

CHAPTER ONE: ARITHMETIC AND ALGEBRA

Thorough understanding and fluency of the concepts and methods of arithmetic and algebra, covered in this chapter and the next, is a cornerstone to success in this course and most future math courses as well. This chapter focuses on the fundamental concepts of numbers (including fractions, decimals, and negatives), exponents, and functions.

1-A Fractions and Decimals

scientific notation

- ➊ Reduce a fraction.
- ➋ Multiply or divide a fraction.
- ➌ Add or subtract fractions with the same denominator.
- ➍ Add or subtract fractions with different denominators.
- ➎ Divide zero or by zero.
- ➏ Convert a percentage to a decimal.
- ➐ Convert calculator notation to scientific notation and to standard notation.

1-B Order of Operations and Negatives

sign

- ➊ Subtract, multiply, or divide by a negative.
- ➋ Apply order of operations for negatives and parentheses.
- ➌ Apply order of operations for basic arithmetic and powers.
- ➍ Use a calculator to evaluate a fraction.

1-C Properties of Exponents

- ➊ Simplify an expression using properties of exponents.
- ➋ Rewrite an expression without negative exponents.

1-D Functions and their Graphs

x -axis • y -axis • origin • coordinates • domain • range • argument • function

- ➊ Estimate coordinates on a graph.
- ➋ Use function notation.
- ➌ Sketch a function by plotting points.
- ➍ Identify the domain and range of a graphed function.
- ➎ Read expressions in function notation, and identify the arguments.
- ➏ Identify the domain of a function in function notation.
- ➐ Identify whether or not a relation is a function.
- ➑ Identify whether or not a graph represents a function.

1-A Fractions and Decimals

1 Reduce a fraction.

1. Divide the numerator and denominator by a number that divides evenly into both.
2. Repeat step 1 until no number divides evenly into both the numerator and the denominator.

1 Reduce $\frac{140}{350}$.

1. 10 divides evenly into 140 and into 350. $140 \div 10 = 14$, and $350 \div 10 = 35$. $\frac{140}{350} = \frac{14}{35}$
2. 7 divides evenly into 14 and into 35. $14 \div 7 = 2$, and $35 \div 7 = 5$. $\frac{14}{35} = \frac{2}{5}$

It also would have worked to divide 140 and 350 both by 70 in step 1 instead of doing two separate steps.

2 Multiply or divide a fraction.

1. If multiplying or dividing by a whole number n , rewrite the number as $\frac{n}{1}$.
2. If dividing, change the divisor (the second number) to its reciprocal. The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$, and the reciprocal of the whole number a is $\frac{1}{a}$.
3. Multiply the numerators together.
4. Multiply the denominators together.

2 a) $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$

b) $\frac{2}{5} \times 3 = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$

c) $\frac{2}{5} \div \frac{4}{3} = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$

d) $\frac{2}{5} \div 3 = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

3 Add or subtract fractions with the same denominator.

1. Add or subtract the numerators.
2. Keep the same denominator.

3 $\frac{12}{20} - \frac{5}{20} = \frac{7}{20}$

4 Add or subtract fractions with different denominators.

1. Multiply the first fraction by $\frac{b}{b}$, where b is the denominator of the second fraction.
2. Multiply the second fraction by $\frac{a}{a}$, where a is the denominator of the first fraction.
3. Add or subtract (see 1).

4 $\frac{3}{5} - \frac{1}{4}$

1. $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$

2. $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$

3. $\frac{12}{20} - \frac{5}{20} = \frac{7}{20}$

It is impossible to divide by zero. Attempting to do so yields a result of “undefined”. Zero divided by any other number is zero.

5 Divide zero or by zero.

1. Anything divided by zero is undefined (there is no answer).
2. Zero divided by anything other than itself is equal to zero.

5 a) $\frac{2}{0}$ is undefined

b) $\frac{0}{2} = 0$

c) $\frac{0}{0}$ is undefined

$x\%$ means $\frac{x}{100}$. 100% is the same as $\frac{100}{100}$ or 1.

6 Convert a percentage to a decimal.

1. Move the decimal point two places to the left, and remove the % symbol.

6 a) 9% = .09

b) .9% = .009

SCIENTIFIC NOTATION is $a \times 10^b$, where $1 \leq a < 10$ and b is an integer.

Many calculators use the notation $a\text{E}b$ instead of $a \times 10^b$ to display scientific notation. Do not write numbers in calculator notation.

7 Convert calculator notation to scientific notation and to standard notation.

1. To convert $a\text{E}b$ to scientific notation, change “E” into “ $\times 10$ ”, and make b an exponent.
2. To convert $a \times 10^b$ to standard notation, move the decimal point right b spaces (which will be left if b is negative). Fill in 0's as needed.

