Chapter One

Arithmetic and Algebra

Fractions and Decimals

Order of Operations and Negatives

Properties of Exponents

Functions and their Graphs

Rewriting Fractions

Objective	Approach	Simple example	Tricky example
Write a whole number as a fraction.	Put "1" in the denominator.	$10 = \frac{10}{1}$	
Reduce a fraction.	Divide the numerator and denominator by the same number. Repeat if possible.	$\frac{22}{60} \div \frac{2}{2} = \frac{11}{30}$	$\frac{24}{60} \div \frac{2}{2} = \frac{12}{30}$ $\frac{12}{30} \div \frac{6}{6} = \frac{2}{5}$
Convert a percentage to a decimal or fraction.	Move the decimal point left two spaces to make it a decimal, or put the percent amount over 100 to make it a fraction.	$3\% = .03$ $3\% = \frac{3}{100}$	$0.3\% = .003$ $0.3\% = \frac{0.3}{100} = \frac{3}{1000}$

Multiplying and Dividing Fractions

Objective	Approach	Simple example	Tricky example
Multiply a fraction.	Multiply the numerators together, and multiply the denominators together.	$\frac{5}{6} \times \frac{2}{3} = \frac{10}{18}$	$\frac{2}{3} \times 5 =$ $\frac{2}{3} \times \frac{5}{1} = \frac{10}{3}$
Divide a fraction.	Multiply by the reciprocal of the second fraction.	$\frac{5}{6} \div \frac{3}{2} =$ $\frac{5}{6} \times \frac{2}{3} = \frac{10}{18}$	$\frac{2}{3} \div 5 =$ $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

Adding and Subtracting Fractions

Objective	Approach	Simple example	Tricky example
Add or subtract fractions with the same denominator.	Add or subtract the numerators, but don't change the denominator.	$\frac{4}{9} + \frac{1}{9} = \frac{5}{9}$	
Add or subtract a fraction.	Multiply the numerator and denominator of each fraction by the denominator of the other fraction, and then add or subtract the numerators.	$\frac{4}{9} + \frac{1}{6} =$ $\frac{4}{9} \times \frac{6}{6} + \frac{1}{6} \times \frac{9}{9} =$ $\frac{24}{54} + \frac{9}{54} = \frac{33}{54}$	$3 + \frac{1}{6} =$ $\frac{3}{1} \times \frac{6}{6} + \frac{1}{6} \times \frac{1}{1} =$ $\frac{18}{6} + \frac{1}{6} = \frac{19}{6}$

Division with Zero

Dividing *A* by *B* means finding how many *B*'s need to be added together to reach *A*. This definition shows how to divide when zero is involved.

Objective	Result	Explanation, based on definition of division
Divide zero by a nonzero.	zero	You're starting at zero, so you don't need to add anything to get to zero.
Divide a nonzero by zero.	undefined	No matter how many zeros you add together, you will never reach any other number.
Divide zero by zero.	undefined	Any number times zero would work to get zero, so there is not one specific answer.

Scientific Notation

A number in **scientific notation** is of the form $a \times 10^b$, where b is an integer and a is at least 1 and less than 10.

To write a large number in scientific notation, make it smaller by moving the decimal to the left by b spaces, and then increase it back to its actual value by multiplying by 10^b. For numbers less than 1, the decimal will move to the right b spaces, and the exponent will be -b instead of b.

Many calculators use their own notation to express scientific notation, most commonly "*a*E*b*". Change this to actual scientific notation before writing it. The *a* part usually will need to be rounded.

Original	TI-84 Notation	Scientific Notation
9051	9.051E3	9.051×10^3
.009051	9.051E-3	9.051×10^{-3}
9051^3	7.414633597E11	7.415×10^{11}

Be careful to notice if a calculator's answer is expressed as scientific notation. Don't be the person who thinks that 9051 to the third power is a little more than seven.

Arithmetic with Negatives

A negative sign (-) means "multiply by negative one." It is generally smaller than a subraction sign (–).

The **sign** of a number is whether it is positive, negative, or zero. "Changing the sign" means switching between positive and negative, that is, multiplying by negative one.

