Test 4 [61 marks]

1. [Maximum mark: 7] SPM.1.AHL.TZ0.8

The plane *Π* has the Cartesian equation $2x + y + 2z = 3$

The line *L* has the vector equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ p \end{pmatrix}$, $\mu, p \in \mathbb{R}$. The acute angle between the line *L* and the plane *Π* is 30°.

Find the possible values of p. [7]

Markscheme

recognition that the angle between the normal and the line is 60° (seen anywhere) ***R1***

attempt to use the formula for the scalar product ***M1***

$\cos 60° = \frac{\left|\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -2 \\ p \end{pmatrix}\right|}{\sqrt{9} \times \sqrt{1+4+p^2}}$ ***A1***

$\frac{1}{2} = \frac{|2p|}{3\sqrt{5+p^2}}$ ***A1***

$3\sqrt{5+p^2} = 4|p|$

attempt to square both sides ***M1***

$9(5+p^2) = 16p^2 \Rightarrow 7p^2 = 45$

$p = \pm 3\sqrt{\frac{5}{7}}$ (or equivalent) ***A1A1***

[7 marks]

2. [Maximum mark: 8] 25M.2.AHL.TZ2.9

A line L_1 has vector equation $\boldsymbol{r} = \begin{pmatrix} 0 \\ 0 \\ \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ \end{pmatrix}$ where $t \in \mathbb{R}$

A line L_1 has vector equation $\mathbf{r} = \begin{pmatrix} \\ 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} \\ 0 \\ 1 \end{pmatrix}$ where $t \in \mathbb{R}$.

The plane Π_1 contains the line L_1 and passes through the point $(2,\ 1,\ 5)$.

(a) Show that the Cartesian equation of the plane Π_1 is $x + y - z = -2$. [4]

Markscheme

METHOD 1

correct direction vector, **e.g.** $\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ (or equivalent) ***A1***

attempt to find direction normal using vector product ***(M1)***

$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ (or equivalent) ***A1***

correct substitution of a point on the line to obtain d ***A1***

$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = d \Rightarrow d = -2$ **OR** $\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = d \Rightarrow d = -2$

OR $\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = d \Rightarrow d = 2$ (or equivalent)

hence equation of the plane Π_1 is $x + y - z = -2$ ***AG***

METHOD 2

attempt to form parametric equations of a plane through point $(0,\ 0,\ 2)$ and two vectors in the plane $\begin{pmatrix}1\\0\\1\end{pmatrix}$ and $\begin{pmatrix}2\\1\\3\end{pmatrix}$ ***M1***

$$\underset{\sim}{r} = \begin{pmatrix}0\\0\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\1\end{pmatrix} + \mu\begin{pmatrix}2\\1\\3\end{pmatrix}$$ ***A1***

$x = \lambda + 2\mu$

$y = \mu$

$z = 2 + \lambda + 3\mu$

$\lambda = x - 2y$

$z = 2 + \lambda + 3y$ ***A1***

$z = 2 + x - 2y + 3y$ ***A1***

$x + y - z = -2$ ***AG***

[4 marks]

Examiners report

Part (a): This part was generally well attempted. Most candidates correctly used the vector product of two vectors in the plane to find the normal vector, followed by calculating the value of d to complete the plane equation. However, a few candidates mistakenly attempted to verify the plane equation rather than derive it, which did not earn marks.

Part (b): Two common methods were observed: eliminating one variable and row reduction. Despite these valid approaches, only a small percentage of candidates successfully obtained both correct values, indicating challenges with solving systems of equations accurately.

Consider the three planes

$\Pi_1 : x + y - z = -2$

$\Pi_2 : 2x + by - z = 3$

$\Pi_3 : x - y + 2z = d$

where $b,\ d \in \mathbb{Q}^+$.

The three planes intersect in a line.

