Chapter Two

Fundamentals of Geometry

Geometric Notation

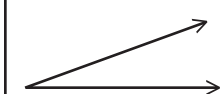
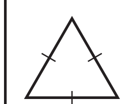
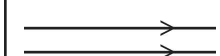
Circles

Quadrilaterals

Scale Factors

Key Geometric Formulas

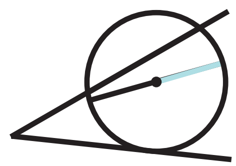
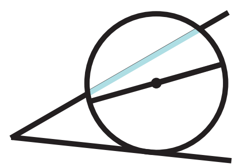
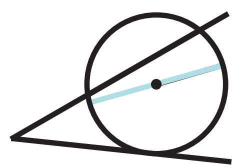
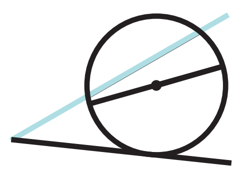
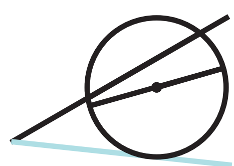
Basic Geometric Terminology and Notation

Term	Notation	Definition	Sketch
Point	A	zero-dimensional object	
Line	$\overleftrightarrow{AB}$	one-dimensional object extending infinitely in both directions	
Ray	$\overrightarrow{AB}$	one-dimensional object extending infinitely in one direction	
Line Segment	$\overline{AB}$	finite one-dimensional object	
Length	AB	distance from one point to another	
Angle	$\angle A$	two rays that share a common endpoint, called the vertex	
Congruent	$\cong$	equal in size and shape	
Parallel	$\parallel$	extending forever in a plane but never intersecting	
Perpendicular	$\perp$	intersecting at a 90° angle	

Components of Circles

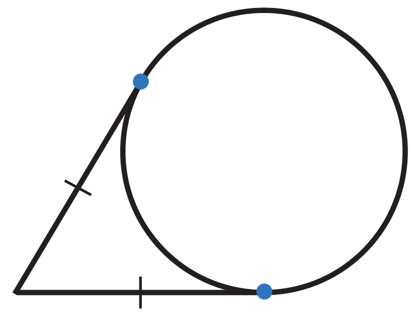
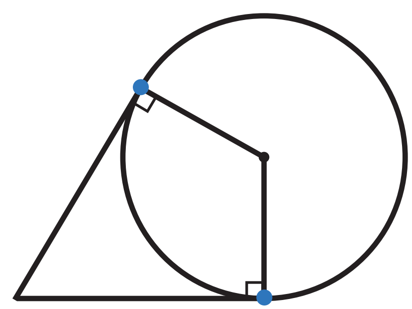
A circle can be defined as the set of points that are a set distance from a given point.

There are special names for each of the types of lines and line segments that intersect a circle.

Line or Line Segment	Definition	Example
Radius	line segment from the center to a point on the circle	
Chord	line segment from a point on the circle to another point on the circle	
Diameter	chord passing through the center	
Secant	line that contains a chord	
Tangent	line, ray, or line segment that intersects the circle at exactly one point without entering it	

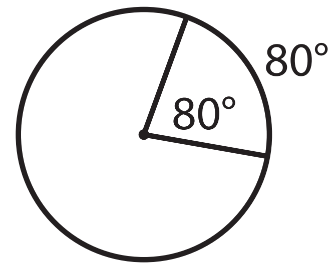
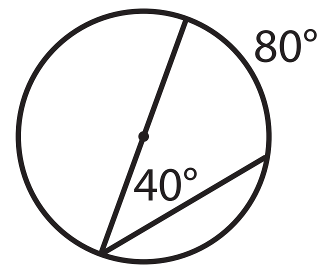
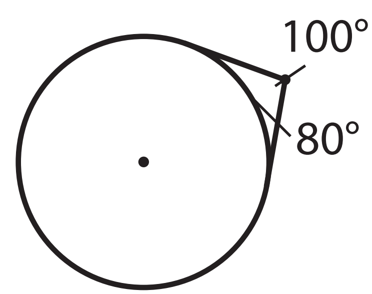
Tangency Theorems

Every point outside a circle is on two lines tangent to the circle. Each of these lines intersect the circle at a **point of tangency**.

Theorem	Description	Sketch
External tangent congruence	The two line segments connecting a given point outside the circle to a point of tangency are congruent.	
Tangent line to circle	A tangent is perpendicular to the radius it intersects.	

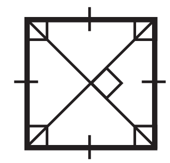
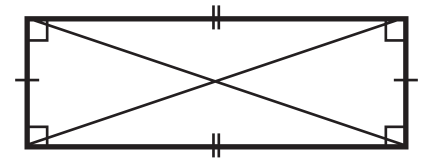
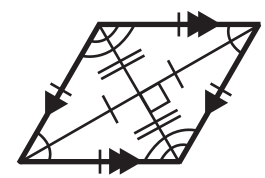
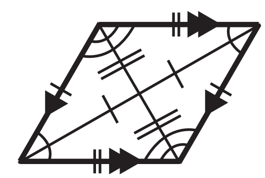
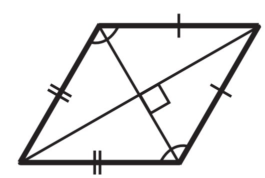
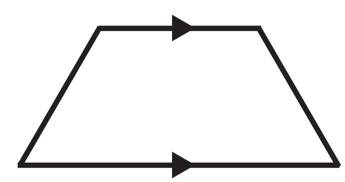
Angles In and Around Circles

Three types of angles intersecting circles have special names.