7 a) 2.57E3

b) 2.57E-3

1. 2.57×10^3

2.57×10^{-3}

2. 2570

.00257

1-B Order of Operations and Negatives

The SIGN of a real number is one of three things: positive, negative, or zero.

“Changing the sign” of a number generally refers to making a positive number negative or vice versa.

Multiplying or dividing by a negative number changes the sign of the original number.

1 Subtract, multiply, or divide by a negative.

1. Multiplying or dividing by a negative number changes the sign.
2. Subtracting a negative number is the same as adding the positive version of the number.

$$\text{1 a) } 20 \times (-10) = -200 \quad \text{b) } 20 \div (-10) = -2 \quad \text{c) } -20 \div (-10) = 2 \quad \text{d) } 20 - (-10) = 30$$

2 Apply order of operations for negatives and parentheses.

1. A negative number to an odd power stays negative.
2. A negative number to an even power becomes positive.
3. If a negative sign is not in parentheses, there is only one of them. This means multiply a positive number by negative one: $-5^n = -1(5^n)$, which is negative and not the same as $(-5)^n$, which is positive.

$$\text{2 a) } -2^3 = -8 \quad \text{b) } (-2)^3 = -8 \quad \text{c) } -2^4 = -16 \quad \text{d) } (-2)^4 = 16$$

When evaluating an expression with no parentheses, exponents are done first, followed by multiplication and division, and then addition and subtraction.

When evaluating an expression with parentheses in it, each expression within parentheses is evaluated on its own first, following the order above, before the overall expression is evaluated.

Numerators and denominators always have parentheses around them, even if they are not written. For example, $\frac{1+3}{3+5}$ is actually $\frac{(1+3)}{(3+5)}$. These parentheses only have to be shown if the numerator is not written above the denominator. For example, on a calculator, $\frac{1+3}{3+5}$ can be typed as $(1+3)/(3+5)$, but not as $1+3/3+5$.

3 Apply order of operations for basic arithmetic and powers.

1. Apply the steps below to everything within parentheses, including hidden parentheses.
2. Calculate exponents.
3. Calculate multiplication and division, including multiplying by -1 for negative signs.
4. Calculate addition and subtraction.

$$\begin{array}{lll} \text{3 a) } 4 + 5 \times 2 & \text{b) } 4 + 5(2)^3 & \text{c) } -5(-2)^3 + 3 \\ 4 + 10 & 4 + 5(8) & -5(-8) + 3 \\ 14 & 4 + 40 & 40 + 3 \\ & 44 & 43 \end{array}$$

4 Use a calculator to evaluate a fraction.

1. Put parentheses around the numerator, and put parentheses around the denominator.
2. Close any parentheses that are opened, such as for the argument of a square root function.
3. Use the minus button – for subtraction and the negative button (-) for a negative number.

$$\text{4 } \frac{-8 + 2\sqrt{100}}{\sqrt{49} - 5}$$

$$(-8 + 2(\sqrt{(100)})) / (\sqrt{(49)} - 5) = 6$$

1-C Properties of Exponents

The rules below are valid in almost all contexts, including any time a and b are positive or x and y are integers.

Property

Rule

Example

Power of a Product

$$(ab)^x = a^x b^x$$

$$(2x)^3 = 2^3 x^3 = 8x^3$$

Power of a Quotient

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3} = \frac{x^3}{8}$$

Power of a Power

$$(b^x)^y = b^{xy}$$

$$(x^5)^3 = x^{15}$$

Product of Powers

$$b^x b^y = b^{x+y}$$

$$x^5 x^3 = x^8$$

Quotient of Powers

$$\frac{b^x}{b^y} = b^{x-y}$$

$$\frac{x^5}{x^3} = x^2$$

Zero Exponent

$$b^0 = 1$$

$$2^0 = 1$$

Negative Exponent

$$b^{-x} = \frac{1}{b^x}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

1 Simplify an expression using properties of exponents.

1. Use the properties above as needed.

2. Reduce fractions if needed.

1 Simplify.

a) $a^6 a^3 = a^9$

b) $(a^6)^3 = a^{18}$

c) $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

d) $(ab)^6 = a^6 b^6$

c) $a^0 = 1$

d) $\frac{a^6}{a^2} = a^4$

g) $\frac{a^2}{a^6} = \frac{1}{a^4}$

h) $a^{-3} = \frac{1}{a^3}$

e) $ab^3(a^2b)^5$

f) $\left(\frac{5a}{2}\right)^3$

k) $\left(\frac{5a}{2}\right)^{-3}$

l) $10\left(\frac{5a}{2}\right)^{-3}$

$$ab^3(a^{10}b^5)$$

$$\frac{5^3 a^3}{2^3}$$

$$\left(\frac{2}{5a}\right)^3$$

$$10\left(\frac{8}{125a^3}\right)$$

$$a^{11}b^8$$

$$\frac{125a^3}{8}$$

$$\frac{8}{125a^3}$$

$$\frac{80}{125a^3}$$

1 Simplify $\frac{(2a^4b)^3 b^6}{a^{12} b^2 c^2}$, and write it without a fraction. State each property of exponents used.