(b) Find the value of b and the value of d. [4]

Markscheme

METHOD 1

attempt to form an equation to eliminate one of the variables ***M1***

EITHER

$(b-2)y + z = 7$ ***A1***

$2y - 3z = -2 - d$ ***A1***

OR

$x + (b-1)y = 5$ ***A1***

$3x + y = d - 4$ ***A1***

THEN

$(3b-4)y = 19-d$

$y = \frac{19-d}{3b-4}$

$b = \frac{4}{3}$ and $d = 19$ ***A1***

Note: Award ***M1A1*** for $b = \frac{4}{3}$ seen anywhere

METHOD 2

$$\left.\begin{matrix} 1 & 1 & -1 \\ 2 & b & -1 \\ 1 & -1 & 2 \end{matrix}\right| \begin{matrix} -2 \\ 3 \\ d \end{matrix}$$

attempt to use row reduction ***M1***

$$\left.\begin{matrix} 1 & 1 & -1 \\ 0 & b-2 & 1 \\ 0 & -2 & 3 \end{matrix}\right| \begin{matrix} -2 \\ 7 \\ d+2 \end{matrix}$$ or equivalent

$$\left.\begin{matrix} 1 & 1 & -1 \\ 0 & b-2 & 1 \\ 0 & 0 & 3+\frac{2}{b-2} \end{matrix}\right| \begin{matrix} -2 \\ 7 \\ d+2+\frac{14}{b-2} \end{matrix}$$ or equivalent ***(A1)***

$3+\frac{2}{b-2} = 0$ and $d+2+\frac{14}{\frac{4}{3}-2} = 0$

$b = \frac{4}{3}$ and $d = 19$ ***A1***

Note: Award ***M1A1*** for $b = \frac{4}{3}$ seen anywhere

METHOD 3

attempt to use determinant and equate it to zero ***(M1)***

$$\left.\begin{matrix} 1 & 1 & -1 \\ 0 & b & -1 \\ 1 & -1 & 2 \end{matrix}\right| \begin{matrix} -2 \\ 3 \\ d \end{matrix}$$

$$\det\left(\begin{bmatrix} 1 & 1 & -1 \\ 2 & b & -1 \\ 1 & -1 & 2 \end{bmatrix}\right) = 1 \times \begin{vmatrix} b & -1 \\ -1 & 2 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + (-1) \times \begin{vmatrix} 2 & b \\ 1 & -1 \end{vmatrix}$$

$\det = 3b - 4$

$3b - 4 = 0$

$b = \frac{4}{3}$ ***A1***

$$\det\left(\begin{bmatrix} 1 & -1 & -2 \\ 2 & -1 & 3 \\ 1 & 2 & d \end{bmatrix}\right) = 0$$ ***(A1)***

$d = 19$ ***A1***

Note: Award ***(M1)A1*** for $b = \frac{4}{3}$ seen anywhere

[4 marks]

3. [Maximum mark: 5] 25M.1.AHL.TZ2.2

The line L_1 is defined by the Cartesian equation $\frac{x-1}{2} = \frac{y+2}{3} = z$.

(a) Find a vector equation of L_1. [2]

Markscheme

attempt to set equal to a parameter or to add detail to cartesian form ***(M1)***

$\frac{x-1}{2} = \frac{y+2}{3} = z = \lambda \Rightarrow x = 2\lambda + 1,\ y = 3\lambda - 2,\ z = \lambda$ **OR** $\frac{x-1}{2} = \frac{y-(-2)}{3} = \frac{z-0}{1}$

$\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ **OR** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ (or equivalent)

A1

Note: Award ***A0*** if $\boldsymbol{r} =$ **OR** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ is omitted.

[2 marks]

Examiners report

This question was generally well attempted. In part (a), a significant number of candidates lost the final accuracy mark due to incorrect use of notation, such as writing $L_1 =$ instead of the required $\boldsymbol{r} =$. The vector intersection in part (b) was also well attempted, with many completely correct answers seen.

A second line L_2 is defined by the vector equation $\boldsymbol{r} = \begin{pmatrix} 0 \\ 4 \\ -8 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, where $t \in \mathbb{R}$.