Type of angle	Position of vertex	Intercepted arc	Sketch
Central	center of circle	equal to central angle	
Inscribed	on circle	double inscribed angle	
Circumscribed	outside circle	180° minus circumscribed angle	

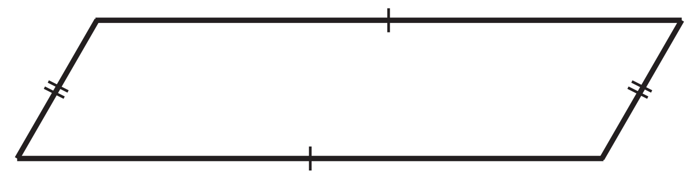
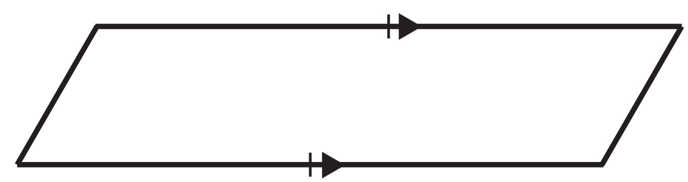
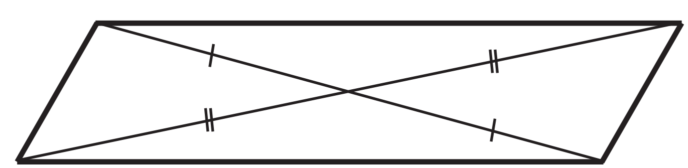
Quadrilaterals

A **quadrilateral** is a four-sided figure that may or may not match one or more of the descriptions below.

Type	Description	Features	Sketch
Square	All four sides are congruent, and all four angles are congruent (90°).	It is a rectangle, rhombus, parallelogram, and regular quadrilateral.	
Rectangle	All four angles are congruent (90°).	It is a parallelogram.	
Rhombus	All four sides are congruent.	It is a parallelogram, and the diagonals are perpendicular.	
Parallelogram	Both pairs of sides are parallel.	The diagonals bisect each other. Opposite sides and angles are congruent.	
Kite	Each side is congruent with exactly one consecutive side.	The diagonals are perpendicular. One pair of opposite angles are congruent.	
Trapezoid	Two sides are parallel.	The midsegment length is half the sum of the lengths of the bases.	

Parallelograms

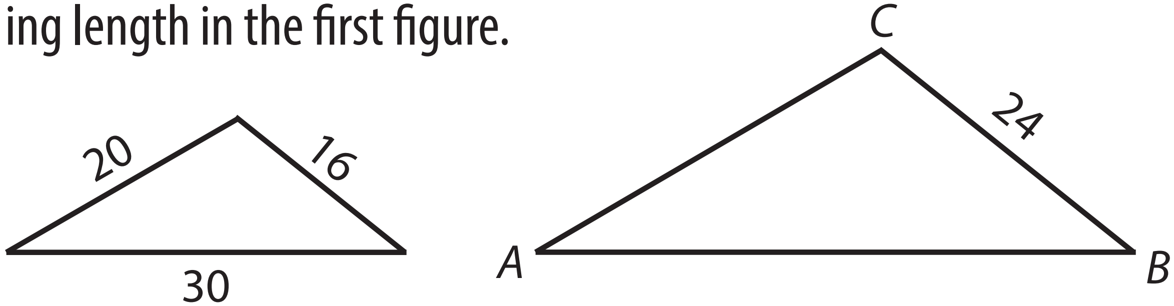
Each of the following criteria define a **parallelogram**.

Criterion	Sketch
Both pairs of opposite angles are congruent.	
Both pairs of opposite sides are congruent.	
One pair of opposite sides is congruent and parallel.	
The diagonals bisect each other.	

Scale Factors

Two figures that are exactly the same shape and size are **congruent**. Two figures that are exactly the same shape but not necessarily the same size are **similar**.

A **scale factor** is the factor by which lengths in one figure can be multiplied to find the corresponding lengths in a similar figure. It can be found by dividing a length in the second figure by the corresponding length in the first figure.



If the triangles above are similar, the scale factor from the small one to the big one is $24 \div 16 = \frac{24}{16} = \frac{3}{2}$.

Therefore, $AB = 30 \cdot \frac{3}{2} = 45$, and $AC = 20 \cdot \frac{3}{2} = 30$.

The scale factor from the large triangle to the small triangle is the reciprocal of the opposite scale factor: $16 \div 24 = \frac{16}{24} = \frac{2}{3}$.

Key Geometric Formulas

The following formulas involve two points, (x_1, y_1) and (x_2, y_2). It does not matter which point is which.

Title	Equation	Example
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	The slope between the points (-3, 7) and (5, 2) is $m = \frac{2-7}{5-(-3)} = \frac{-5}{8}$.
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	The midpoint between the points (-3, 7) and (5, 2) is $M = \left(\frac{-3+5}{2}, \frac{7+3}{2}\right) = (1, 4.5)$.
Distance	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	The distance between the points (-3, 7) and (5, 2) is $AB = \sqrt{(7-2)^2 + (-3-5)^2} = \sqrt{89} \approx 9.4$.

The following formulas involve the slope *m* of a line.

Title	Equation	Example
Line	$y = mx + b$	The equation of the line with slope 4 and *y*-intercept -6 is $y = 4x - 6$.
Perpendicular Slope	$m_2 = -\frac{1}{m}$	The slope perpendicular to $m = \frac{3}{4}$ is $m_2 = -\frac{1}{3/4} = -\frac{4}{3}$.