

INTERNATIONAL BACCALAUREATE
Mathematics: analysis and approaches
Math AA

EXERCISES [Math-AA 3.6]
TRIGONOMETRIC EQUATIONS
Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 8] **[with / without GDC]**

Complete the following table with the values of $\sin^{-1} a$ and $\cos^{-1} a$ in **degrees**.

	$\sin^{-1}(1) =$	
	$\sin^{-1}\left(\frac{1}{2}\right) =$	30°
	$\sin^{-1}(0) =$	
	$\sin^{-1}\left(-\frac{1}{2}\right) =$	
	$\sin^{-1}(-1) =$	
	$\cos^{-1}(1) =$	
	$\cos^{-1}\left(\frac{1}{2}\right) =$	60°
	$\cos^{-1}(0) =$	
	$\cos^{-1}\left(-\frac{1}{2}\right) =$	
	$\cos^{-1}(-1) =$	

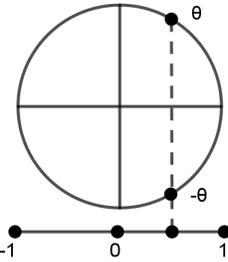
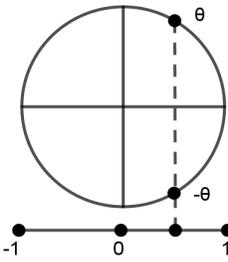
2. [Maximum mark: 28] **[without GDC]**

Write down the **general solutions** of the equations below

	we find first the principal solution $\theta = \sin^{-1} a$	
	in degrees	in radians
$\sin x = a$		
$\sin x = \frac{1}{2}$	$x = 30^\circ + 360^\circ k$ $x = 150^\circ + 360^\circ k$	$x = \frac{\pi}{6} + 2k\pi$ $x = \frac{5\pi}{6} + 2k\pi$
$\sin x = \frac{\sqrt{2}}{2}$		
$\sin x = \frac{\sqrt{3}}{2}$		
$\sin x = -\frac{1}{2}$	$x = -30^\circ + 360^\circ k$ $x = -150^\circ + 360^\circ k$	$x = -\frac{\pi}{6} + 2k\pi$ $x = -\frac{5\pi}{6} + 2k\pi$
$\sin x = -\frac{\sqrt{2}}{2}$		
$\sin x = -\frac{\sqrt{3}}{2}$		
$\sin x = 0$		
$\sin x = 1$		
$\sin x = -1$		