(b) Find the coordinates of the point where L_1 and L_2 intersect. [3]

Markscheme

attempt to equate at least one component ***(M1)***

$1+2\lambda=t$ **OR** $-2+3\lambda=4$ **OR** $\lambda=-8+2t$

$\lambda=2$ **OR** $t=5$ ***(A1)***

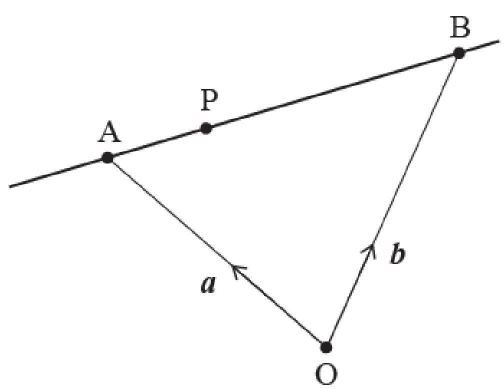
intersection point $=(5,4,2)$ ***A1***

Note: Condone coordinates given in vector form.

[3 marks]

4. [Maximum mark: 8] 24N.1.AHL.TZ0.8

The following diagram shows two points A and B such that $\overrightarrow{\mathrm{OA}}=\boldsymbol{a},\ \overrightarrow{\mathrm{OB}}=\boldsymbol{b}$.



The point P lies on (AB) so that $\overrightarrow{\mathrm{AP}}=\lambda\overrightarrow{\mathrm{AB}}$ where $0<\lambda<1$.

(a) Show that $\overrightarrow{\mathrm{OP}}=(1-\lambda)\boldsymbol{a}+\lambda\boldsymbol{b}$. [1]

Markscheme

$\overrightarrow{\mathrm{OP}}=\boldsymbol{a}+\lambda(\boldsymbol{b}-\boldsymbol{a})$ ***A1***

$= (1-\lambda)\boldsymbol{a} + \lambda\boldsymbol{b}$ ***AG***

[1 mark]

Examiners report

Many correct responses to part (a) were seen.

Part (b) proved to be a good discriminator, with the better students reaching $\lambda = \frac{1}{6}$ through some carefully presented work. Slips were seen from some students who were able to 'multiple out' their scalar product correctly, but went on to make errors when collecting terms. A number of students often scored a maximum of one mark in this question part, usually by just appreciating $\overrightarrow{\mathrm{OP}}.\overrightarrow{\mathrm{AB}} = 0$ or recognising that $\cos \mathrm{A\hat{O}B} = \frac{1}{8}$.

It is given that $|\boldsymbol{a}| = 1,\ \ |\boldsymbol{b}| = 2$ and $\boldsymbol{a} \cdot \boldsymbol{b} = \frac{1}{4}$.

(b) In the case that $\overrightarrow{\mathrm{OP}}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$, find the value of λ. [7]

Markscheme

METHOD 1

recognition that $\overrightarrow{\mathrm{OP}} \cdot \overrightarrow{\mathrm{AB}} = 0$ (may be seen anywhere) ***(M1)***

$[(1-\lambda)\boldsymbol{a} + \lambda\boldsymbol{b}] \cdot [\boldsymbol{b} - \boldsymbol{a}] \quad (=0)$ ***A1***

attempt to multiply out scalar product ***M1***

$(1-\lambda)\boldsymbol{a} \cdot \boldsymbol{b} + \lambda\boldsymbol{b} \cdot \boldsymbol{b} - (1-\lambda)\boldsymbol{a} \cdot \boldsymbol{a} - \lambda\boldsymbol{b} \cdot \boldsymbol{a} \quad (=0)$ ***(A1)***

attempt to substitute for $\boldsymbol{a} \cdot \boldsymbol{b}$ and $|\boldsymbol{a}|$ and $|\boldsymbol{b}|$ ***(M1)***

$\frac{1}{4}(1-\lambda) + 4\lambda - (1-\lambda) - \frac{\lambda}{4} \quad (=0)$ ***(A1)***

$1 - \lambda + 16\lambda - 4 + 4\lambda - \lambda = 0$

$18\lambda - 3 = 0$

$\lambda = \frac{1}{6}$ ***A1***

METHOD 2

$\cos AOB = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} = \frac{1}{8}$ ***A1***

attempt to use cosine rule to find AB ***M1***

$|AB|^2 = 1^2 + 2^2 - 2(1)(2)\left(\frac{1}{8}\right)$

$AB = \frac{3\sqrt{2}}{2}$ ***A1***

attempt to apply Pythagoras'Theorem twice: ***M1***

$|OP|^2 + \left(\frac{3\sqrt{2}}{2}\lambda\right)^2 = 1$ and

$|OP|^2 + \left(\frac{3\sqrt{2}}{2}(1-\lambda)\right)^2 = 4$ ***A1***

attempt to solve simultaneously: ***M1***

$\frac{9}{2}(1-\lambda)^2 - \frac{9}{2}\lambda^2 = 3$

$\lambda = \frac{1}{6}$ ***A1***

[7 marks]

