CHAPTER TWO: FUNDAMENTALS OF GEOMETRY — Review October 29 ⤳ Test November 6

Terminology and notation are fundamental for any subject. This chapter sets up basic concepts for describing, sketching, and solving geometric scenarios.

2-A Geometric Notation — Monday • 10/12

point • line • ray • line segment • plane • parallel • perpendicular • angle • vertex • congruent • midpoint

❶ Identify and label geometric components.
❷ Refer to angles by name.

2-B Circles — Thursday • 10/15

circle • radius • diameter • chord • secant • tangent • point of tangency • arc • major arc • minor arc • semicircle • central angle • inscribed angle • circumscribed angle

❶ Identify types of lines and line segments intersecting circles.
❷ Calculate the measure of an angle that intercepts two points on a circle.

2-C Quadrilaterals — Monday • 10/19

quadrilateral • trapezoid • rhombus • kite • parallelogram

❶ Identify types of quadrilaterals.
❷ Calculate values based on angles within a parallelogram.
❸ Calculate values based on lengths within a parallelogram.
❹ Identify the coordinates of a vertex of a parallelogram, given the coordinates of the other vertices.

2-D Scale Factors — Thursday • 10/22

similar • scale factor

❶ Determine a scale factor that maps a figure onto a similar figure.
❷ Calculate lengths in similar figures based on a scale factor.
❸ Find the perimeter of a similar figure.
❹ Find the area of a similar figure.

2-E Key Geometric Formulas — Tuesday • 10/27

❶ Write the equation of a line, given two points on the line.
❷ Write the equation of a perpendicular bisector of a line, given two points on the line.
❸ Find the distance between two points.

2-A Geometric Notation

The basic building blocks of geometry are points, lines, and planes.

A zero-dimensional object is a POINT. Points are typically labeled with a capital letter.

A one-dimensional object is a LINE if it extends infinitely in both directions (indicated by arrowheads), a RAY if it extends infinitely in only one direction, and a Line SEGMENT if it does not extend infinitely in either direction.

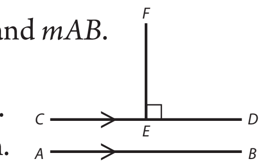
Lines, rays, and line segments can be labeled by two points and a bar drawn above. For lines, the bar has arrows on each side, such as $\overleftrightarrow{AB}$. For rays, the bar has an arrow on the side that extends infinitely, such as $\overrightarrow{AB}$. For line segments, the bar has no arrows, such as $\overline{AB}$.

A pair of points without a bar, such as AB, represents the length of the line segment. Other variations of this notation include $|AB|$ and mAB.

A two-dimensional object is a PLANE if it extends infinitely in all directions.

Lines in a plane that never intersect are PARALLEL, written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and indicated graphically with arrowheads within the lines.

Lines that intersect at a 90° angle are PERPENDICULAR, written $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$ and indicated graphically with a box at the intersection.



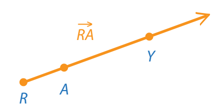
Lines and planes can also be labeled by a single script letter, such as ℓ. In this case, lowercase is used for lines and uppercase is used for planes.

Letters used to represent points, lines, and other geometric objects, as well as variables, are italicized when typed.

❶ Identify and label geometric components.

1. Label each point with a capital letter. If they are typed, italicize them.
2. Use commonly accepted notation, such as described above.

❶ Sketch and label a ray that starts at R and passes through A and Y.



An ANGLE is formed by a pair of rays sharing the same endpoint, called the VERTEX.

An angle can be labeled with a number or letter written inside it, or by its vertex, such as A.

When labeling an angle by its vertex, use the angle symbol, such as $\angle A$, to clarify that you are referring to angle A and not point A.

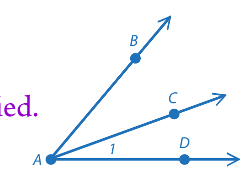
If a point is the vertex of more than one angle, use three letters to clarify which angle is intended, with the vertex in the middle, such as $\angle BAC$.

