}{52}\right) = 34.5$

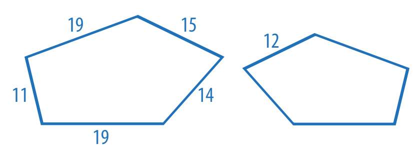
b) The scale factor from the small triangle to the big triangle is $\frac{52}{39}$

$y = 24\left(\frac{52}{39}\right) = 32$

Since the perimeter of a figure is simply the total of its lengths, the perimeter of a similar figure can be found by multiplying the original perimeter by the scale factor, just like for any one side.

❸ Find the perimeter of a similar figure.

1. Determine the scale factor (see ❶).
2. Determine the perimeter of the original figure.
3. Multiply the perimeter of the original figure by the scale factor.



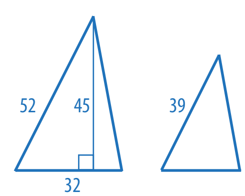
❸ Find the perimeter of the smaller pentagon, given it is similar to the larger one.

1. The scale factor is $\frac{12}{15}$.
2. The perimeter of the larger pentagon is $11 + 19 + 14 + 15 + 19 = 78$.
3. The perimeter of the smaller triangle is $78\left(\frac{12}{15}\right) = 62.4$.

Since area is two-dimensional and scale factors are one-dimensional, the area of a similar figure can be found by multiplying the original area by the scale factor twice. Likewise, the volume of a similar object can be found by multiplying the original volume by the scale factor three times.

❹ Find the area or volume of a similar figure.

1. Determine the scale factor (see ❶).
2. Determine the area or volume of the original figure.
3. Multiply the area of the original figure by the scale factor twice for area or three times for volume.



❹ Find the area of the smaller triangle, given it is similar to the larger triangle.

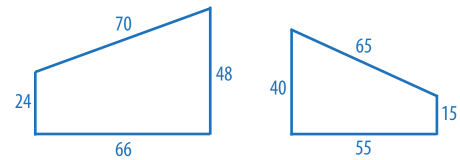
1. The scale factor is $\frac{39}{52}$.
2. The area of the larger triangle is $\frac{1}{2}(32)(45) = 720$.
3. The area of the smaller triangle is $720\left(\frac{39}{52}\right)\left(\frac{39}{52}\right) = 405$.

Two figures are similar if every pair of corresponding angles is congruent or if the same scale factor applies to every pair of corresponding lengths.

❺ Use lengths to determine whether or not two figures are similar.

1. Identify the corresponding length in the image for each length in the pre-image.
2. Determine the scale factor for each pair of lengths.
3. The figures are similar if and only if the scale factor is the same every time.

❺ Are the quadrilaterals at right similar?



1. 66 corresponds with 55
   48 corresponds with 40
   70 corresponds with 65
   24 corresponds with 15

2. $\frac{66}{55} = 1.2$
   $\frac{48}{40} = 1.2$
   $\frac{70}{65} \approx 1.1$
   $\frac{24}{15} = 1.6$

3. The quadrilaterals are not similar.

To prove similarity, it is not needed to verify the last angle pair. Likewise, it is not needed to verify the last scale factor if all the corresponding angle pairs other than the two touching the unknown side are shown to be congruent.

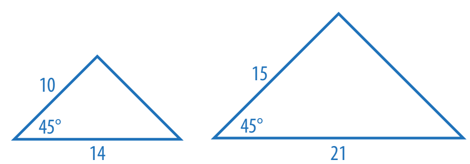
For triangles, these methods work out to three situations in which two triangles must be similar.

- SSS: All three pairs of corresponding sides are proportional (see ➊).
- AA: Two pairs of corresponding angles are congruent.
- SAS: Two pairs of corresponding sides are proportional, and the corresponding angles between them are congruent.

❻ Determine whether or not two triangles are similar.

1. Identify the measures of the angles, if known. If two of the angles in the first triangle are congruent to two of the angles in the second triangle, they are similar by AA.
2. Otherwise, identify the lengths of the sides, if known, and divide to find the scale factor. If the scale factor is the same for all three pairs of corresponding sides (SSS), or if it is the same for two pairs of corresponding sides and the corresponding angles between these sides are congruent (SAS), then the triangles are similar.

❻ Are the triangles at right similar?



2. $\frac{15}{10} = 1.5$
   $\frac{21}{14} = 1.5$
   The angle between these sides is the same for each triangle (45°).
   The triangles are similar by the SAS theorem of similarity.