INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 3.10-3.12]

BASIC ALGEBRA OF VECTORS

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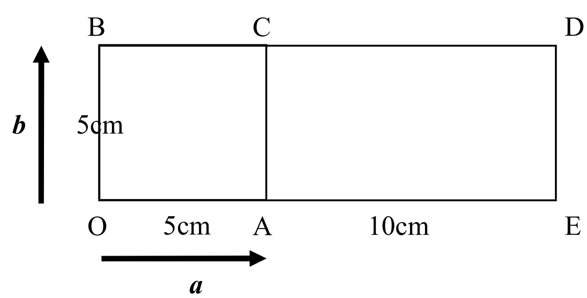
O. Practice questions

1. [Maximum mark: 12] ***[without GDC]***

In the following diagram OBCA is a square of side 5cm, while ACDE is a rectangle of length 10cm and width 5cm. Let $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{OB}$.



(a) Determine whether the following expressions in terms of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are true or false

$\overrightarrow{AC} = \boldsymbol{b}$	**true**	$\overrightarrow{BC} = \boldsymbol{a}$		$\overrightarrow{BD} = 3\boldsymbol{a}$	
$\overrightarrow{DE} = \boldsymbol{b}$		$\overrightarrow{BC} = \boldsymbol{b}$		$\overrightarrow{CD} = 2\boldsymbol{b}$	
$\overrightarrow{ED} = \boldsymbol{a}$		$\lvert\overrightarrow{BC}\rvert = \lvert\boldsymbol{b}\rvert$		$\overrightarrow{CD} = 2\boldsymbol{a}$	

[4]

(b) Express the following vectors in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

$\overrightarrow{OC} =$	$\overrightarrow{AB} =$
$\overrightarrow{OD} =$	$\overrightarrow{CE} =$
$\overrightarrow{AD} =$	$\overrightarrow{BE} =$
$\overrightarrow{BA} =$	$\overrightarrow{EC} =$

[8]

2. [Maximum mark: 20] ***[without GDC]***

Let $\boldsymbol{a} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find

magnitude of $\boldsymbol{a}$	
magnitude of $\boldsymbol{b}$	
unit vector corresponding to $\boldsymbol{a}$	
unit vector corresponding to $\boldsymbol{b}$	
$\boldsymbol{a} + \boldsymbol{b}$	
$\boldsymbol{a} - \boldsymbol{b}$	
$2\boldsymbol{a}$	
$\boldsymbol{a} - 2\boldsymbol{b}$	
$\lvert \boldsymbol{a} - 2\boldsymbol{b} \rvert$	
a vector $\boldsymbol{c}$ parallel to $\boldsymbol{a}$ of magnitude 39	

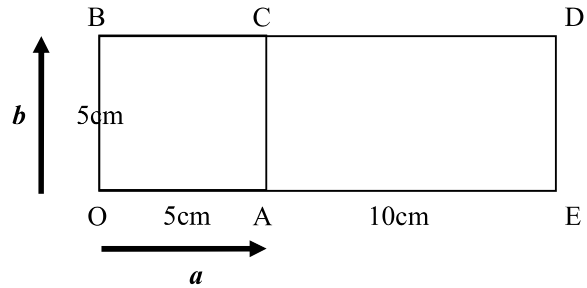
3. [Maximum mark: 16] ***[without GDC]***

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Find

magnitude of $\boldsymbol{a}$	
magnitude of $\boldsymbol{b}$	
unit vector corresponding to $\boldsymbol{a}$	
unit vector corresponding to $\boldsymbol{b}$	
$\boldsymbol{a}+\boldsymbol{b}$	
$\boldsymbol{a}+2\boldsymbol{b}$	
$\lvert\boldsymbol{a}+2\boldsymbol{b}\rvert$	
a vector $\boldsymbol{c}$ parallel to $\boldsymbol{a}$ of magnitude 6	

4. [Maximum mark: 12] ***[without GDC]***

In the following diagram OBCA is a square of side 5cm, while ACDE is a rectangle of length 10cm and width 5cm. Let $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{OB}$.