The measure of an angle of an angle can be notated by putting m before the angle, such as $m\angle BAC$.

❷ Refer to angles by name.

1. If the angle itself is labeled, use this number or letter.
2. If it is not labeled, use the $\angle$ symbol and the vertex.
3. If the vertex is the vertex of other angles in the diagram as well, use additional points to clarify which rays are in the angle specified.

❷ Name each angle shown at right.



$\angle BAC$ is the top angle. It could also be labeled as $\angle CAB$.

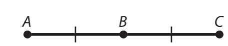
$\angle CAD$ is the bottom angle. It could also be labeled as $\angle DAC$ or $\angle 1$.

$\angle BAD$ is the big angle, that is, the other two angles put together. It could also be labeled as $\angle DAB$.

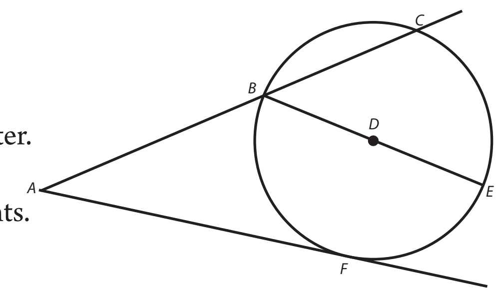
Objects that are exactly the same size and shape (although not necessarily with the same position or orientation) are CONGRUENT, symbolized by "≅".

The MIDPOINT of a Line Segment divides the line segment into two congruent segments.

Tick marks or otherwise identical marks drawn through different objects identify these objects as congruent.



2-B Circles



A CIRCLE is formally defined as the set of points that are all a set distance from a given center.

A RADIUS is a line segment or distance from the center of a circle to the circle itself. $\overline{BD}$ and $\overline{DE}$ are radii.

A CHORD is a line segment from any point on a circle to any other point on the circle. $\overline{BC}$ and $\overline{BE}$ are chords.

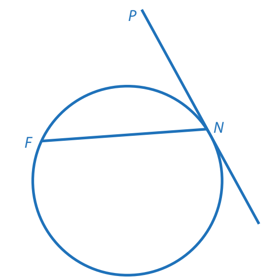
A DIAMETER is a chord that passes through the center of the circle, making it equal to two radii. $\overline{BE}$ is a diameter.

A SECANT is a line that contains a chord. $\overleftrightarrow{BC}$ is a secant.

A TANGENT is line, ray, or line segment that intersects a circle at exactly one point. $\overline{AF}$, $\overrightarrow{AF}$, and $\overleftrightarrow{AF}$ are tangents.

❶ Identify types of lines and line segments intersecting circles.

1. If it goes from the center to the circle, it is a radius.
2. If it touches the circle at two points, it is a chord if it does not leave the circle, or a secant if it does continue infinitely outside the circle. If it is a chord passing through the center, it is a diameter.
3. If it touches the circle but goes past it without entering it, like a road under a wheel, then it is a tangent.



❶ Identify the following in the diagram at right.

a) $\overleftrightarrow{PN}$

3. It touches the circle but keeps going past it without entering it, so it is a tangent.

b) $\overline{FN}$

2. It touches the circle twice, but does not go outside the circle, so it is a chord.

The point at which a tangent intersects a circle is a POINT OF TANGENCY.

Every point outside a circle is on two tangent lines to the circle and is equidistant from the two points of tangency.

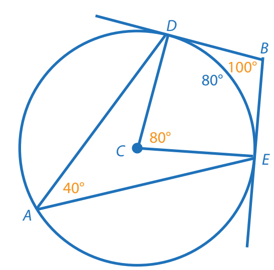
A tangent is perpendicular to the radius at the point of tangency.

An ARC is a portion of a circle. It is a SEMICIRCLE if it is exactly half a circle, a MINOR Arc if smaller, or a MAJOR Arc if larger.

A CENTRAL Angle has its vertex at the center of the circle. It has the same measure as the arc it intercepts.

