Chapter Two

Terms and Factors

Terms versus Factors

Polynomial Addition and Multiplication

Solving Equations

Terms versus Factors

A **term** is one or more factors multiplied together. A **Factor** is one or more terms added together.

The effects of coefficents and exponents depend on if they are applied to terms or to factors.

	Terms	Factors
Consist of	one or more factors $5x$ is one term (but two factors)	one or more terms $x + 5$ is one factor (but two terms)
Combined by	addition $5x + 2$ combines the terms $5x$ and 2.	multiplication $2(x + 5)$ combines the factors 2 and $x + 5$.
Coefficients	apply to each term individually $2(x + y) = 2x + 2y$	do not apply to each factor individually $2(xy) \neq (2x)(2y)$
Exponents	do not apply to each term individually $(x + y)^2 \neq x^2 + y^2$	apply to each factor individually $(xy)^2 = x^2y^2$

Terms and Degrees of Polynomials

A **polynomial** is the sum of one or more terms with nonnegative integer exponents. Polynomials with up to three terms have special names.

Type	Number of terms	Example
Monomial	1	$2x^2$
Binomial	2	$2x^2 + 9x$
Trinomial	3	$2x^2 + 9x + 3$

The **degree** of a polynomial in one variable is the highest exponent. A polynomial of degree n is called an n^{th} degree polynomial, although polynomials of low degree are usually referred to by the names below. Exponents of 1 or 0 are normally not written, such as $9x$ instead of $9x^1$ or 9 instead of $9x^0$.

Type	Degree	Example
Constant	0	9
Linear	1	$9x$
Quadratic	2	$9x^2$
Cubic	3	$9x^3$

Combining **like terms** means adding all terms together of the same degree. Once this is done, a single-variable polynomial is in **standard form** if its terms are in order from highest exponent to lowest.

A term's **coefficient** is the constant multiplier, such as 2 for $2x^3$ or ¾ for $\frac{3x^3}{4}$. The **leading coefficient** of an expression is the coefficient of the term with the highest exponent.

Distributing

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial. Keep in mind that negative times negative is positive.

Example	Work	Result
$4x(x^2 - 5x + 2)$	$4x(x^2) + 4x(-5x) + 4x(2)$	$4x^3 - 20x^2 + 8x$
$-4x(x^2 - 5x + 2)$	$-4x(x^2) + -4x(-5x) + -4x(2)$	$-4x^3 + 20x^2 - 8x$

A monomial does not distribute into denominators, arguments, or more than one factor within a term.

Example	Incorrect	Correct	Reason
$2x\left(\frac{4x+10}{3}\right)$	$\frac{8x^2 + 20x}{6x}$	$\frac{8x^2 + 20x}{6x}$	Do not distribute $2x$ into the denominator. $2x$ is $\frac{2x}{1}$, not $\frac{2x}{2x}$.
$2x(4x + \sqrt{3})$	$8x^2 + \sqrt{6x}$	$8x^2 + 2x\sqrt{3}$	3 is an argument, not a term. The term is $\sqrt{3}$.
$2x(4x + 3(5x))$	$8x^2 + 6x(10x^2)$	$8x^2 + 6x(5x)$	$3(5x)$ is a single term. Do not multiply it twice.

Simplifying Fractions

A fraction can be simplified by dividing the same expression out of each term one time.

Example	Incorrect	Reason	Correct
$\frac{8x+9y}{6x+5z}$	$\frac{4+9y}{3+5z}$	Cannot divide some terms and not others.	can't be simplifed
$\frac{8x+9y}{6x+6z}$	$\frac{4+3y}{3+2z}$	Cannot divide some terms by $2x$ and some terms by 3.	can't be simplifed
$\frac{8x^2+6x(4x)}{6x^2+8x}$	$\frac{4x+3(2)}{3x+4}$	$6x(4x)$ is a single term and cannot be divided more than once.	$\frac{4x+3(4x)}{3x+4}$
$\frac{4x+8\sqrt{6x}}{8x^2+6x}$	$\frac{2+8\sqrt{3}}{4x+3}$	$6x$ is an argument, not a term.	$\frac{2x+4\sqrt{6x}}{4x^2+3x}$
$\frac{12x^2+2x}{8x^2+6x}$	$\frac{6x}{4x+3}$	$2x \div 2x$ is 1, not 0.	$\frac{6x+1}{4x+3}$

Simplifying Powers and Roots

To take the power or root of a term, take the power or root of each factor of the term.