Use the geometric definition of the dot product to find the following

$\boldsymbol{a} \cdot \boldsymbol{b} =$
$\boldsymbol{a}^2 =$
$\boldsymbol{b}^2 =$
$\overrightarrow{OE} \cdot \overrightarrow{AC} =$
$\overrightarrow{OA} \cdot \overrightarrow{AE} =$
$\overrightarrow{OA} \cdot \overrightarrow{OC} =$

5. [Maximum mark: 8] ***[without GDC]***

Let $\boldsymbol{a} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Use the algebraic definition of the dot product to find

$\boldsymbol{a} \cdot \boldsymbol{b}$	
$\boldsymbol{a}^2$	
$\boldsymbol{b}^2$	
cosine of angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$	

6. [Maximum mark: 8] ***[without GDC]***

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Use the algebraic definition of the dot product to find

$\boldsymbol{a} \cdot \boldsymbol{b}$	
$\boldsymbol{a}^2$	
$\boldsymbol{b}^2$	
cosine of angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$	

7. [Maximum mark: 10] ***[without GDC]***

Let $\boldsymbol{u} = \begin{pmatrix} 3 \\ 1 \\ b \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} a \\ 4 \\ 8 \end{pmatrix}$. Find the values of a and b for each case below.

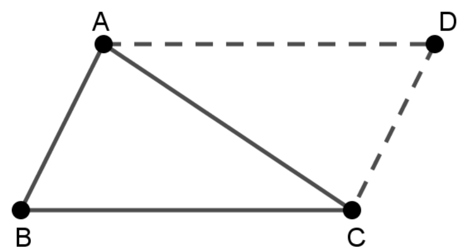
(a) if $2\boldsymbol{u} - 3\boldsymbol{v} = \begin{pmatrix} 3 \\ -10 \\ -4 \end{pmatrix}$. [3]

(b) if $\boldsymbol{u}$ is parallel to $\boldsymbol{v}$ [3]

(c) if $a = -2b$ and $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ **.** [4]

8. [Maximum mark: 18] ***[without GDC]***

Consider the points $A(2,1,3)$, $B(5,0,4)$, $C(4,7,3)$.



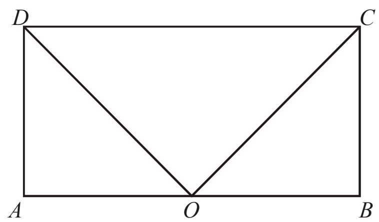
Find

position vector of point A	
vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$	
magnitude of the vector $\overrightarrow{AB}$	
magnitude of the vector $\overrightarrow{AC}$	
distance between A and B	
$\overrightarrow{AB}\cdot\overrightarrow{AC}$	
size of the angle $B\hat{A}C$	
area of the triangle ABC	
the coordinates of point D given that $ABCD$ is a parallelogram	

A. Exam style questions (SHORT)

9. [Maximum mark: 6] ***[without GDC]***

ABCD is a rectangle and *O* is the midpoint of [*AB*].



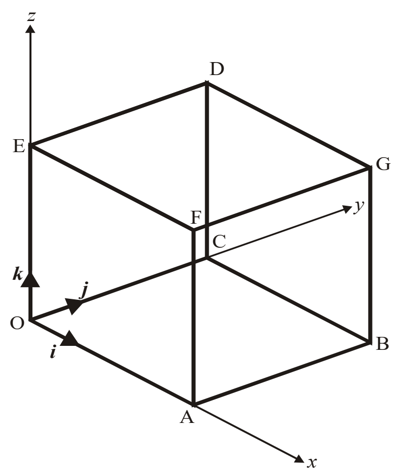
Express each of the following vectors in terms of $\overrightarrow{OC}$ and $\overrightarrow{OD}$

(a) $\overrightarrow{CD}$ (b) $\overrightarrow{OA}$ (c) $\overrightarrow{AD}$

10. [Maximum mark: 6] ***[without GDC]***

The diagram shows a cube, OABCDEFG where the length of each edge is 5cm.