An INSCRIBED Angle has its vertex on the circle. It has half the measure as the arc it intercepts.

A CIRCUMSCRIBED Angle has sides that are tangent to the circle. Its measure is 180° minus the measure of the arc it intercepts.



❷ Calculate the measure of an angle that intercepts two points on a circle.

1. If the vertex is at the center, the angle has the same measure as the intercepted arc.
2. If the vertex is on the circle, divide the measure of the intercepted arc by 2.
3. If the vertex is inside the circle, subtract the measure of the intercepted arc from 180°.

❷ Find the measures of the following angles in the diagram at right.

a) $\angle DCE$

1. The vertex is at the center.

$\angle DCE = 80°$

b) $\angle DAE$

2. The vertex is on the circle.

$\angle DAE = \frac{1}{2}(80°) = 40°$

c) $\angle DBE$

4. The vertex is outside the circle.

$\angle DBE = 180° - 80° = 100°$

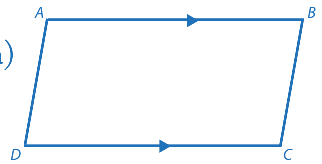
2-C Quadrilaterals

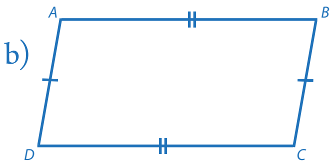
A QUADRILATERAL is a shape with four sides.
A TRAPEZOID is quadrilateral with a pair of parallel sides.
A RHOMBUS is a parallelogram in which all four sides are congruent.
A KITE is a quadrilateral that has two pairs of consecutive congruent sides but is not a rhombus.
The angles of a kite that are between noncongruent sides are congruent.
The diagonals of a rhombuses and of kites are perpendicular.

➊ Identify types of quadrilaterals.

1. If there are two pairs of congruent sides, none of which are parallel, it is a kite.
2. If there is one pair of parallel sides, it is a trapezoid.
3. If there is a pair of congruent parallel sides, two pairs of parallel sides, two pairs of congruent opposite sides, or two pairs of congruent opposite angles, then it is a parallelogram (see 2-B ➌), and:
   - If all the angles are congruent (90°), the parallelogram is a rectangle.
   - If all the sides are congruent, if all of the angles are bisected by the diagonals, or if the diagonals are perpendicular, the parallelogram is a rhombus.
   - If it is both a rectangle and a rhombus, it is a square.

➊ Identify the quadrilaterals at right.

a)


b)


a) $\overline{AB}$ is parallel with $\overline{CD}$, so $ABCD$ is a trapezoid.
b) There are two pairs of opposite congruent sides, so it is a parallelogram.
Not all four angles are congruent, so it is not a rectangle.
Not all four sides are congruent, so it is not a rhombus.

A PARALLELOGRAM is a quadrilateral with two pairs of parallel sides.

Parallelograms have many special properties. The following are true for all parallelograms but not for any other quadrilaterals.

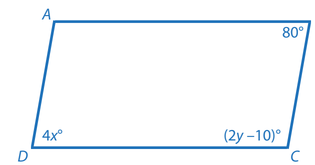
- Both pairs of opposite sides are congruent.
- Both pairs of opposite angles are congruent.
- The diagonals bisect each other.
- Consecutive angles are supplementary.

A quadrilateral is a parallelogram if one pair of sides is parallel and congruent.

❷ Calculate values based on angles within a parallelogram.

1. Each angle is congruent to the opposite angle.
2. Each angle measures 180° minus the measure of an angle next to it.
3. Alternate interior angles created by a diagonal are congruent.

❷ Find x and y, given $ABCD$ is a parallelogram.



1. $m\angle ADC = m\angle CBA$

$4x = 80$

$x = 20$

2. $m\angle DCB + m\angle CBA = 180°$

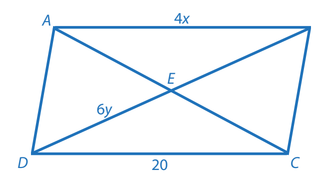
$(2y - 10) + 80 = 180$

$y = 55$

❸ Calculate values based on lengths within a parallelogram.