$$(2a^4b)^3 = 2^3(a^4)^3 b^3$$

Power of a Product

$$\frac{8(a^4)^3 b^3 b^6}{a^{12} b^2 c^2}$$

$$(a^4)^3 = a^{12}$$

Power of a Power

$$\frac{8a^{12} b^9 b^6}{a^{12} b^2 c^2}$$

$$b^3 b^6 = b^9$$

Product of Powers

$$\frac{8a^{12} b^9}{a^{12} b^2 c^2}$$

$$\frac{a^{12} b^9}{a^{12} b^2} = a^0 b^7$$

Quotient of Powers

$$\frac{8a^0 b^7}{c^2}$$

$$a^0 = 1$$

Zero Exponent

$$\frac{8(1) b^7}{c^2}$$

$$\frac{1}{c^2} = c^{-2}$$

Negative Exponent

$$8b^7 c^{-2}$$

A factor to the power of -1 is the reciprocal of that factor. This means moving the numerator to the denominator and vice versa.

2 Rewrite an expression without negative exponents.

1. If a factor in the numerator has a negative exponent, move the factor to the denominator and change the exponent to positive.

2. If a factor in the denominator has a negative exponent, move the factor to the numerator and change the exponent to positive.

3. If a fraction has a negative exponent, take the reciprocal of the fraction (switch the numerator with the denominator), and change the exponent to positive.

Make sure not to move any factors that do not have a negative exponent, such as the 4 in $4x^{-1}$.

2 Rewrite with no parentheses or negative exponents.

a) $\frac{a^{-2}b}{c^3d}$

b) $\frac{a^2b}{c^{-3}d}$

c) $\frac{ab^{-2}}{c^3d}$

d) $\frac{(ab)^{-2}}{c^3d}$

$$\frac{b}{a^2 c^3 d}$$

$$\frac{a^2 c^3 b}{d}$$

$$\frac{a}{b^2 c^3 d}$$

$$\frac{1}{a^2 b^2 c^3 d}$$

e) $\left(\frac{a^2b}{c^3d}\right)^{-2}$

f) $\left(\frac{a^2b}{c^3d}\right)^{-2}$

g) $(2a)^{-1}$

h) $2a^{-1}$

$$\left(\frac{c^3d}{a^2b}\right)^2$$

$$\left(\frac{c^3d}{a^2b}\right)^2$$

$$\frac{1}{2a}$$

$$\frac{2}{a}$$

$$\frac{c^6 d^2}{a^4 b^2}$$

$$\left(\frac{a^4 c^6 d^2}{b^2}\right)^2$$

$$\frac{1}{2a}$$

$$\frac{2}{a}$$

1-D Functions and their Graphs

Thursday • 8/18

The X-AXIS is the horizontal line where y is zero. It is the same as the number line.

The Y-AXIS is the vertical line where x is zero.

The ORIGIN is where the x -axis and y -axis meet, which is the point $(0, 0)$.

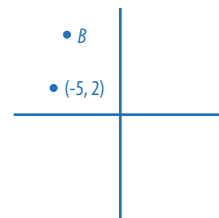
The x and y values of a point on a graph are the point's COORDINATES and are listed alphabetically in parentheses: (x, y) .

1 Estimate coordinates on a graph.

1. Negative x values are left of the y -axis, and positive x -values are right of the y -axis.
2. Negative y -values are below the x -axis, and positive y -values are above the x -axis.
3. The closer a coordinate is to zero, the closer the point is to the origin.

1 Estimate the coordinates of point B at right.

1. It is left of the y -axis, so x is negative.
2. It is above the x -axis, so y is positive.
3. x is approximately -5 but a little closer to zero, and y is much higher than 2.
 $(-4, 6)$ is a good estimate.



Functions are written in the form $f(x)$, where x is the independent variable (the input) and $f(x)$ is the dependent variable (the output). Letters other than f can be used as well, especially to distinguish between different functions. Variables other than x can be used as well.

A formula for $f(x)$ is given in terms of x , such as $f(x) = 2x$. Values of $f(x)$ can be found by using different values of x in the formula.

In addition to numbers, algebraic expressions can be used in functions.