Express the following vectors in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.



(a) $\overrightarrow{\text{OG}}$ ....

(b) $\overrightarrow{\text{BD}}$ ....

(c) $\overrightarrow{\text{EB}}$ ....

11. [Maximum mark: 4] ***[without GDC]***

The vectors $\boldsymbol{u}$, $\boldsymbol{v}$ are given by $\boldsymbol{u} = 3\boldsymbol{i} + 5\boldsymbol{j}$, $\boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j}$. Find scalars a, b such that

$$a(\boldsymbol{u}+\boldsymbol{v}) = 8\boldsymbol{i} + (b-2)\boldsymbol{j}.$$

12. [Maximum mark: 4] ***[without GDC]***

The vectors $\vec{i}$, $\vec{j}$ are unit vectors along the x-axis and y-axis respectively.

The vectors $\vec{u} = -\vec{i} + 2\vec{j}$ and $\vec{v} = 3\vec{i} + 5\vec{j}$ are given.

(a) Find $\vec{u} + 2\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$. [2]

A vector $\vec{w}$ has the same direction as $\vec{u} + 2\vec{v}$, and has a magnitude of 26.

(b) Find $\vec{w}$ in terms of $\vec{i}$ and $\vec{j}$. [2]

13. [Maximum mark: 6] ***[without GDC]***

Consider the vectors $\boldsymbol{c} = 3\boldsymbol{i} + 4\boldsymbol{j}$ and $\boldsymbol{d} = 5\boldsymbol{i} - 12\boldsymbol{j}$.

(a) Calculate the scalar product $\boldsymbol{c} \cdot \boldsymbol{d}$. [2]

(b) Find the vector $\boldsymbol{c} + \boldsymbol{d}$. [1]

(c) Find the value of $|\boldsymbol{c}| + |\boldsymbol{d}|$. [3]

14. [Maximum mark: 6] ***[without GDC]***

Find the cosine of the angle between the two vectors $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

15. [Maximum mark: 4] ***[with GDC]***

Find the size of the angle between the two vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$. Give your answer to the nearest degree.

16. [Maximum mark: 4] ***[with GDC]***

Find the angle between the following vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, giving your answer to the nearest degree.

$$\boldsymbol{a} = -4\boldsymbol{i} - 2\boldsymbol{j} \qquad \boldsymbol{b} = \boldsymbol{i} - 7\boldsymbol{j}$$

17. [Maximum mark: 7] ***[with GDC]***

(a) Find the scalar product of the vectors $\begin{pmatrix} 60 \\ 25 \end{pmatrix}$ and $\begin{pmatrix} -30 \\ 40 \end{pmatrix}$. [2]

(b) Two markers are at the points $\text{P}(60, 25)$ and $\text{Q}(-30, 40)$.

(i) Find the distance between the two markers.

(ii) A surveyor stands at $\text{O}\ (0, 0)$ and looks at marker P. Find the angle she turns through to look at marker Q. [5]

18. [Maximum mark: 5] ***[with GDC]***

Find the angle between the vectors $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{w} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$. Give your answer in radians.

19. [Maximum mark: 6] ***[with GDC]***

The position vectors of points P and Q are $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix}$ respectively. The origin is at O

(a) Find the angle $\mathrm{P\hat{O}Q}$; [4]

(b) Find the area of the triangle OPQ. [2]

20. [Maximum mark: 6] ***[with GDC]***

A triangle has its vertices at A(–1, 3), B(3, 6) and C(–4, 4).

(a) Show that $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}} = -9$. [3]

(b) Show that, to three significant figures, $\cos \mathrm{B\hat{A}C} = -0.569$. [3]

21. [Maximum mark: 6] ***[with GDC]***

A triangle has its vertices at A(–1, 3, 2), B(3, 6, 1) and C(–4, 4, 3).

