CHAPTER TWO: TERMS AND FACTORS — Test: Thursday, October 5

For an expression E, terms are expressions that equal E when added together, and factors are expressions that equal E when multiplied together. Most algebra is based upon this distinction, including polynomial multiplication and solving.

2-A Terms versus Factors — Thursday • 9/14

❶ Identify terms and factors of an expression.
❷ Identify a coefficient.
❸ Add or subtract expressions.
❹ Classify a polynomial in one variable.
❺ Multiply a polynomial by a monomial.
❻ Distribute a monomial.
❼ Simplify a fraction with multiple terms in the numerator or denominator.
❽ Take a power or root of an expression.

2-B Polynomial Multiplication — Tuesday • 9/19

❶ Multiply two polynomials.
❷ Multiply more than two polynomials.
❸ Multiply a binomial by its conjugate.
❹ Square a binomial.

2-C Solving Equations — Thursday • 9/21

equation • solution • solve • inverse

❶ Identify an equation.
❷ Apply an operation to both sides of an equation.
❸ Identify notation errors in applying an operation to both sides of an equation.
❹ Identify the inverse of an operation by definition.
❺ Apply inverses to solve an equation.
❻ Express a numerical answer.

2-A Terms versus Factors

A TERM is the product of one or more factors. The terms of an expression are the components of the expression that equal the whole expression when added together.

A FACTOR is the sum of one or more terms. The factors of an expression are the components of the expression that equal the whole expression when multiplied together.

❶ Identify terms and factors of an expression.

1. Components of an expression separated by addition are terms. Addition can be notated by "+" or by "–" (because "–" means add the opposite).
2. Components of an expression separated by multiplication are factors. Multiplication can be shown in many ways, including ×, •, parentheses, or no symbol.
3. If an expression can be rewritten as an expression multiplied by one or more other expressions, each of these expressions are factors of the original expression (See chapter 5).
4. A term can consist of more than one factor, and a factor can consist of more than one term.

❶ List the factors of the expression $5x^2(3x + 4)(x^2 - 9x + 2)$, and state how many terms each factor has.

2. The factors are $5x^2$, $(3x + 4)$, and $(x^2 - 9x + 2)$.
4. $5x^2$ is one term, $(3x + 4)$ is the two terms $3x$ and 4, and $(x^2 - 9x + 2)$ is the the three terms x^2, $-9x$, and 2.

❶ State the terms and the factors of the expression $5a - 5b$.

1. The terms are $5a$ and $-5b$, because $5a + -5b$ is equal to $5a - 5b$.
3. The factors are 5 and $(a - b)$, because $5 \times (a - b)$ is equal to $5a - 5b$.

The COEFFICIENT of a term is the constant factor of the term. In other words, it is a number and not a variable. It is normally written at the beginning of the term.

❷ Identify a coefficient.

1. The coefficient of each term is the number that the rest of the term is multiplied by.
2. If the term is subtracted, the coefficient is negative.
3. If no number is written, then the coefficient is 1 (or -1).
4. If the term is a fraction, the coefficient is the numerical part of the fraction.
5. If the term is a constant, then it is its own coefficient.

❷ Identify the coefficients of the expression $5x^6 + x^3 + \frac{8x^2}{7} - 9x + 2$.

1. The coefficent of $5x^6$ is 5, because 5 times x^6 is $5x^6$.
3. The coefficient of x^3 is 1, because 1 times x^3 is x^3.
4. The coefficient of $\frac{8x^2}{7}$ is $\frac{8}{7}$, because $\frac{8}{7}$ times x^2 is $\frac{8x^2}{7}$.
2. The coefficient of $-9x$ is -9, because -9 times x is $-9x$.
5. The coefficient of 2 is 2, because there is no variable.

LIKE Terms are terms that are identical to each other, not includeing the coefficient.

❸ Add or subtract expressions.

1. Group each term with any other term that is identical, not including the coefficient.
2. Add or subtract the coefficients in each group, leaving the rest of the term the same.

❸ $(4x^2 + 10x - 5) + (x^2 + 2)$

1. $(4x^2 + x^2) + (10x) + (-5 + 2)$
2. $5x^2 + 10x - 3$

A MONOMIAL is a nonzero coefficient multiplied by a variable to a power. The power, which can be any nonnegative integer, is the DEGREE of the monomial.