5. [Maximum mark: 5] 25M.2.AHL.TZ1.7

At $09:00$ a helicopter is located at a point $(10,\ 3,\ 0.5)$ relative to a point O on horizontal ground. The x-direction is due east, the y-direction is due north and the z-direction is vertically upwards.

All distances are measured in kilometres.

The helicopter is flying at a constant height.

The helicopter's position relative to the point O is given by

$\boldsymbol{r} = \begin{pmatrix} 10 \\ 3 \\ 0.5 \end{pmatrix} + 4t \begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix}$, where t represents the time in hours since $09:00$.

(a) Find the speed of the helicopter. [2]

Markscheme

recognition that the speed is the magnitude of the velocity *(M1)*

speed $= 4\sqrt{10^2 + (-25)^2}$

$= 107.703\ldots$

$= 108 \ \left(= 20\sqrt{29}\right)$ (km/h) ***A1***

[2 marks]

Examiners report

Most students knew that they had to find the magnitude of the velocity vector to find the speed required in part (a), but many did not read the question carefully and omitted the factor of 4.

Part (b) proved to be very challenging, with very few students able to interpret motion in 3 dimensions successfully. Many started by finding the position of the helicopter at $10:00$, and then tried to this new position vector to find the angle of descent rather than considering a new velocity vector for the second part of the journey.

At $10:00$ the helicopter begins to descend.

During descent the helicopter's vertical height decreases at a constant rate of 16 km h^{-1} and its horizontal velocity remains unchanged.

The angle of descent, β, is defined as the angle between the helicopter's direction of travel and the horizontal.

(b) Find β, giving your answer in degrees. [3]

Markscheme

METHOD 1

attempt to use right-angled triangle with the horizontal speed found in (a) ***(M1)***

$$\tan\beta = \frac{16}{20\sqrt{29}}\left(= \frac{4}{5\sqrt{29}}\right)$$ ***(A1)***

$$\beta = 8.44984\,(^\circ)$$

$$\beta = 8.45\,(^\circ)$$ ***A1***

Note: Award ***M1A1A0*** for answer of 0.147 radians.

METHOD 2

attempt to find angle between $\begin{pmatrix}10\\-25\\-4\end{pmatrix}$ and $\begin{pmatrix}10\\-25\\0\end{pmatrix}$ ***(M1)***

$$\cos\beta = \frac{\begin{pmatrix}10\\-25\\-4\end{pmatrix}\cdot\begin{pmatrix}10\\-25\\0\end{pmatrix}}{\sqrt{10^2+(-25)^2+(-4)^2}\sqrt{10^2+(-25)^2}}\left(= \frac{725}{\sqrt{741}\sqrt{725}} = 0.989144\ldots\right)$$

OR

$$\sin\beta = \frac{\left|\begin{pmatrix}10\\-25\\-4\end{pmatrix}\times\begin{pmatrix}10\\-25\\0\end{pmatrix}\right|}{\sqrt{10^2+(-25)^2+(-4)^2}\sqrt{10^2+(-25)^2}} = \frac{\left|\begin{pmatrix}-100\\-40\\0\end{pmatrix}\right|}{\sqrt{741}\sqrt{725}}\left(= \frac{4}{\sqrt{741}} = 0.146943\ldots\right)$$

(A1)

$$\beta = 8.44984\,(^\circ)$$

$\beta = 8.45\,(^\circ)$ ***A1***

METHOD 3

attempt to find angle between $\begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix}$ and a plane parallel to $z = 0$ ***(M1)***

$$\sin \beta = \frac{\left| \begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right|}{\sqrt{10^2 + (-25)^2 + (-4)^2}} \left(= \frac{4}{\sqrt{741}} = 0.146943\ldots \right)$$ ***(A1)***

