Chapter One

Fundamentals of Algebra

Fractions

Expressions

Solving Equations

Properties of Exponents

Addition, Subtraction, and Multiplication of Polynomials

Rewriting Fractions

Objective	Approach	Simple example	Tricky example
Write a whole number as a fraction.	Put "1" in the denominator.	$10 = \frac{10}{1}$	
Reduce a fraction.	Divide the numerator and denominator by the same number. Repeat if possible.	$\frac{22}{60} \div \frac{2}{2} = \frac{11}{30}$	$\frac{24}{60} \div \frac{2}{2} = \frac{12}{30}$ $\frac{12}{30} \div \frac{6}{6} = \frac{2}{5}$
Convert a percentage to a decimal or fraction.	Move the decimal point left two spaces to make it a decimal, or put the percent amount over 100 to make it a fraction.	$2\% = .02$ $2\% = \frac{2}{100}$	$0.2\% = .002$ $0.2\% = \frac{0.2}{100} = \frac{2}{1000}$

Multiplying and Dividing Fractions

Objective	Approach	Simple example	Tricky example
Multiply by a fraction.	Multiply the numerators together, and multiply the denominators together.	$\frac{5}{6} \times \frac{2}{3} = \frac{10}{18}$	$5 \times \frac{2}{3} = \frac{5}{1} \times \frac{2}{3} = \frac{10}{3}$
Divide by a fraction.	Multiply by the reciprocal of the fraction.	$\frac{5}{6} \div \frac{3}{2} = \frac{5}{6} \times \frac{2}{3} = \frac{10}{18}$	$5 \div \frac{3}{2} = \frac{5}{1} \times \frac{2}{3} = \frac{10}{3}$

Adding and Subtracting Fractions

Objective	Approach	Simple example	Tricky example
Add or subtract fractions with the same denominator.	Add or subtract the numerators, but don't change the denominator.	$\frac{4}{9} + \frac{1}{9} = \frac{5}{9}$	
Add or subtract a fraction.	Multiply the numerator and denominator of each fraction by the denominator of the other fraction, and then add or subtract the numerators.	$\frac{4}{9} + \frac{1}{6} =$ $\frac{4}{9} \times \frac{6}{6} + \frac{1}{6} \times \frac{9}{9} =$ $\frac{24}{54} + \frac{9}{54} = \frac{33}{54}$	$3 + \frac{1}{6} =$ $\frac{3}{1} \times \frac{6}{6} + \frac{1}{6} \times \frac{1}{1} =$ $\frac{18}{6} + \frac{1}{6} = \frac{19}{6}$

Parts of Equations

Component	Definition	Comment	$y = 8x - 3 + 2\sqrt{x+9}$
Equation	two expressions set equal to each other	An equation has an equals sign.	$y = 8x - 3 + 2\sqrt{x+9}$
Expression	one or more terms added together	An expression does not have an equals sign.	y $8x - 3 + 2\sqrt{x+9}$
Term	the product of a coefficint (a number) and any number of variable values	An operation on a term applies only once for the whole term.	y $8x$ -3 $2\sqrt{x+9}$
Function	an operation that results in a single value (or no value) for any given input	Most functions are followed by their argument, which must be in parentheses unless it is only one term.	$\sqrt{\ }$
Argument	the value input into a function	An operation on the function does not affect the function's argument.	$x + 9$

Many functions in advanced math and computer languages take more than one argument.

Terms and Degrees of Polynomials

Polynomials with up to three terms have special names.

Type	Number of terms	Example
Monomial	1	$2x^2$
Binomial	2	$2x^2 + 9x$
Trinomial	3	$2x^2 + 9x + 3$

The **degree** of a polynomial in one variable is the highest exponent. A polynomial of degree n is called an n^{th} degree polynomial, although polynomials of low degree are usually referred to by the names below. Note that exponents 1 and 0 are normally not written, such as $9x$ instead of $9x^1$ or 9 instead of $9x^0$.

Type	Degree	Example
Constant	0	9
Linear	1	$9x$
Quadratic	2	$9x^2$
Cubic	3	$9x^3$

A polynomial in a single variable is in **standard form** if its terms are in order from highest degree to lowest.

A term's **coefficient** is the constant multiplier, such as 2 for $2x^3$ or ¾ for $\frac{3x^3}{4}$. The **leading coefficient** of an expression is the coefficient of the highest-degree term.

Scientific Notation

A number in **scientific notation** is of the form $a \times 10^b$, where b is an integer and a is at least 1 but less than 10.

To write a large number in scientific notation, make it smaller by moving the decimal to the left by x spaces, and then increase it back to its actual value by multiplying by 10^x. For numbers less than 1, the decimal will move to the right x spaces, and the exponent will be $-x$ instead of x.