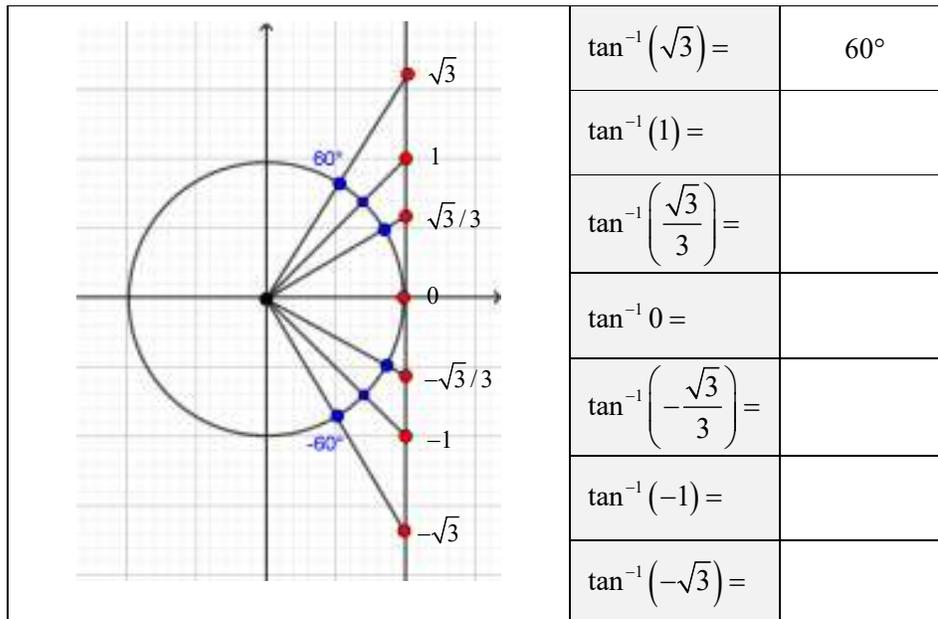
3. [Maximum mark: 28] **[without GDC]**

Write down the **general solutions** of the equations below

	we find first the principal solution $\theta = \cos^{-1} a$	
	in degrees	in radians
$\cos x = a$		
$\cos x = \frac{1}{2}$	$x = 60^\circ + 360^\circ k$ $x = -60^\circ + 360^\circ k$	$x = \frac{\pi}{3} + 2k\pi$ $x = -\frac{\pi}{3} + 2k\pi$
$\cos x = \frac{\sqrt{2}}{2}$		
$\cos x = \frac{\sqrt{3}}{2}$		
$\cos x = -\frac{1}{2}$	$x = 120^\circ + 360^\circ k$ $x = -120^\circ + 360^\circ k$	$x = \frac{2\pi}{3} + 2k\pi$ $x = -\frac{2\pi}{3} + 2k\pi$
$\cos x = -\frac{\sqrt{2}}{2}$		
$\cos x = -\frac{\sqrt{3}}{2}$		
$\cos x = 0$		
$\cos x = 1$		
$\cos x = -1$		

4. [Maximum mark: 6] **[without GDC]**

Complete the following table with the values of $\tan^{-1} a$ in **degrees**.



5. [Maximum mark: 24] **[without GDC]**

Write down the **general solutions** of the equations below

$\tan x = a$	we only need the principal solution $\theta = \tan^{-1} a$	
	in degrees	in radians
	$x = \theta + 180^\circ k$	$x = \theta + k\pi$
$\tan x = \sqrt{3}$	$x = 60^\circ + 180^\circ k$	$x = \frac{\pi}{3} + k\pi$
$\tan x = 1$		
$\tan x = \frac{\sqrt{3}}{3}$		
$\tan x = 0$		
$\tan x = -\frac{\sqrt{3}}{3}$		
$\tan x = -1$		
$\tan x = -\sqrt{3}$		

6. [Maximum mark: 24] **[without GDC]**

Solve the following equations, within the given domain.

	<i>in degrees</i> $-180^\circ \leq x \leq 180^\circ$	<i>in radians</i> $-\pi \leq x \leq \pi$
$\sin x = \frac{1}{2}$	$x = 30^\circ, \quad x = 150^\circ$	$x = \frac{\pi}{6}, \quad x = \frac{5\pi}{6}$
$\sin x = \frac{\sqrt{2}}{2}$		
$\sin x = \frac{\sqrt{3}}{2}$		
$\sin x = -\frac{1}{2}$	$x = -30^\circ, \quad x = -150^\circ$	$x = -\frac{\pi}{6}, \quad x = -\frac{5\pi}{6}$
$\sin x = -\frac{\sqrt{2}}{2}$		
$\sin x = -\frac{\sqrt{3}}{2}$		

[Confirm the results with your GDC, using SolveN]

7. [Maximum mark: 24] **[without GDC]**

Solve the following equations, within the given domain.

	<i>in degrees</i> $-180^\circ \leq x \leq 180^\circ$	<i>in radians</i> $-\pi \leq x \leq \pi$
$\cos x = \frac{1}{2}$	$x = 60^\circ, \quad x = -60^\circ$	$x = \frac{\pi}{3}, \quad x = -\frac{\pi}{3}$
$\cos x = \frac{\sqrt{2}}{2}$		
$\cos x = \frac{\sqrt{3}}{2}$		
$\cos x = -\frac{1}{2}$	$x = 120^\circ, \quad x = -120^\circ$	$x = \frac{2\pi}{3}, \quad x = -\frac{2\pi}{3}$
$\cos x = -\frac{\sqrt{2}}{2}$		
$\cos x = -\frac{\sqrt{3}}{2}$		

[Confirm the results with your GDC, using SolveN]

8. [Maximum mark: 24] **[without GDC]**

Solve the following equations, within the given domain.

	in degrees		in radians	
	$0^\circ \leq x \leq 360^\circ$	$-180^\circ \leq x \leq 180^\circ$	$0 \leq x \leq 2\pi$	$-\pi \leq x \leq \pi$
$\sin x = 0$				
$\sin x = 1$				
$\sin x = -1$				
$\cos x = 0$				
$\cos x = 1$				
$\cos x = -1$				

[Confirm the results with your GDC, using SolveN]

9. [Maximum mark: 24] **[with GDC]**

Solve the following equations, within the given domain.

	in degrees		in radians	
	$-180^\circ \leq x \leq 360^\circ$		$-\pi \leq x \leq 2\pi$	
$\sin x = \frac{1}{3}$				
$\sin x = -\frac{1}{3}$				
$\cos x = \frac{1}{3}$				
$\cos x = -\frac{1}{3}$				

10. [Maximum mark: 10] **[without GDC]**

Solve the following equations [in degrees].