Note: This could also be written as $\cos\,(90^\circ - \beta) = \ldots$

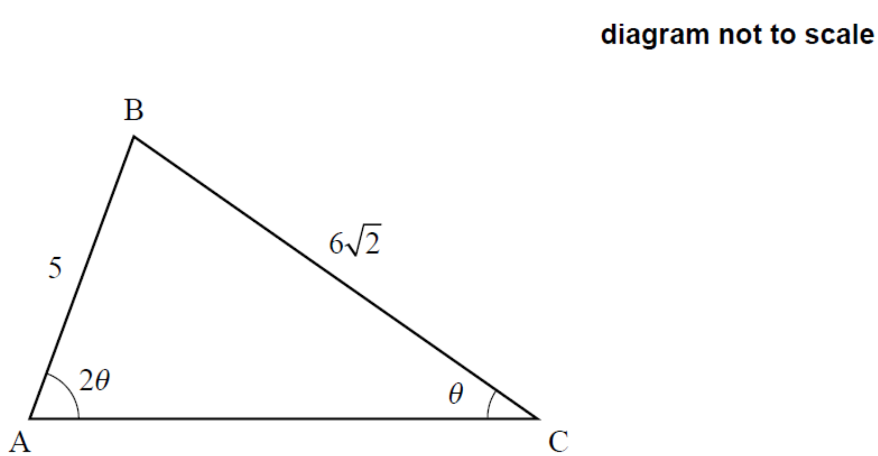
$\beta = 8.44984\,(^\circ)$

$\beta = 8.45\,(^\circ)$ ***A1***

[3 marks]

6. [Maximum mark: 7] 25M.1.AHL.TZ2.3

The following diagram shows a non-right angled triangle ABC.



$\mathrm{AB} = 5,\ \mathrm{BC} = 6\sqrt{2},\ \mathrm{A\widehat{C}B} = \theta$ and $\mathrm{B\widehat{A}C} = 2\theta$, where $0 < \theta < \frac{\pi}{2}$.

(a) Using the sine rule, show that $\cos\,\theta = \frac{3\sqrt{2}}{5}$. [3]

Markscheme

correct substitution in sine rule ***(A1)***

$\frac{\sin\,\theta}{5} = \frac{\sin\,2\theta}{6\sqrt{2}}$ (or equivalent)

attempt to use double angle rule for $\sin\,2\theta$ ***(M1)***

$\frac{\sin\,\theta}{5} = \frac{2\,\sin\,\theta\,\cos\,\theta}{6\sqrt{2}}$

$6\sqrt{2}\,\sin\,\theta = 10\,\sin\,\theta\,\cos\,\theta$ **OR** $\frac{1}{5} = \frac{2\,\cos\,\theta}{6\sqrt{2}}$ **OR** equivalent ***A1***

$\cos\,\theta = \frac{3\sqrt{2}}{5}$ ***AG***

[3 marks]

Examiners report

SL:

Candidates were able to use a rule from the formula booklet for each part of this question, and this was done well. Errors were typically arithmetic, rather than conceptual, with some candidates unsure how to manipulate surds/radicals.

HL:

This was one of the better-answered questions on the paper. Part (a) was well done, with almost all candidates confidently applying double angle identities to arrive at the given result. Part (b) was also well attempted, with many completely correct answers. A few candidates made arithmetic errors or failed to simplify their final answer to an integer denominator. Part (c) was again well answered with many gaining follow-through marks and demonstrating good manipulation of surds.

(b) Hence, find $\sin\,\theta$. [2]

Markscheme

valid attempt to find $\sin\theta$ ***(M1)***

$\sin^2\theta + \left(\frac{3\sqrt{2}}{5}\right)^2 = 1$ **OR** right triangle with adjacent side and hypotenuse labelled

$\sin\theta = \frac{\sqrt{7}}{5}$ ***A1***

[2 marks]

Point D is located on $[\mathrm{AC}]$ such that the area of triangle BCD is $2\sqrt{14}$.