Example	Power or root of each factor	Simplified
$\sqrt{9x^2y^{10}}$	$\sqrt{9}\sqrt{x^2}\sqrt{y^{10}}$	$3xy^5$
$(3xy^5)^2$	$(3^2)(x^2)((y^5)^2)$	$9x^2y^{10}$
$\left(\dfrac{3x}{y^5}\right)^2$	$\dfrac{(3^2)(x^2)}{(y^5)^2}$	$\dfrac{9x^2}{y^{10}}$

Common Mistakes with Terms and Factors

Multiplication and division apply to terms. Powers and roots apply to factors.

Expression	Multiplication or Division	Powers or Roots
Sum of Terms	Multiply or divide each term. $2(x+4) = 2(x) + 2(4) = 2x + 8$	**Do not** take the power or root of each term. $(x+4)^2 \neq x^2 + 4^2$
Product of Factors	**Do not** multiply or divide each factor. $2(4x) \neq 2(4) \cdot 2(x)$	Take the power or root of each factor. $(3x)^2 = 3^2x^2 = 9x^2$

Multiplying Polynomials

To multiply a polynomial by a polynomial, multiply each term of one polynomial by each term of the other polynomial, and add the products. Simplify by combining like terms.

Step	Work
Original problem	$(4x + 3)(x^2 - 5x + 2)$
Multiply	$4x(x^2 - 5x + 2) + 3(x^2 - 5x + 2)$
Distribute	$(4x^3 - 20x^2 + 8x) + (3x^2 - 15x + 6)$
Combine like terms	$4x^3 + (-20x^2 + 3x^2) + (8x - 15x) + 6$
Write as a polynomial	$4x^3 - 17x^2 - 7x + 6$

Special Products of Binomials

The conjugate of a binomial $a + b$ is $a - b$.

Multiplying a binomial by itself or by its conjugate is most easily done by following the patterns below.

Multiplier	Pattern	Proof	Example
same	$(a+b)^2 = a^2 + 2ab + b^2$	$(a+b)^2$ $= a(a+b) + b(a+b)$ $= a^2 + ab + ab + b^2$ $= a^2 + 2ab + b^2$	$(3x+10)^2$ $= (3x)^2 + 2(3x(10)) + 10^2$ $= 9x^2 + 60x + 100$
conjugate	$(a+b)(a-b) = a^2 - b^2$	$(a+b)(a-b)$ $= a(a-b) + b(a-b)$ $= a^2 - ab + ab - b^2$ $= a^2 - b^2$	$(3x+10)(3x-10)$ $= (3x)^2 - 10^2$ $= 9x^2 - 100$

The expression $a + b$ consists of two terms, not two factors. **$(a+b)^2$ does not equal $a^2 + b^2$.**

Solving Equations

An **expression** is one or more terms added together. An **equation** is one expression set equal to another.

Mathematical notation is the written language of math, so it is important to use clear, valid notation when solving equations. **The following are required when solving equations in this course.**

Rule	Details
Do not write anything that is not an equation with a variable.	Make sure each equation has a variable. If you do scratchwork that is not an equation with a variable, do it on scratch paper and not on the paper that will be graded.
Make sure expressions on each side of an equal sign are equal.	If you are going to do an operation to each side, you cannot write it only on one side. If you have a string of expressions with equal signs between them, *all* of the expressions must be equal to each other.
Neatly write each step directly below the previous step.	Don't do some work on one part of a page and the next step in a different place on the page. Don't use arrows to indicate answers or next steps.
Make sure each symbol has its intended position and size.	Fraction bars are under the whole numerator but not under equal signs or anything else. Square root signs are over the whole radicand and nothing else. Exponents are small and raised.

Inverse Functions

An **inverse** function does the opposite of the original function. One-step equations can be solved by applying the inverse function on each side.