1. Each side is congruent to the opposite side.
2. Each diagonal is cut in half by the other diagonal.

❸ Find x and y, given $ABCD$ is a parallelogram and $DB = 24$.



1. $AB = DC$

$4x = 20$

$x = 5$

2. $DB = 2DE$

$24 = 2(6y)$

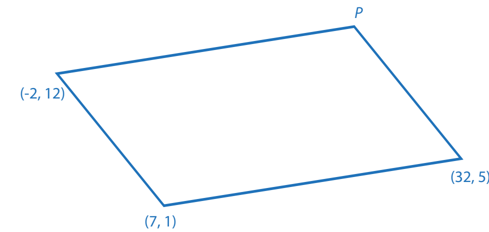
$y = 2$

The difference between *x*-coordinates is the same on one side of a parallelogram as the other. The same is true for *y*-coordinates of a parallelogram.

❹ Identify the coordinates of a vertex of a parallelogram, given the coordinates of the other vertices.

1. Write an equation showing the *x*-value of any vertex minus the *x* value of a neighboring vertex is equal to the difference of the *x*-values in the other two vertices (subtracted in the same order).
2. Solve the equation.
3. Repeat steps 1 ad 2 for the *y*-values.

❹ Identify the coordinates of *P* in the parallelogram at right.



1. $x - 32 = -2 - 7$
2. $x = 23$
3. $y - 5 = 12 - 1$

   $y = 16$

   $P = (23, 16)$

2-D Scale Factors

A figure is SIMILAR to another figure if it is exactly the same shape (although not necessarily the same size or direction).

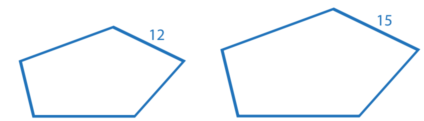
A SCALE FACTOR is the factor by which any length in a figure can be multiplied to find the corresponding length in a similar figure.

❶ Determine a scale factor that maps a figure onto a similar figure.

1. Identify a length in the original figure and the corresponding length in the similar figure.
2. Divide the second length by the original length.

❶ Determine the scale factor that maps the pentagon on the left onto the similar pentagon on the right.

1. The side of 12 in the original pentagon corresponds with the side of 15 in the similar pentagon.
2. $15 \div 12 = \frac{15}{12}$



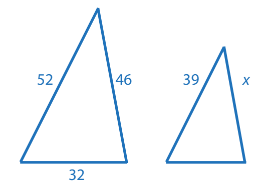
A scale factor can be used to determine lengths in similar figures.

❷ Calculate lengths in similar figures based on a scale factor.

1. To find a length in the similar figure, multiply the corresponding length in the original figure by the scale factor.
2. To find a length in the original figure, divide the corresponding length in the similar figure by the scale factor.

❷ Find x, given the triangles are similar.

1. $x = 46 \bullet \left(\frac{39}{52}\right) = 34.5$



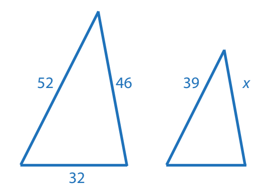
Since the perimeter of a figure is simply the total of its lengths, the perimeter of a similar figure can be found by multiplying the original perimeter by the scale factor, just like for any one side.

❸ Find the perimeter of a similar figure.

1. Determine the scale factor (see ❶).
2. Determine the perimeter of the original figure.
3. Multiply the perimeter of the original figure by the scale factor.

❸ Find the perimeter of the smaller triangle, given it is similar to the larger triangle.

1. The scale factor is $\frac{39}{52}$.
2. The perimeter of the larger triangle is $52 + 32 + 46 = 130$.
3. The perimeter of the smaller triangle is $130\left(\frac{39}{52}\right) = 97.5$.