(a) Show that $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}} = -10$. [3]

(b) Show that, to three significant figures, $\cos \mathrm{B\hat{A}C} = -0.591$. [3]

22. [Maximum mark: 6] ***[without GDC]***

Consider the points A(5, 8), B(3, 5) and C(8, 6).

(a) Find (i) $\overrightarrow{\mathrm{AB}}$; (ii) $\overrightarrow{\mathrm{AC}}$. [3]

(b) (i) Find $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}}$. (ii) Find the size of the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. [3]

23. [Maximum mark: 5] ***[without GDC]***

The quadrilateral $OABC$ has vertices with coordinates $O(0,0)$ $A(5,1)$ $B(10,5)$ and $C(2,7)$

(a) Find the vectors $\overrightarrow{OB}$ and $\overrightarrow{AC}$. [2]

(b) Find the angle between the diagonals of the quadrilateral $OABC$. [3]

24. [Maximum mark: 5] ***[with / without GDC]***

The vectors $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$ and $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$ are perpendicular for two values of x.

Find the two values of x.

25*. [Maximum mark: 5] ***[without GDC]***

Let α be the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, where $\boldsymbol{a} = (\cos\theta)\boldsymbol{i} + (\sin\theta)\boldsymbol{j}$,

$\boldsymbol{b} = (\sin\theta)\boldsymbol{i} + (\cos\theta)\boldsymbol{j}$ and $0 < \theta < \frac{\pi}{4}$. Express α in terms of θ.

26. [Maximum mark: 6] ***[without GDC]***

Consider the vectors $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = 4\boldsymbol{i} + \boldsymbol{j} - p\boldsymbol{k}$.

(a) Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ find the value of p. [3]

(b) Given that $q|\mathbf{u}| = 14$, find the value of q. [3]

27. [Maximum mark: 7] ***[without GDC]***

Let $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$. The vector $\boldsymbol{v} + p\boldsymbol{w}$ is perpendicular to $\boldsymbol{w}$.

Find the value of p.

28. [Maximum mark: 6] ***[without GDC]***

The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} 2 \\ y \\ 3 \end{pmatrix}$.

Given that $\boldsymbol{c}$ is perpendicular to $2\boldsymbol{a} - \boldsymbol{b}$, find the value of y.

29. [Maximum mark: 6] ***[without GDC]***

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$, $\boldsymbol{c} = \begin{pmatrix} 3 \\ 1 \\ \lambda \end{pmatrix}$, $\boldsymbol{d} = \begin{pmatrix} \mu \\ -2 \\ 1 \end{pmatrix}$

Let $\boldsymbol{s} = (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{c} + \boldsymbol{d}$, where $\boldsymbol{s}$ is perpendicular to $\boldsymbol{a}$.

Find an expression for λ in terms of μ.

30. [Maximum mark: 6] ***[with GDC]***

The angle between the vector $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ and the vector $\boldsymbol{b} = 3\boldsymbol{i} - 2\boldsymbol{j} + m\boldsymbol{k}$ is $30°$.

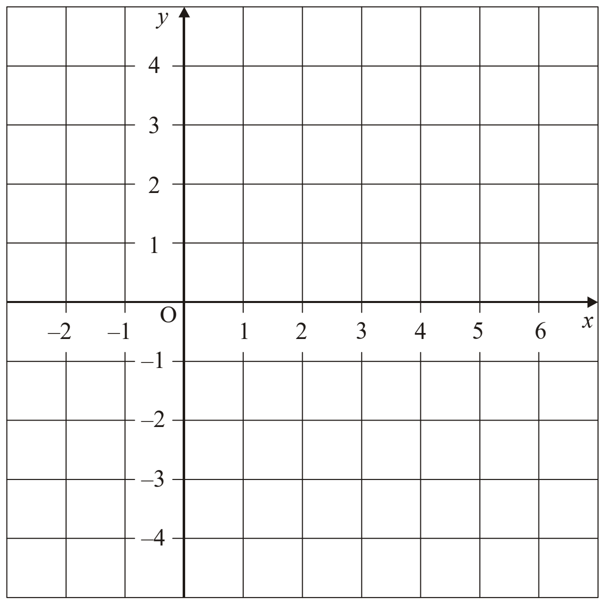
Find the values of m.