(a) $\sin x = \frac{1}{2}$, for $0^\circ \leq x \leq 720^\circ$. [5]

(b) $\sin 3x = \frac{1}{2}$, for $0^\circ \leq x \leq 360^\circ$. [5]

[Confirm the results with your GDC, using SolveN]

11. [Maximum mark: 10] **[without GDC]**

Solve the following equations [in radians].

(a) $\sin x = \frac{1}{2}$, for $0 \leq x \leq 4\pi$. [5]

(b) $\sin 3x = \frac{1}{2}$, for $0 \leq x \leq 2\pi$. [5]

[Confirm the results with your GDC, using SolveN]

12. [Maximum mark: 10] **[without GDC]**

Solve the following equations [in degrees].

(a) $\cos x = \frac{1}{2}$, for $0^\circ \leq x \leq 720^\circ$. [5]

(b) $\cos 3x = \frac{1}{2}$, for $0^\circ \leq x \leq 360^\circ$. [5]

[Confirm the results with your GDC, using SolveN]

13. [Maximum mark: 10] **[without GDC]**

Solve the following equations [in radians].

(a) $\cos x = \frac{1}{2}$, for $0 \leq x \leq 4\pi$. [5]

(b) $\cos 3x = \frac{1}{2}$, for $0 \leq x \leq 2\pi$. [5]

[Confirm the results with your GDC, using SolveN]

14. [Maximum mark: 5] **[without GDC]**

Solve the equation $\sin 3x = -\frac{1}{2}$, for $-90^\circ \leq x \leq 180^\circ$ [in degrees]

[Confirm the results with your GDC, using SolveN]

15. [Maximum mark: 5] **[without GDC]**

Solve the equation $\sin 3x = -\frac{1}{2}$, for $-\frac{\pi}{2} \leq x \leq \pi$ [in radians]

[Confirm the results with your GDC, using SolveN]

16. [Maximum mark: 5] **[without GDC]**

Solve the equation $\cos 3x = -\frac{1}{2}$, for $-90^\circ \leq x \leq 180^\circ$ [in degrees]

[Confirm the results with your GDC, using SolveN]

17. [Maximum mark: 5] **[without GDC]**

Solve the equation $\cos 3x = -\frac{1}{2}$, for $-\frac{\pi}{2} \leq x \leq \pi$ [in radians]

[Confirm the results with your GDC, using SolveN]

18. [Maximum mark: 8] **[without GDC]**

Solve the equation $\sin 3x = 1$ (i) for $-90^\circ \leq x \leq 180^\circ$ (ii) for $-\frac{\pi}{2} \leq x \leq \pi$

19. [Maximum mark: 8] **[without GDC]**

Solve the equation $\cos 3x = 1$ (i) for $-90^\circ \leq x \leq 180^\circ$ (ii) for $-\frac{\pi}{2} \leq x \leq \pi$

20. [Maximum mark: 36] **[without GDC]**

Solve the following equations, within the given domain.

	<i>in degrees</i> $-180^\circ \leq x \leq 180^\circ$	<i>in radians</i> $-\pi \leq x \leq \pi$
$\tan x = \sqrt{3}$	$x = 60^\circ$, $x = -120^\circ$	$x = \frac{\pi}{3}$, $x = -\frac{2\pi}{3}$
$\tan x = 1$		
$\tan x = \frac{\sqrt{3}}{3}$		
$\tan x = 0$		
$\tan x = -\frac{\sqrt{3}}{3}$		
$\tan x = -1$		
$\tan x = -\sqrt{3}$		

21. [Maximum mark: 6] **[without GDC]**

Solve the equation $\tan x = 0$

- (i) For $0^\circ \leq x \leq 720^\circ$ [in degrees] (ii) For $0 \leq x \leq 4\pi$ [in radians]

22. [Maximum mark: 15] **[without GDC]**

Solve the following equations [in degrees]

- (a) $\tan 3x = 0$, $-90^\circ \leq x \leq 180^\circ$ [5]
 (b) $\tan 3x = 1$, $-90^\circ \leq x \leq 180^\circ$ [5]
 (c) $\tan 3x = -1$, $-90^\circ \leq x \leq 180^\circ$ [5]

[Confirm the results with your GDC, using SolveN]

23. [Maximum mark: 15] **[with / without GDC]**

Solve the following equations [in radians].