Objective	Equivalent to	Examples
Subtract a negative.	Adding the positive version.	$8 - (-2) = 8 + 2 = 10$ $-8 - (-2) = -8 + 2 = -6$
Multiply or divide by a negative.	Multiplying or dividing by the positive version, and changing the sign.	$8 \times (-2) = -(8 \times 2) = -16$ $-8 \div (-2) = -(-8 \div 2) = 4$

Every time you multiply by a negative, the sign changes. Therefore, and even number of negatives (such as from an exponent) turns out positive.

Example	Sign	Explanation
$(-2)^3$	negative	an odd number of negatives: (-2)(-2)(-2)
$(-2)^4$	positive	an even number of negatives: (-2)(-2)(-2)(-2)
-2^4	negative	only one negative: -(2)(2)(2)(2)
$-(2)^4$	negative	same as -2^4

Order of Operations

Operations within parentheses always happen before other operations.

Some parentheses do not need to be written because of the formatting of the expression, but they still need to be included in the calculations.

Parentheses	Example	Simplified	On Calculator
Written	$2(1+5)$	$2(6)$	`2(1+5)`
Unwritten around numerator or denominator	$\frac{7-1}{9+2}$	$\frac{6}{11}$	`(7-1)/(9+2)`
Unwritten around argument	$2\sqrt{16+9}+1$	$2\sqrt{25}+1$	`2√(16+9)+1`

After parentheses, operations are done in the following order.

Operation	Example: $1+2\times 3^4$
Exponents	$1+2\times 81$
Multiplication and division	$1+162$
Addition and subtraction	163

The above order (parentheses, exponents, multiplication and division, addition and subtraction) is often referred to by its acronym, **PEMDAS**.

Properties of Exponents

The properties below form the basis of algebra.

Property	Rule	Example
Power of a Product	$(ab)^x = a^x b^x$	$(2x)^3 = 2^3 x^3 = 8x^3$
Power of a Quotient	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	$\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3} = \frac{x^3}{8}$
Power of a Power	$(b^x)^y = b^{xy}$	$(x^5)^3 = x^{15}$
Product of Powers	$b^x b^y = b^{x+y}$	$x^5 x^3 = x^8$
Quotient of Powers	$\frac{b^x}{b^y} = b^{x-y}$	$\frac{x^5}{x^3} = x^2$
Zero Exponent	$b^0 = 1$	$2^0 = 1$
Negative Exponent	$b^{-x} = \frac{1}{b^x}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

These properties are not valid for a negative value to a fractional exponent or zero to the power of zero.

Parts of Equations

Component	Definition	Comment	$y = 8x - 3 + 2\sqrt{x+9}$
Equation	two expressions set equal to each other	An equation has an equals sign.	$y = 8x - 3 + 2\sqrt{x+9}$
Expression	one or more terms added together	An expression does not have an equals sign.	y, $8x - 3 + 2\sqrt{x+9}$
Term	the product of one or more constant or variable values	The sum of all the terms of an expression is the expression itself.	y, $8x$, -3, $2\sqrt{x+9}$
Function	an operation that results in a single value for any valid input	Functions are normally followed by their argument	$\sqrt{}$
Argument	the value input into a function	An operation on the function does not affect the function's argument.	$x + 9$

Many functions in advanced math and computer languages take more than one argument.

Function Notation

***f*(*x*)** is function notation and is read "*f* of *x*". It means that whatever *x* is will be the input of the function. Despite the parentheses, it does not mean multiplication.

Notation	Meaning
f	the name of the function, which can be any letter
x	the input of the function in general, called its argument
***f*(*x*)**	the output of the function in general, which is *y* on a graph
***f*(5)**	the specific value of the function when the argument is 5

For example, for the function $g(x) = x^2 + 4x - 1$, $g(3) = 3^2 + 4(3) - 1 = 20$.

Graphing Points from Function Notation

The closer an *x* value is to zero, the closer the point is to the *y*-axis, and vice versa.