31. [Maximum mark: 5] ***[without GDC]***

The triangle ABC is defined by the following information

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad \overrightarrow{AB} \cdot \overrightarrow{BC} = 0, \quad \overrightarrow{AC}, \text{ is parallel to } \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

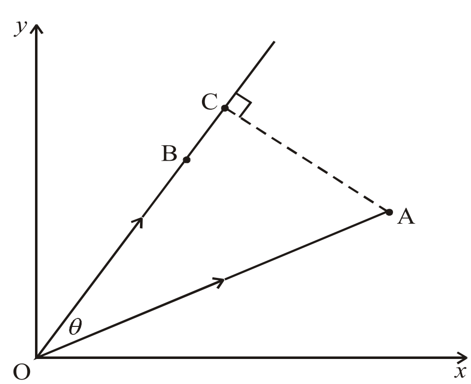
(a) On the grid below, draw an accurate diagram of triangle ABC. [3]



(b) Write down the vector $\overrightarrow{OC}$. [2]

32. [Maximum mark: 8] ***[with GDC]***

The following diagram shows the point O with coordinates (0, 0), the point A with position vector $\boldsymbol{a} = 12\boldsymbol{i} + 5\boldsymbol{j}$, and the point B with position vector $\boldsymbol{b} = 6\boldsymbol{i} + 8\boldsymbol{j}$. The angle between (OA) and (OB) is θ.



(a) Find $|\,\boldsymbol{a}\,|$; [2]

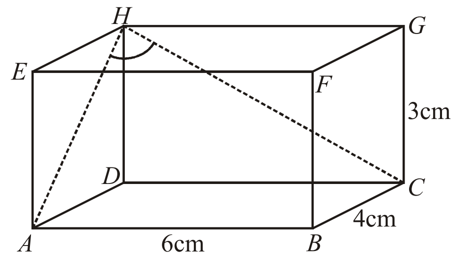
(b) Find a unit vector in the direction of $\boldsymbol{b}$; [2]

(c) Find the **exact** value of $\cos\theta$ in the form $\frac{p}{q}$, where, $p, q \in \mathbb{Z}$. [2]

(d) Show that $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{b}||OC|$. [2]

33. [Maximum mark: 10] ***[with GDC]***

The rectangle box shown in the diagram has dimensions 6 cm × 4 cm × 3 cm.



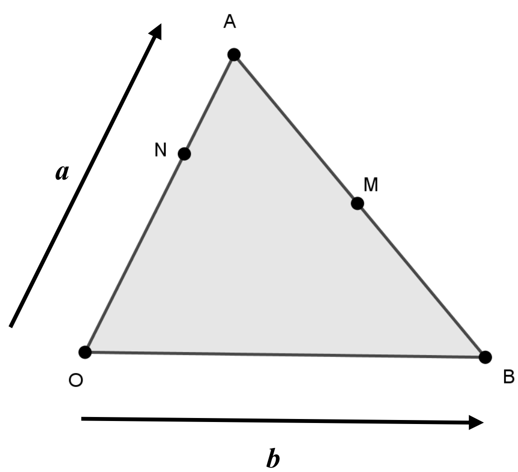
Find, correct to the nearest one-tenth of a degree, the size of the angle $A\hat{H}C$ by following two different methods

(a) Using the cosine rule on the triangle AHC. [5]

(b) Assuming that $A(0,0,0)$, B is on x-axis, D on y axis, E on z-axis and using the vectors $\overrightarrow{HA}$ and $\overrightarrow{HC}$. [5]

34. [Maximum mark: 8] ***[without GDC]***

In the following triangle OAB, let $\boldsymbol{a} = \overrightarrow{\mathrm{OA}}$ and $\boldsymbol{b} = \overrightarrow{\mathrm{OB}}$.