(c) Find DC. [2]

Markscheme

$\frac{1}{2} \times 6\sqrt{2} \times \mathrm{DC} \times \frac{\sqrt{7}}{5} = 2\sqrt{14}$ ***(A1)***

$\mathrm{DC} = \frac{10}{3}$ ***A1***

[2 marks]

7. [Maximum mark: 5] 25M.1.AHL.TZ3.3

Solve the equation $2\cos 2\theta - 5\cos\theta + 2 = 0$, where $\pi \le \theta \le 2\pi$. [5]

Markscheme

recognizing to use $\cos 2\theta = 2\cos^2\theta - 1$ ***(M1)***

$2\left(2\cos^2\theta - 1\right) - 5\cos\theta + 2\ (= 0)$ ***A1***

$4\cos^2\theta - 5\cos\theta\ \ (= 0)$

choosing an appropriate method to solve their quadratic equation ***(M1)***

$\cos\,\theta(4\cos\,\theta - 5)$ **OR** $\frac{5\pm\sqrt{(-5)^2-4\times4\times0}}{2\times4}$ ***(A1)***

$\cos\,\theta = 0$

$\theta = \frac{3\pi}{2}$ ***A1***

Note: Do not award final ***A1*** if any extra solutions given.

[5 marks]

Examiners report

There were a significant number of good solutions to this question, although there were a number of students that did not pay attention to the range of values for the solutions, and others who included an extra solution of $\cos^{-1}\frac{5}{4}$. A significant number of students approached the quadratic equation found, by dividing both sides by $\cos\,\theta$, resulting in a loss of most of the marks.

8. [Maximum mark: 4] 25M.2.AHL.TZ1.2

Consider the function $f(x) = a\tan\,(2x) + b$, where $x \neq \frac{(2n+1)\pi}{4}$, $n \in \mathbb{Z}$ and $a,\ b \in \mathbb{R}$.

(a) Write down the period of f. [1]

Markscheme

period is $\frac{\pi}{2}(= 1.57079\ldots = 1.57)$ ***A1***

[1 mark]

Examiners report

SL:

In part (a), the vast majority of students stated the period as π, not realizing that the tangent function does not have the same period as sine and cosine.

In part (b), students were quite successful using analytical approaches. It is worth noting that the system resulting from substituting the given points can be solved using a GDC's simultaneous equation solver (but needs to be in radian mode to do so).

HL:

It was very common for students to give an answer of π to part (a), suggesting unfamiliarity with the period of the tangent function. This was a shame as access to a calculator would have allowed for this to be verified.

Conversely, part (b) was often well done. Where students found the correct equations, it was disappointing to see a lot of step-by-step algebraic methods for solving linear simultaneous equations when this can be done on the calculator. Sometimes this led to unnecessary errors. Occasionally students did not double the input when making the substitution.

The graph of $y = f(x)$ passes through the points $\left(\frac{\pi}{12},\ 5\right)$ and $\left(\frac{\pi}{3},\ 7\right)$.

(b) Find the value of a and the value of b. [3]

Markscheme

attempt to substitute $x = \frac{\pi}{12},\ f(x) = 5$ and $x = \frac{\pi}{3}, f(x) = 7$ to obtain two equations ***(M1)***

Note: accept work where x values have been converted into degrees

$a\tan\left(\frac{\pi}{6}\right) + b = 5$ and
$a\tan\left(\frac{2\pi}{3}\right) + b = 7\ \left(\Rightarrow \frac{a}{\sqrt{3}} + b = 5 \text{ and } -a\sqrt{3} + b = 7\right)$