2 Use function notation.

1. Substitute a value of x (in parentheses) for each x in the formula.
2. Simplify.

2 Given $f(x) = 5x$ and $g(x) = x^2 + 2x - 10$, find the following.

- a) $f(4) = 5(4) = 20$ b) $g(4) = (4)^2 + 2(4) - 10 = 14$ c) $f(x+2) = 5(x+2) = 5x + 10$

On a graph, $f(x)$ represents y . A graph can be sketched by calculating the value of $f(x)$ for each of several values of x and connecting the points.

3 Sketch a function by plotting points.

1. Choose a value of x and calculate the value of $f(x)$ for this x value.
2. Plot the point $(x, f(x))$ on the graph.
3. Repeat steps 1 and 2 until the shape of the graph starts taking place. For best results, make sure to include the highest and lowest points on the graph (if there are any) and points on each side of them.
4. Connect the points in a curve.

3 $f(x) = x^2 - 2x - 3$

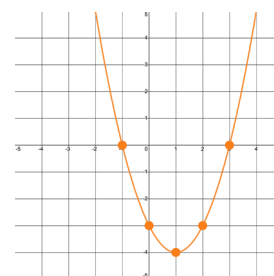
$$f(0) = (0)^2 - 2(0) - 3 = -3$$

$$f(1) = (1)^2 - 2(1) - 3 = -4$$

$$f(2) = (2)^2 - 2(2) - 3 = -3$$

$$f(3) = (3)^2 - 2(3) - 3 = 0$$

$$f(-1) = (-1)^2 - 2(-1) - 3 = 0$$



The set of all possible values that can be used as input for a function is called the DOMAIN of the function. On a graph, the domain is the set of all x -values.

The set of all possible values that could be an output of a function is called the RANGE of the function. On a graph, the range is the set of all y -values.

4 Identify the domain and range of a graphed function.

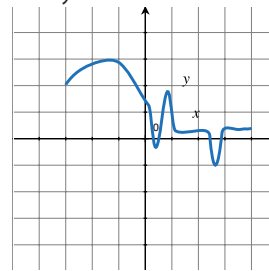
1. The domain is all x values the graph reaches.

2. The range is all y values the graph reaches.

4 Identify the domain and range of the function graphed at right.

1. The domain is approximately $-3 \leq x \leq 4$.

2. The range is approximately $-1 \leq y \leq 3$.



An ARGUMENT of a Function is an expression input into the function.

Arguments are in parentheses, although in some cases the parentheses do not need to be written, such as in a square root function. These parentheses mean “of”, but they do not indicate multiplication.

5 Read expressions in function notation, and identify the arguments.

1. The arguments are the expressions inside the functions, and are usually in parentheses.

2. Read it as *function “of” argument*, not as multiplication.

5 Read the notation $f(x) = \sqrt{x+1}$, and identify the arguments.

x is the argument of function f , and $x+1$ is the argument of the square root function.

f of x equals the square root of $x+1$.

Many functions can have any real number as their arguments, but some functions result in error for certain inputs. Numbers that would cause an error in a given function are not part of the domain of that function. In Math 2, this only occurs when dividing by zero or taking the square root of a negative.

Some functions take nonnumerical arguments.

6 Identify the domain of a function in function notation.

1. If the function takes numerical input, the domain is all real numbers except:

• If there is a denominator D , the domain is limited by $D \neq 0$.

• If there is a radical \sqrt{R} , the domain is limited by $R \geq 0$.

2. If the function is not numerical, the domain is all arguments that make sense in the context of the problem.

6 a) $a(x) = 6x^2 - x + 4$

x can be any real number

b) $b(x) = \frac{x+8}{2x-6}$

$2x - 6 \neq 0$

$x \neq 3$

c) $c(x) = 4\sqrt{2x-6}$

$2x - 6 \geq 0$

$x \geq 3$

d) $d(x) = x$'s student ID number

x can be any student

The definition of a FUNCTION f is a relation such that $f(x)$ is equal to one specific value (if anything) for any single value of x . By definition, it is impossible to get two different outputs for one function input. If a relation has more than one output for any one input, it is not a function.

7 Identify whether or not a relation is a function.

1. It is a function unless there exists a possible input that results in more than one output. In other words, in a function, there is only one answer to any question.

7 State whether or not the following relations are functions. For ones that are not functions, write an x value and two different y values for that x value.

a) $y = 5x^2 + 9x - 10$

function

b) $y = \pm\sqrt{x}$

not a function: $y = 3$ or -3 for $x = 9$

c) $y = \text{age of person } x$

function

d) $y = \text{brother of person } x$

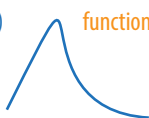
not a function: $y = \text{Jon or Ed for } x = \text{Mia}$

The graph of a function cannot have two different points with the same y value, since that would mean there are two different values of $f(x)$ for that value of x .

8 Identify whether or not a graph represents a function.

1. It is a function unless there is a point on the graph that is directly above another point on the graph.

8 a) function



b) not a function