It is also given that M is the midpoint of AB, while N divides OA in ratio ON:NA = 2:1

Express the following vectors in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $\overrightarrow{\mathrm{AB}}$ [1]

(b) $\overrightarrow{\mathrm{OM}}$ [2]

(c) $\overrightarrow{\mathrm{NB}}$ [2]

(d) $\overrightarrow{\mathrm{MN}}$ [3]

35. [Maximum mark: 6] ***[without GDC]***

Three distinct non-zero vectors are given by $\overrightarrow{OA} = \boldsymbol{a}, \overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OC} = \boldsymbol{c}$.

(a) Express the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. [3]

(b) If $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$ and $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{CA}$, show that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{AB}$. [5]

36. [Maximum mark: 10] ***[without GDC]***

Given two non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ show that

(a) $|\boldsymbol{a}| = |\boldsymbol{b}|$ if and only if $\boldsymbol{a} + \boldsymbol{b} \perp \boldsymbol{a} - \boldsymbol{b}$ [3]

(b) $\boldsymbol{a} \perp \boldsymbol{b}$ if and only if $|\boldsymbol{a} + \boldsymbol{b}| = |\boldsymbol{a} - \boldsymbol{b}|$ [3]

(c) Give the geometric interpretation

(i) of the result (a)

(ii) of the result (b)

(iii) of the results (a) and (b) together. [4]

B. Exam style questions (LONG)

37. [Maximum mark: 13] ***[without GDC]***

Consider the points A(1, 5, 4), B(3, 1, 2), D(3, k, 2), with (AD) perpendicular to (AB).

(a) Find

(i) $\overrightarrow{AB}$; (ii) $\overrightarrow{AD}$, giving your answer in terms of k. [3]

(b) Show that $k = 7$. [3]

The point C is such that $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$.

(c) Find the position vector of C. [4]

(d) Find $\cos A\hat{B}C$. [3]

38. [Maximum mark: 18] ***[with GDC]***

The diagram shows a parallelogram ABCD.

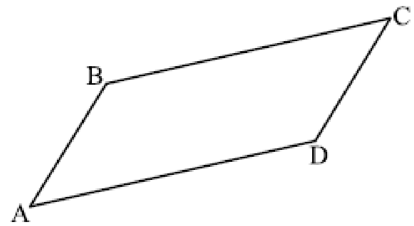


diagram not to scale

The coordinates of A, B and D are A(1, 2, 3), B(6, 4, 4) and D(2, 5, 5).

(a) (i) Show that $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$. (ii) Find $\overrightarrow{AD}$. (iii) **Hence** show that $\overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$. [5]

(b) Find the coordinates of point C. [3]

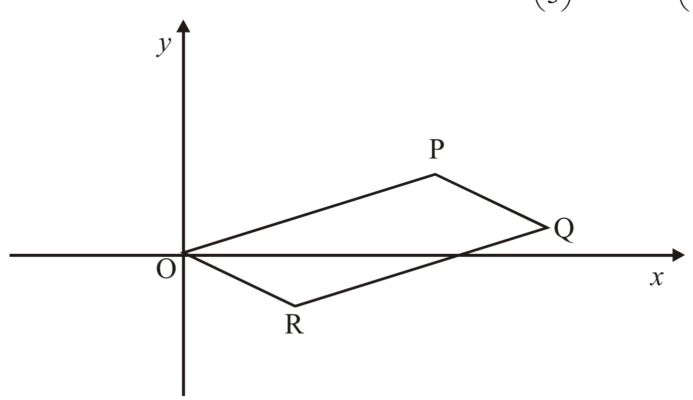
(c) (i) Find $\overrightarrow{AB} \bullet \overrightarrow{AD}$.