$a = -\frac{\sqrt{3}}{2}(= -0.866025\ldots = -0.866)$ ***A1***

$b = \frac{11}{2} (= 5.5)$ ***A1***

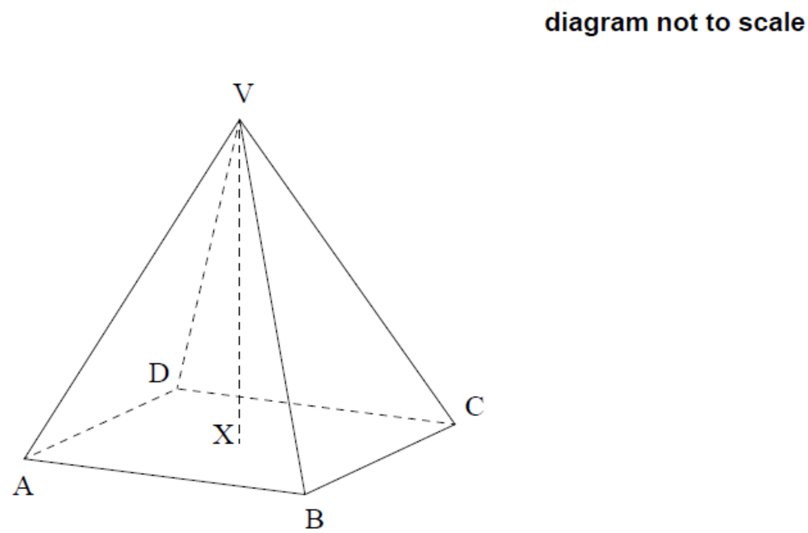
Note: These ***A1*** marks may be awarded independently.

[3 marks]

9. [Maximum mark: 6] 25M.2.AHL.TZ2.2

The following diagram shows a square-based right-pyramid with vertex $\text{V}\,(1,\ 7,\ 0)$.

Point $\text{X}\,(-3,\ 4,\ 2)$ is the centre of the base ABCD.



(a) Find VX. [2]

Markscheme

attempt to find distance between two points ***(M1)***

$\sqrt{(-3-1)^2 + (4-7)^2 + (2-0)^2}\ (= 5.38516\ldots)$

$\text{VX} = 5.39\ \left(= \sqrt{29}\right)$ ***A1***

[2 marks]

Examiners report

SL:

Parts (a) and (b) did not prove difficult for candidates, and most were able to find both lengths.

Part (c) caused most difficulty. Some students could not identify the angle they needed to find. Others did not realize that VXC was a right-angled triangle. Another difficulty that students met was not being able to see how to use their values from parts (a) and (b) to solve for the angle using the tangent formula, as well as recognizing that the length of side XC was half of the length of AC. Some attempted to use the sine rule to find the angle. Many attempted to find the incorrect angle XVC.

There were also quite a few lengthy calculations in (c). Many candidates used Pythagoras' Theorem to find VC and then the sine rule to find the required angle. Even though it was a valid method, it sometimes led to accuracy errors due to premature rounding.

HL:

In part (a), some candidates misinterpreted the notation, which led to incorrect responses despite understanding the underlying concept. Part (b), on the other hand, was very well answered, indicating that candidates were confident and well-prepared for that section. However, part (c) posed a significant challenge for several candidates, as many were unsure which trigonometric formula to apply when calculating an angle in a three-dimensional context. This suggests a need for more targeted practice with spatial reasoning and the application of trigonometry in 3D problems.

The square base has side length $5\ \mathrm{cm}$.

(b) Find AC. [2]

Markscheme

attempt to use Pythagoras' theorem *(M1)*

$\sqrt{5^2+5^2}\ (=7.07106\ldots)$

$\mathrm{AC}=7.07\ \left(=5\sqrt{2}\right)$ ***A1***

[2 marks]

(c) Find the size of the angle between the edge $[\mathrm{VC}]$ and the base of the pyramid. [2]

Markscheme

valid attempt to use trig ratio in triangle VXC *(M1)*

$\tan\theta=\frac{\sqrt{29}}{\frac{5\sqrt{2}}{2}}\left(=\frac{5.38516\ldots}{3.53553\ldots}\right)$

$56.7138\ldots^\circ$ **OR** $0.989842\ldots$ rad

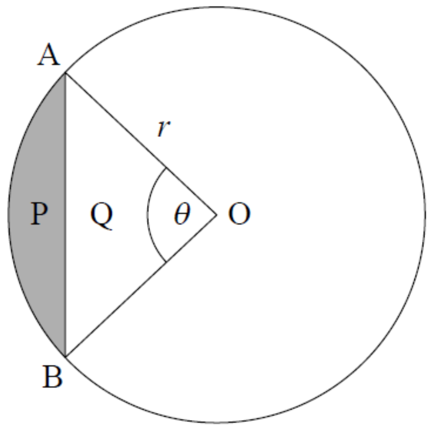
$\theta=56.7^\circ$ **OR** $\theta=0.990\,\mathrm{rad}$ (accept $\theta=0.99$) ***A1***

[2 marks]

10. [Maximum mark: 6] 25M.2.AHL.TZ2.6

The following diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and $\mathrm{A\hat{O}B}=\theta$ radians.