Many calculators use their own notation to express scientific notation, most commonly "*a*E*b*". Change this to actual scientific notation before writing it.

Original	TI-84 notation	Scientific Notation
9051	9.051E3	9.051×10^3
.009051	9.051E-3	9.051×10^{-3}
9051^3	7.414633597E11	7.415×10^{11}

Be careful to notice if a calculator's answer is expressed as scientific notation. Don't be the person who thinks that 9051 to the third power is a little more than seven.

Solving Equations

An **expression** is one or more terms added together. An **equation** is an expression set equal to another.

An **inverse** is the opposite of the original. For example, the inverse of adding 5 is subtracting 5. Equations are solved by applying one or more inverses equally to the expression on each side of the equals sign.

Notation is the written language of math, and is important to do clearly and correctly when solving equations. **The following are required when solving equations in this course.**

Rule	Details
Do not write anything that is not an equation with a variable.	Make sure each equation has a variable. If you do scratchwork that is not an equation with a variable, such as "$30 \times 4 = 120$" or "$+ 8$", do it on scratch paper and not on the paper that will be graded.
Make sure the expressions on each side of an equals sign are equal.	If you are going to do an operation to each side, you cannot write it only on one side. If you have a string of expressions with equals signs between them, all the expressions must be equal.
Neatly write each step directly below the previous step.	Don't do some work on one part of a page and the next step in a different place on the page. Don't use arrows to indicate answers or next steps.
Make sure each symbol has the intended position and size.	Fraction bars are under the whole numerator but not under equals signs or anything else. Square root signs are over the whole radicand and nothing else. Exponents are small and raised.

Common Notation Issues

Equation	Incorrect	Reason	Correct
$2x = 8$	$\frac{2x = 8}{2}$	An equals sign is not part of an expression and cannot be operated on.	$\frac{2x}{2} = \frac{8}{2}$
$x - 5 = 9$	$x - 5 = 9 + 5$	The equation is not true if 5 is added on one side and not the other.	$x - 5 + 5 = 9 + 5$
$x - 5 = 9$	$\underset{+5}{x - 5} = \underset{+5}{9}$	"+ 5" is not part of an equation. (It's ok to show it like above, but not needed.)	$x = 14$
$x^2 = 2/3$	$x = \pm\frac{\sqrt{2}}{3}$	A square root was applied to one side but only to part of the other side.	$x = \pm\sqrt{\frac{2}{3}}$
Al's age is 3 more than half of 8.	$8 \div 2 = 4 + 3 = 7$	$8 \div 2$ does not equal $4 + 3$, and there was no variable.	$A = 8 \div 2 + 3 = 7$
$x^2 = 3$	$x = 3\sqrt{\ }$	The function should be before its argument, not after it.	$x = \sqrt{3}$

Common Solving and Simplifying Issues

To solve an equation, one or more operations must each be done once to each entire side of the equation.

Equation	Incorrect	Correct	Reason
$2x = 10(8x) + 1$	$x = 5(4x) + 1$	$x = 5(8x) + 1$	$10(8x)$ is a single term, so it should only be divided by 2 one time.
$2x = 8 \sin 6x$	$x = 4 \sin 3x$	$x = 4 \sin 6x$	$6x$ is an argument within a term, not a separate term.
$\frac{1}{2}x = \frac{5}{x+4}$	$x = \frac{10}{2x+8}$	$x = \frac{10}{x+4}$	To multiply a fraction by 2, multiply by $\frac{2}{1}$. Multiplying by $\frac{2}{2}$ is multiplying by 1.

To reduce a fraction, the same expression must be divided out of every term one time.

Fraction	Incorrect	Correct	Reason
$\frac{4x+6y}{8x+18z}$	$\frac{1+y}{2+6z}$	$\frac{2x+3y}{4x+9z}$	The same operation must be done to every term, rather than dividing some terms by $2x$ and some terms by 6.
$\frac{4x+6\sqrt{6}}{8x+18z}$	$\frac{2x+3\sqrt{3}}{4x+9z}$	$\frac{2x+3\sqrt{6}}{4x+9z}$	The 6 under the square root sign is an argument, not a separate term, so it should not be divided separately.

Writing Answers

There are many different ways to express a solution that is not a whole number.