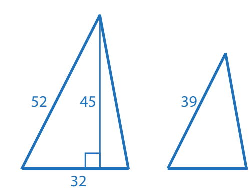
Since area is two-dimensional and scale factors are one-dimensional, the area of a similar figure can be found by multiplying the original area by the scale factor twice.

❹ Find the area of a similar figure.

1. Determine the scale factor (see ❶).
2. Determine the area of the original figure.
3. Multiply the area of the original figure by the scale factor twice.

❺ Find the area of the smaller triangle, given it is similar to the larger triangle.

1. The scale factor is $\frac{39}{52}$.
2. The area of the larger triangle is $\frac{1}{2}(32)(45) = 720$.
3. The area of the smaller triangle is $720\left(\frac{39}{52}\right)\left(\frac{39}{52}\right) = 405$.



2-E Key Geometric Formulas

$y = mx + b$ is the equation of a line, where m is the slope, b is the y-intercept, and x and y are variables.

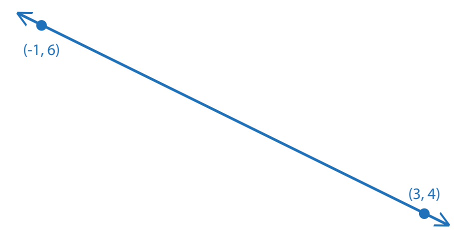
The slope between two points can be found by dividing the vertical distance between them by the horizontal distance between them: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

If the slope is known, the y-intercept of a line can be calculated by plugging in the x value and the y value of any point on the line and solving for b.

❶ Write the equation of a line, given two points on the line.

1. Use the slope formula above to calculate m. Make sure to subtract the x values in the same direction as you subtracted the y values.
2. In the equation $y = mx + b$, plug in the calculated value of m and the given values of x and y from one of the two points.
3. Solve for b.
4. Write the equation using the calculated value of m and of b.

❶ Write an equation of the line passing through the points (-1, 6) and (3, 4).



1. $m = \frac{4 - 6}{3 - (-1)} = \frac{-2}{4} = -\frac{1}{2}$
2. $4 = -\frac{1}{2}(3) + b$
3. $\frac{11}{2} = b$
4. $y = -\frac{1}{2}x + \frac{11}{2}$

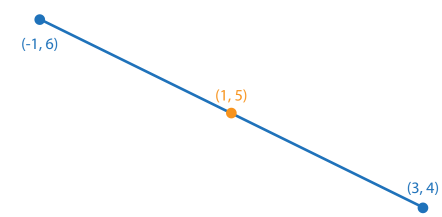
The midpoint of the line segment between two points is the point that averages the two x-coordinates and averages the two y-coordinates: $M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.

The slope of a perpendicular is the negative reciprocal of the original slope: $m_2 = -\frac{1}{m_1}$.

❷ Find the midpoint of a line segment.

1. Identify the coordinates of the two endpoints (x_1, y_1) and (x_2, y_2) of the line segment.
2. Find the average of the x coordinates: $x = \frac{x_1 + x_2}{2}$.
3. Find the average of the y coordinates: $y = \frac{y_1 + y_2}{2}$.
4. The midpoint is the point with the averages of the coordinates (x, y).

❷ Find the midpoint of the line segment shown at right.



1. The endpoints are (-1, 6) and (3, 4).
2. $x = \frac{-1 + 3}{2} = 1$
3. $y = \frac{6 + 4}{2} = 5$
4. The midpoint is (1, 5).

The distance between two points can be found using the Pythagorean theorem $c^2 = a^2 + b^2$: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

❸ Find the distance between two points.

1. Find the horizontal distance between the points, $a = x_2 - x_1$.
2. Find the vertical distance between the points, $b = y_2 - y_1$.
3. Use the Pythagorean theorem. Be sure to put any negative number in parentheses before squaring it.

❸ Find the distance between the points (-1, 6) and (3, 4).

1. $a = 3 - (-1) = 4$
2. $b = 4 - 6 = -2$
3. $AB = \sqrt{4^2 + (-2)^2} = \sqrt{20} \approx 4.5$