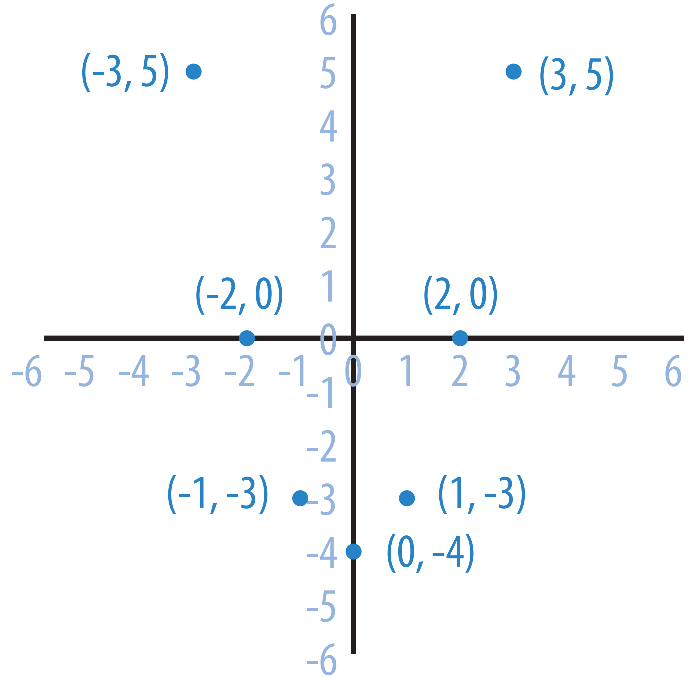
Coordinate	Negative	Zero	Positive
x	left of *y*-axis	on *y*-axis	right of *y*-axis
y	below *x*-axis	on *x*-axis	above *x*-axis

A point on a graph is represented as (x, y). The horizontal coordinate *x* is before the vertical coordinate *y*.

Points on the graph of *f* can be found by calculating values of $f(x)$ for values of *x*. The $f(x)$ value for a particular *x* value is the *y* value for that (x, y) point.

For example, $f(x) = x^2 - 4$:

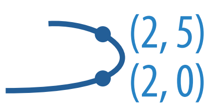
x	*f*(*x*)
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5



Definition of a Function

A **relation** is a set of paired values. Each pair has one input value (often called *x*) and one output value (often called *y*).

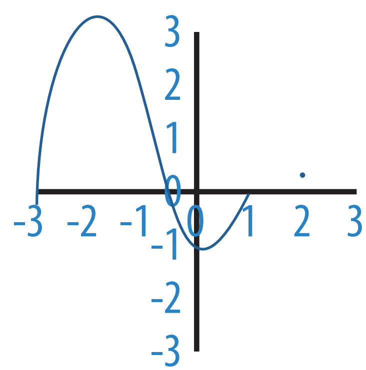
A relation is a **function** if every input value has exactly one corresponding output value.

Relation Example	Function?	Explanation
$\{(1, 1), (2, 1), (5, 9)\}$	yes	Each possible input (1, 2, and 5) has one specific output.
$\{(1, 1), (2, 1), (2, 9)\}$	no	An input of 2 could have an output of either 1 or 9.
$a(x) = x + 2$	yes	No matter what *x* is, there is exactly one resulting value when 2 is added.
$b(x) = \pm\sqrt{x}$	no	When *x* is 9, the result is both 3 and -3.
c(*x*) = current height of person *x*	yes	Every person has one current height.
d(*x*) = country visited by person *x*	no	For example, Marcus visited both Spain and Italy.
	no	For example, when *x* is 2, *y* is both 0 and 5.

Domain

The **domain** of a function is the set of all possible inputs. In basic contexts, this means all values of *x* that have a *y* value on a graph and don't cause an error in an expression.

Basic mathematical functions have a domain of all real numbers. Common exceptions are shown below.

Scenario	Limitation	Example	Domain of Example
Polynomial	none	$a(x) = 4x^2 + x - 9$	all real numbers
Fraction	denominator cannot be zero	$b(x) = \dfrac{x-5}{x-3}$	all real numbers except $x = 3$
Square Root	argument cannot be negative	$c(x) = \sqrt{x-3}$	all real numbers greater than or equal to 3
Graph	*x* must be part of graph		2 and all real numbers from -3 to 1