Degree	Name	Example
0	CONSTANT	5 (that is, $5x^0$)
1	LINEAR	$5x$ (that is, $5x^1$)
2	QUADRATIC	$5x^2$
3	CUBIC	$5x^3$
n	n^{th} degree	$5x^{12}$

A POLYNOMIAL is an expression of the sum of one or more monomials, after like terms are combined. The degree of a polynomial in one variable is the highest exponent.

A BINOMIAL is a polynomial with two terms.

A TRINOMIAL is a polynomial with three terms.

A polynomial in one variable with its terms written in order from highest exponent to lowest is in STANDARD Form.

The LEADING Coefficient of a Polynomial is the coefficient of the term with the highest exponent.

❹ Classify a polynomial in one variable.

1. A single term is a monomial, two terms is a binomial, and three terms is a trinomial.
2. Consider the term with the highest exponent:

   If there is no variable at all, the polynomial is a constant.

   If there is a variable but no exponent (that is, there is an unwritten exponent of 1), the polynomial is linear.

   If the variable is to the second power, the polynomial is quadratic.

   If the variable is to the third power, the polynomial is cubic.

   If the variable is to the fourth power or higher, look up what it is called, or simply refer to the polynomial as "n^{th} degree", where n is the exponent.

❹ Classify the following polynomials, write them in standard form, and identify the leading coefficient.

a) $x + 4x^3$
cubic binomial
$4x^3 + x$
4

b) $-15x$
linear monomial
$-15x$
-15

c) $8x^2 - 2x^9 + 3$
9^{th} degree trinomial
$-2x^9 + 8x^2 + 3$
-2

d) $2 - \frac{7x^3}{5} + 6x^2 + x$
cubic polynomial
$-\frac{7x^3}{5} + 6x^2 + x + 2$
$-\frac{7}{5}$

To DISTRIBUTE a monomial is to multiply an expression by it. To do so, multiply each term in the expression by the monomial.

❺ Multiply a polynomial by a monomial.

1. Multiply each term in the expression by the monomial.

❺ $2(5x^3 - 4x + 10)$

$10x^3 - 8x + 20$

When distributing a monomial, each term is only multiplied once. This means if a term consists of multiple factors, such as $(x + 3)(2x - 1)$, only the terms in one of the factors are multiplied.

When multiplying a fraction by a monomial, only multiply the terms in the numerator (see 1-A ❷).

A function and its argument make up a single factor. The argument itself is not a factor.

❻ Distribute a monomial.

1. Identify one factor of each term of the expression. Ignore the denominator if there is one.
2. Multiply each term of these factors by the monomial.

❻ Multiply the expression $((x + 3)(2x - 1) + 4x(9x^2 + 3) + \sqrt{5})$ by 10.

1. $(x + 3)$, $4x$, and $\sqrt{5}$
2. $(10x + 30)(2x - 1) + 40x(9x^2 + 3) + 10\sqrt{5}$

❻ Identify the error, if any, in each of the following attempts to multiply $\frac{11(4x) - \sqrt{10x}}{5}$ by 2.

a) $\frac{22(4x) - \sqrt{20x}}{5}$ 10x is an argument and should not be multiplied.

b) $\frac{22(4x) - 2\sqrt{10x}}{10}$ The denominator should not be multiplied.

c) $\frac{22(8x) - 2\sqrt{10x}}{5}$ The term 11(3x) should be multiplied by 2 one time, but it was multiplied by 2 once on the 11 and again on the 4x.

d $\frac{22(4x) - 2\sqrt{10x}}{5}$ This is correct, because each term in the numerator was multiplied by 2 one time.

❼ Simplify a fraction with multiple terms in the numerator or denominator.

1. Identify the terms of the numerator and the denominator. An argument by itself is not a term.
2. Identify a factor that divides evenly into all of the terms.
3. Rewrite the fraction with the factor divided out of each term. If it is variable, specify that it cannot be zero.

❼ $\frac{6x^2 - 9x\sqrt{30x}}{6x^2 - 9x}$

1. The terms are $6x^2$ and $-9x\sqrt{30x}$ in the numerator, and $6x^2$ and $-9x$ in the denominator.
2. $3x$ divides evenly into each of these terms:

   $6x^2 \div 3x = 2x$

   $-9x\sqrt{30x} \div 3x = -3\sqrt{30x}$ (not $-3\sqrt{10}$)

   $6x^2 \div 3x = 2x$

   $-9x \div 3x = -3$

3. $\frac{11(4x) - 3\sqrt{10x}}{5}$, $x \neq 0$ (We divided by $3x$, so our answer doesn't work if $3x$ is zero.)