(ii) **Hence** find angle A. `` [7]

(d) Hence, or otherwise, find the area of the parallelogram. [3]

39. [Maximum mark: 14] ***[with GDC]***

The diagram shows a parallelogram OPQR in which $\overrightarrow{OP} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$.



(a) Find the vector $\overrightarrow{OR}$. [3]

(b) Use the scalar product of two vectors to show that $\cos \mathrm{O\hat{P}Q} = -\frac{15}{\sqrt{754}}$. [4]

(c) (i) Explain why $\cos \mathrm{P\hat{Q}R} = -\cos \mathrm{O\hat{P}Q}$.

(ii) Hence show that $\sin \mathrm{P\hat{Q}R} = \frac{23}{\sqrt{754}}$.

(iii) Find the area of the parallelogram OPQR, giving your answer as an integer. [7]

40. [Maximum mark: 15] ***[without GDC]***

The points A and B have the position vectors $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$ respectively.

(a) Find (i) the vector $\overrightarrow{AB}$. (ii) $|\overrightarrow{AB}|$. [4]

The point D has position vector $\begin{pmatrix} d \\ 23 \end{pmatrix}$

(b) Find the vector $\overrightarrow{AD}$ in terms of d. [2]

The angle $B\hat{A}D$ is $90°$.

(c) (i) Find d. (ii) Write down the position vector of the point D. [3]

The quadrilateral ABCD is a rectangle.

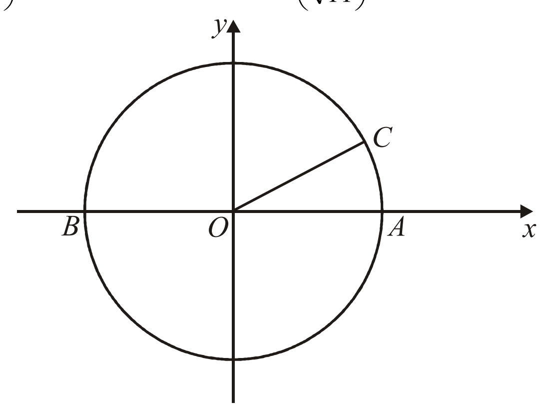
(d) Find the position vector of the point C. [4]

(e) Find the area of the rectangle ABCD. [2]

41. [Maximum mark: 12] ***[without GDC]***

The circle shown has centre O and radius 6. $\overrightarrow{OA}$ is the vector $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$,

$\overrightarrow{OB}$ is the vector $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$ and $\overrightarrow{OC}$ is the vector, $\begin{pmatrix} 5 \\ \sqrt{11} \end{pmatrix}$.



(a) Verify that A, B and C lie on the circle. [3]

(b) Find the vector $\overrightarrow{AC}$. [2]

(c) Using an appropriate scalar product, or otherwise, find the cosine of angle OAC. [3]

(d) Find the area of triangle ABC, giving your answer in the form $a\sqrt{11}$, where $a \in \mathbb{N}$. [4]

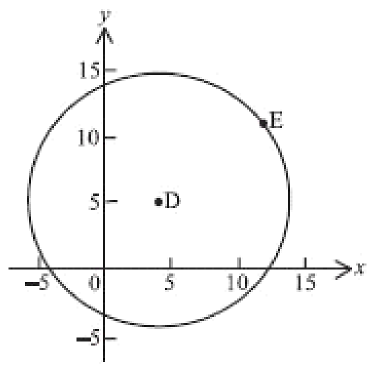
42. [Maximum mark: 12] ***[without GDC]***

Consider the point D with coordinates (4, 5), and the point E, with coordinates (12, 11).

(a) Find $\overrightarrow{DE}$. [2]

(b) Find $\left|\overrightarrow{DE}\right|$. [2]

(c) The point D is the centre of a circle and E is on the circumference as shown in the following diagram.



The point G is also on the circumference. $\overrightarrow{DE}$ is perpendicular to $\overrightarrow{DG}$. Find the possible coordinates of G. [8]