Instruction	Description	Solve $12x = 14$
"Round"	Type it into a calculator, and leave a certain number of digits after the decimal point. Increase the last written digit by 1 if the following digit was 5 or higher.	$x \approx 1.14$
"Answer exactly"	Do not use decimals, unless there are only a few digits after the decimal point and you write all of them.	$x = \frac{14}{12}$
"Simplify"	Answer exactly, and reduce fractions, combine like terms, simplify square roots, etc.	$x = \frac{7}{6}$

If there are no instructions to answer in a certain way, then you can choose whichever one you prefer, so long as it makes sense for the problem. Mathematically, it is better not to round, since rounding changes the answer slightly. However, answers to word problems are often best rounded, such as \$0.67 instead of \$⅔.

Rounding

Consideration	Description	Example
Round up when needed.	Add 1 to the last digit of your answer if the digit after it (the first one getting dropped) is 5 or higher.	For $x = 2.485204$, $x \approx 2.49$, not 2.48.
Use the stated level of precision if there is one.	Tenths are the first place after the decimal point, hundredths are the second, and thousandths are the third.	For $x = 2.485204$, nearest tenth: $x \approx 2.5$ nearest hundredth: $x \approx 2.49$ nearest thousandth: $x \approx 2.485$
Match the context of the problem.	Don't round in a way that doesn't make sense for the units or measurements.	Avoid awkward answers like $83.1 or 24.188291 meters, unless you have a specific reason.
Keep the rounding consistent if there are multiple answers with the same units.	Pick a place to round to, and stick with it.	Avoid answers like "The average score was 10.2 for boys and 9.84 for girls."
Don't round to just one significant figure.	Don't round so much that all or all but one of the digits in the answer are 0 or are 9.	Answers such as .002 or .998 can lead to huge rounding errors if used for later calculations.

Properties of Exponents

The properties below form the basis of algebra.

Property	Rule	Example
Power of a Product	$(ab)^x = a^x b^x$	$(2x)^3 = 2^3 x^3 = 8x^3$
Power of a Quotient	$(\frac{a}{b})^x = \frac{a^x}{b^x}$	$(\frac{x}{2})^3 = \frac{x^3}{2^3} = \frac{x^3}{8}$
Power of a Power	$(b^x)^y = b^{xy}$	$(x^5)^3 = x^{15}$
Product of Powers	$b^x b^y = b^{x+y}$	$x^5 x^3 = x^8$
Quotient of Powers	$\frac{b^x}{b^y} = b^{x-y}$	$\frac{x^5}{x^3} = x^2$
Zero Exponent	$b^0 = 1$	$2^0 = 1$
Negative Exponent	$b^{-x} = \frac{1}{b^x}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

These properties are not valid for a negative value to a fractional exponent or zero to the power of zero.

Multiplying Polynomials

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial. Keep in mind that negative times negative is positive.

Example	Work	Result
$4x(x^2 - 5x + 2)$	$4x(x^2) + 4x(-5x) + 4x(2)$	$4x^3 - 20x^2 + 8x$
$-4x(x^2 - 5x + 2)$	$-4x(x^2) + -4x(-5x) + -4x(2)$	$-4x^3 + 20x^2 - 8x$

To multiply a polynomial by a polynomial, multiply each term of one polynomial by each term of the other polynomial, and add the products. Simplify by combining **like terms**, which are terms that are identical except possibly for the coefficients.

Step	Work
Original problem	$(4x + 3)(x^2 - 5x + 2)$
Multiply	$4x(x^2 - 5x + 2) + 3(x^2 - 5x + 2)$
Distribute	$(4x^3 - 20x^2 + 8x) + (3x^2 - 15x + 6)$
Combine like terms	$4x^3 + (-20x^2 + 3x^2) + (8x - 15x) + 6$
Write as a polynomial	$4x^3 - 17x^2 - 7x + 6$

Special Products of Binomials

The conjugate of a binomial $a + b$ is $a - b$.

Multiplying a binomial by itself or by its conjugate is most easily done by following the patterns below.

Multiplier	Pattern	Proof	Example
same	$(a+b)^2 = a^2 + 2ab + b^2$	$(a+b)^2$ $= a(a+b) + b(a+b)$ $= a^2 + ab + ab + b^2$ $= a^2 + 2ab + b^2$	$(3x+10)^2$ $= (3x)^2 + 2(3x(10)) + 10^2$ $= 9x^2 + 60x + 100$
conjugate	$(a+b)(a-b) = a^2 - b^2$	$(a+b)(a-b)$ $= a(a-b) + b(a-b)$ $= a^2 - ab + ab - b^2$ $= a^2 - b^2$	$(3x+10)(3x-10)$ $= (3x)^2 - 10^2$ $= 9x^2 - 100$

$(a+b)^2$ does not equal $a^2 + b^2$, just like $(1+2)^2$ does not equal $1^2 + 2^2$. It is essential to understand this in order to understand algebra in general, and it is probably the single biggest indicator about a person's algebraic readiness.