Sector OAB is divided into two regions, a shaded segment P and a triangle Q.



[6]

The area of the shaded segment P is $12.8\ \mathrm{cm}^2$.

The areas of P and Q are in the ratio $3 : 5$.

Find the value of r.

Markscheme

METHOD 1

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) ***(M1)***

$\text{area}_{\text{triangle}} = 21.3333\ldots$ **OR** $\text{area}_{\text{sector}} = 34.1333\ldots$ ***A1***

correct equation in r and θ ***(A1)***

$\frac{1}{2}r^2\theta = 34.1333\ldots$ **OR** $\frac{1}{2}r^2 \sin\theta = 21.3333\ldots$ **OR**
$\frac{1}{2}r^2(\theta - \sin\theta) = 12.8$ (seen anywhere)

correct equation in one variable ***A1***

$\frac{1}{2}r^2\left(\frac{68.2666\ldots}{r^2} - \sin\left(\frac{68.2666..}{r^2}\right)\right) = 12.8$ **OR**
$\frac{1}{2}\left(\frac{68.2666..}{\theta}\right)\sin\theta = 21.3333\ldots$ **OR** $\frac{1}{2}\left(\frac{68.2666..}{\theta}\right)(\theta - \sin\theta) = 12.8$

attempt to solve their equation or use of graph ***(M1)***

$\theta = 1.59934\ldots$

$6.53330\ldots$

$r = 6.53$ ***A1***

METHOD 2

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) ***(M1)***

$\text{area}_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin\theta)$ (seen anywhere) ***(A1)***

$\frac{\frac{1}{2}r^2(\theta - \sin\theta)}{\frac{1}{2}r^2\sin\theta} = \frac{3}{5}$

correct equation without r ***A1***

$\frac{(\theta - \sin\theta)}{\sin\theta} = \frac{3}{5}$

$\theta = 1.59934\ldots$ ***(A1)***

attempt to solve for r using their θ ***(M1)***

$\frac{1}{2}r^2(1.59\ldots - \sin 1.59\ldots) = 12.8$

$6.53330\ldots$

$r = 6.53$ ***A1***

METHOD 3

recognising $\text{area}_{\text{segment}} = \frac{3}{8} \times \text{area}_{\text{sector}}$ (seen anywhere) ***(M1)***

$\text{area}_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin\theta)$ (seen anywhere) ***(A1)***

$\frac{1}{2}r^2(\theta - \sin\theta) = \frac{3}{8} \times \frac{1}{2}r^2\theta$

correct equation without r ***A1***

$\frac{1}{2}(\theta - \sin\theta) = \frac{3}{8} \times \frac{1}{2}\theta$

$\theta = 1.59934\ldots$ ***(A1)***

attempt to solve for r using their θ ***(M1)***

$\frac{1}{2}r^2(1.59\ldots - \sin 1.59\ldots) = 12.8$

$6.53330\ldots$

$r = 6.53$ ***A1***

[6 marks]

Examiners report

SL:

In this question the first problem that candidates had was in dealing with the given ratio. Instead of considering $\frac{\text{P}}{\text{Q}} = \frac{3}{5}$, giving $5\text{P} = 3\text{Q}$, many wrote $3\text{P} = 5\text{Q}$. Others did not know how to use this condition.

Of those candidates who started off correctly, most were able to find a correct equation for either the area of the sector, the area of the triangle or the area of the segment. Many students were not able to continue solving after they found two equations in r and θ. Of those who managed to find an equation in one of the variables, only a few continued beyond that point. Some candidates incorrectly assumed that $\theta = 90^\circ$, hence eliminating $\sin\theta$ from the equations.

HL:

This question was attempted by most candidates, and the correct answer was frequently seen, indicating a generally good understanding of the concept. Several different approaches were used to calculate the angle leading to the radius, showcasing flexibility in problem-solving. However, common mistakes included early rounding, confusion between different ratio equations, and incorrectly assuming the angle was a right angle. These errors suggest a need for greater care in interpreting geometric relationships and maintaining precision throughout calculations.