Note that $6x^2$ in the numerator and $6x^2$ in the denominator do not cancel each other out, because there are other terms.

Unlike multiplying or dividing an expression, which results in each individual *term* being multiplied or divided, taking a power or root of an expression results in the power or root of each individual *factor*.

❽ Take a power or root of an expression.

1. Identify the factors of the expression.
2. Take the power or root of each factor. If the root is even, such as a square root, put $\pm$.

❽Simplify.

a) $10xy$ to the third power

1. The factors are 10, x, and y.
2. $1000x^3y^3$

b) the square root of $49x^8y^2$

The factors are 49, x^8, and y^2.

$\pm 7x^4y$

2-B Polynomial Multiplication

➊ Multiply two polynomials.

1. Multiply each term in one polynomial by each term in the other polynomial.
2. Combine like terms.

➊ $(4x - 3)(x + 5)$

1. $4x(x) + 4x(5) + \text{-}3(x) + \text{-}3(5)$
   $4x^2 + 20x - 3x - 15$
2. $4x^2 + 17x - 15$

➋ Multiply more than two polynomials.

1. Multiply two of the polynomials together (see ➊).
2. Multiply the result by the next polynomial.
3. Combine like terms.

➋ $(x + 2)(x + 5)(x - 10)$

1. $(x^2 + 5x + 2x + 10)(x - 10)$
   $(x^2 + 7x + 10)(x - 10)$
2. $x(x^2 + 7x + 10) - 10(x^2 + 7x + 10)$
   $x^3 + 7x^2 + 10x - 10x^2 - 70x - 100$
3. $x^3 - 3x^2 - 60x - 100$

The CONJUGATE of a Binomial is the original binomial except with the sign of one of the terms switched: The conjugate of $a + b$ is $a - b$.

A binomial can be multiplied by itself or by its conjugate the same as multiplying any two binomials (see ➊), or you can follow these patterns:

Binomial times itself (squaring a binomial): $(a + b)^2 = a^2 + 2ab + b^2$

Binomial times its conjugate: $(a + b)(a - b) = a^2 - b^2$

➌ Multiply a binomial by its conjugate.

1. Square each term of the binomial.
2. Subtract the second square from the first.

➌ $(3x - 10)(3x + 10)$

1. $a^2 = (3x)^2 = 9x^2$
   $b^2 = 10^2 = 100$
2. $(3x - 10)(3x + 10) = 9x^2 - 100$

➍ Square a binomial.

1. Square each term of the binomial.
2. Multiply the two terms together, and double this product.
3. Add the three results of the steps above.

➍ $(3x - 10)^2$

1. $a^2 = (3x)^2 = 9x^2$
   $b^2 = (\text{-}10)^2 = 100$
2. $2ab = 2(3x)(\text{-}10) = \text{-}60x$
3. $(3x - 10)^2 = 9x^2 - 60x + 100$

2-C Solving Equations

An EQUATION is one expression set equal to another expression.

An expression on one side of an equal sign must be equal to the expression on the other side.

❶ Identify an equation.

1. If there is no equal sign, there is no equation.
2. If there is one equal sign, what is written is an equation.
3. If there are multiple equal signs, each one is in the middle of an equation that includes everything up to but not including the next equal sign.

❶ a) $x + 5$ b) $x + 5 = 9 + 5$ c) $x + 5 = 9 + 5 = 14$

1. This is not an equation. 2. This is an equation. 3. $x + 5 = 9 + 5$ is an equation, and $9 + 5 = 14$ is an equation.

Any operation can be done to the expression on one side of an equation as long as it is done to the whole expression and also to the whole expression on the other side as well.

❷ Apply an operation to both sides of an equation.

1. For addition or subtraction, apply it at the end of both expressions.
2. For multiplication or division, apply it to each term of both expressions. Use parentheses around each expression if it has more than one term
3. For powers or roots, apply it to each factor of both expressions. Use parentheses around each expression if it has more than one factor.

Never include an equal sign within parentheses.