Function	Inverse	Example	Solve by…
Addition	subtraction	$x + 3 = 12$	subtracting 3 from each side
Multiplication	division	$3x = 12$	dividing each side by 3
Power	root	$x^2 = 12$	taking the square root of each side

If more than one operation has been applied to a variable, the variable can be solved for by applying the inverse of each function, in reverse order. For example, the solutions to $2(5x - 1)^2 + 3 = 21$ are $x = \frac{4}{5}$ and $x = \frac{-2}{5}$, as solved below.

Equation	Last Operation	Solve by…
$2(5x - 1)^2 + 3 = 21$	adding 3	subtracting 3 from each side
$2(5x - 1)^2 = 18$	multiplying by 2	dividing each side by 2
$(5x - 1)^2 = 9$	squaring	taking the square root of each side
$5x - 1 = \pm 3$	subtracting 1	adding 1 to each side
$5x = 4$ or $5x = -2$	multiplying by 5	dividing each side by 5

Common Notation Issues

Equation	Not Valid	Reason	Valid
$2x = 8$	$\frac{2x = 8}{2}$	An equal sign is not part of an expression and cannot be operated on.	$\frac{2x}{2} = \frac{8}{2}$
$x - 5 = 9$	$x - 5 = 9 + 5$	The equation is not true if 5 is added on one side and not the other.	$x - 5 + 5 = 9 + 5$
$x - 5 = 9$	$x \overset{+5}{-5} = \overset{+5}{9}$	"+ 5" is not part of the equation.	$x = 14$
$\frac{x}{4} = 5x + 2$	$4\left(\frac{x}{4}\right) = 5x + 2(4)$	One side was multiplied but only part of the other side was multiplied.	$4\left(\frac{x}{4}\right) = (5x + 2)4$
$x^2 = \frac{2}{3}$	$x = \pm\frac{\sqrt{2}}{3}$	A square root was applied to one side but only to part of the other side.	$x = \pm\sqrt{\frac{2}{3}}$
$x = 2(4) - 3$	$x = 2(4) = 8 - 3 = 5$	2(4) does not equal 8 – 3.	$x = 8 - 3 = 5$
$\frac{x}{4} = 18$	$x = 18 \text{ x } 4$	"x" should not represent multiplication and a variable in the same equation.	$x = 18(4)$

Writing Answers

There are different ways to express a solution that is not a whole number.

Instruction	Description	Solve $12x = 14$
Round	Type it into a calculator, and leave a certain number of digits after the decimal point. Increase the last written digit by 1 if the following digit was 5 or higher. Write "$\approx$" instead of "$=$".	$x \approx 1.14$
Answer exactly	Do not use decimals, unless there are only a few digits after the decimal point and you write all of them.	$x = \frac{14}{12}$
Simplify	Answer exactly, and reduce fractions, combine like terms, simplify square roots, etc.	$x = \frac{7}{6}$

If there are no instructions to answer in a certain way, then you can choose whichever one you prefer, so long as it makes sense for the problem. Mathematically, it is better not to round, since rounding changes the answer slightly. However, answers to word problems are often best rounded, such as \$0.67 instead of \$$\frac{2}{3}$.

Rounding

Consideration	Description	Example
Round up when needed.	Add 1 to the last digit of your answer if the digit after it (the first one getting dropped) is 5 or higher.	For $x = 2.485204$, $x \approx 2.49$, not 2.48.
Use the stated level of precision if there is one.	Tenths are the first place after the decimal point, hundredths are the second, and thousandths are the third.	For $x = 2.485204$, nearest tenth: $x \approx 2.5$ nearest hundredth: $x \approx 2.49$ nearest thousandth: $x \approx 2.485$
Match the context of the problem.	Don't round in a way that doesn't make sense for the units or measurements.	Avoid awkward answers like $83.1 or 24.1882951 meters, unless you have a specific reason.
Keep the rounding consistent if there are multiple answers with the same units.	Pick a place to round to, and stick with it.	Avoid answers like "The average score was 10.2 for sophomores and 9.84 for freshmen."
Don't round to only one significant figure.	Don't round so much that all or all but one of the digits in a value are 0 or are 9.	Rounded values such as .002 or .998 can lead to huge rounding errors.