❷ Do the following to each side of the equation $x + 5 = 8$, and simplify.

a) add two

$x + 5 + 2 = 8 + 2$

$x + 7 = 10$

b) multiply by 2

$2(x + 5) = 2(8)$

$2x + 10 = 16$

c) put it to the power of 2

$(x + 5)^2 = 8^2$

$x^2 + 10x + 25 = 64$

When applying an operation to both sides of an equation, it is not necessary to show this step. However, if you do show it, the following are all incorrect: applying it only on one side, applying only to part of one or both sides, applying it to the equation as a whole, or writing it in an illogical place such as $+2\ 8$ instead of $8 + 2$.

❸ Identify notation errors in applying an operation to both sides of an equation.

1. Operations must be applied to each side, not written on only one side.
2. Symbols for addition, subtraction, multiplication, and division must be between two expressions, not before both of them.
3. An equal sign cannot be operated on. Make sure it is not within parentheses or part of a numerator, denominator, or argument.
4. Multiplying must apply to every term. Use parentheses when multiplying expressions with more than one term.
5. Division must apply to every term. Use appropriately sized fraction bars when dividing expressions with more than one term.
6. Powers must apply to every factor. Use parentheses when applying powers to expressions with more than one factor.
7. Roots must apply to every factor. Use appropriately sized square-root symbols when taking roots of expressions with more than one factor.
8. "x" should not be a variable and a multiplication sign in the same equation.

❸ Identify each error in $\times 4(\frac{1}{4}x = x + 1)$, and rewrite it correctly.

2. The multiplication sign should not be at the beginning.
3. The equal sign cannot be multiplied. It should not be in parentheses.
4. The whole expression $x + 1$ needs to be in parentheses.
8. The multiplication sign should not be the same as the variable.

$4(\frac{1}{4}x) = 4(x + 1)$

The INVERSE of an operation is the operation that cancels the original operation.

❹ Identify the inverse of an operation by definition.

1. The inverse of addition is subtraction.
2. The inverse of multiplication is division.
3. The inverse of a power is a root.

❹ Identify the inverse of each operation in the equation $5x - 9 = 16$.

The inverse of subtracting 9 is adding 9.

The inverse of multiplying by 5 is dividing by 5.

The SOLUTIONS of an Equation are values of the variable that make the equation true. SOLVING an Equation means finding the solutions.

Equations that only have one instance of the variable once like terms are combined (such as $8x^2 + 9 = 20$ or $8x^2 + 9 = 3x^2 + 20$) can be solved by applying one or more inverse operations to each side of the equation. These are done in reverse order of operations.

An equation can have any number of solutions. In particular, since every positive number has two square roots (the square roots of x are $\pm\sqrt{x}$), most quadratic equations have two solutions.

❺ Apply inverses to solve an equation.

1. Identify each operation that took place on the variable, in order by order of operations.
2. On each side of the equation, apply the inverse of each operation, in reverse order.

❺ $2(x-3)^2 + 8 = 40$

1. The following operations took place, in order:
   a) 3 was subtracted from x.
   b) The difference was squared.
   c) The square was multiplied by 2.
   d) 8 was added to to the product.
2. d) Subtract 8 from each side: $2(x-3)^2 = 32$
   c) Divide each side by 2: $(x-3)^2 = 16$
   b) Take the square root of each side: $x - 3 = \pm 4$
   a) Add 3 to each side: $x = 3 \pm 4$

The solutions are $x = -1$ and $x = 7$.

To SIMPLIFY an answer is to express it in its simplest form. For rational answers, this generally means writing it with no fractions or decimals in the numerator or denominator, and reducing. Simplifying irrational answers is explained in chapter 7.

To ROUND an answer is to get a decimal approximation of the answer such as by evaluating it in a calculator.

Simplified answers are the actual answers. Rounded answers are not the actual answers, but are instead approximations of them. Do not round when directions say to simplify.

❻ Express a numerical answer.

1. If the instructions are to round, then round to the indicated place, or choose an appropriate place to round to if none is specified. Write "≈" instead of "=".
2. If the instructions are to simplify, then eliminate any fractions or decimals within fractions, reduce fractions, and simplify square roots (see 7-B).
3. If the instructions do not specify whether to round or simplify, then do either one unless there is a reason in the context of the problem to do one or the other.

❻ Give an appropriate answer for each question.

a) Solve $7x = 9$. $x = \frac{9}{7}$

b) Solve $7x = 9$. Round the answer to the nearest tenth. $x \approx 1.3$

c) If a pack of seven pens costs \$9, what is the price per pen? Each pen costs \$1.29.