- (a) $\tan 3x = 0$, $-\frac{\pi}{2} \leq x \leq \pi$ [5]
 (b) $\tan 3x = 1$, $-\frac{\pi}{2} \leq x \leq \pi$ [5]
 (c) $\tan 3x = -1$, $-\frac{\pi}{2} \leq x \leq \pi$ [5]

[Confirm the results with your GDC, using SolveN]

24. [Maximum mark: 10] **[without GDC]**

Solve the equation, $2\sin^2 x = \sin x$,

- (a) for $0^\circ \leq x \leq 360^\circ$ [in degrees]. [5]
 (b) for $0 \leq x \leq 2\pi$ [in radians]. [5]

25*. [Maximum mark: 15] **[without GDC]**

Solve the equation $3\sin^2 x = \cos^2 x$. $0^\circ \leq x \leq 180^\circ$ in three different ways

- (a) by transforming to an equivalent expression of the form $\sin^2 x = a$. [5]
 (b) by transforming to an equivalent expression of the form $\cos^2 x = b$. [5]
 (c) by transforming to an equivalent expression of the form $\tan^2 x = c$. [5]

26. [Maximum mark: 6] **[with / without GDC]**

Solve the equation $\tan^2 x = 3$, for $-\pi \leq x \leq \pi$

27. [Maximum mark: 5] **[with / without GDC]**

Solve the equation $3\sin x = \sqrt{3}\cos x$, for $-\pi \leq x \leq \pi$.

A. Exam style questions (SHORT)

28. [Maximum mark: 6] **[without GDC]**

Solve the equation $\sin 2x = \frac{1}{2}$, for $0^\circ \leq x \leq 360^\circ$.

29. [Maximum mark: 4] **[with GDC]**

Solve the equation $\sin 2x = \frac{1}{3}$, for $0^\circ \leq x \leq 200^\circ$. Give your answers in 3.s.f.

30. [Maximum mark: 6] **[without GDC]**

Solve the equation $\sin 2x = \frac{1}{2}$, for $0 \leq x \leq 2\pi$

31. [Maximum mark: 4] **[with GDC]**

Solve the equation $\sin^2 x = \frac{1}{3}$, for $0 \leq x \leq \frac{3\pi}{2}$. Give your answers in 3.s.f.

32. [Maximum mark: 6] **[without GDC]**

Solve the equation $\cos 2x = \frac{\sqrt{3}}{2}$, for $0^\circ \leq x \leq 360^\circ$

33. [Maximum mark: 6] **[without GDC]**

Solve the equation $\cos 2x = \frac{\sqrt{3}}{2}$, for $0 \leq x \leq 2\pi$

- 34*. [Maximum mark: 6] **[without GDC]**

Solve the equation $\sin(2x - 10^\circ) = \frac{1}{2}$, for $0^\circ \leq x \leq 360^\circ$

- 35*. [Maximum mark: 6] **[without GDC]**

Solve the equation $\sin(2x - 1) = \frac{\sqrt{2}}{2}$, for $0 \leq x \leq 2\pi$

36. [Maximum mark: 6] **[without GDC]**

Solve the equation $2 \cos x = \sin 2x$, for $0 \leq x \leq 3\pi$.

37. [Maximum mark: 6] **[without GDC]**

Solve the equation $2 \cos^2 x = \sin 2x$ for $0 \leq x \leq \pi$, giving your answers in terms of π .

38. [Maximum mark: 6] **[without GDC]**

Solve $2 \sin x = \tan x$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$

39. [Maximum mark: 4] **[without GDC]**

Solve the equation $\sqrt{3} \cos x = \sin x$, for x in the interval $0^\circ \leq x \leq 360^\circ$.

40. [Maximum mark: 6] **[without GDC]**

Let $f(x) = \sqrt{3}e^{2x} \sin x + e^{2x} \cos x$, for $0 \leq x \leq \pi$. Given that $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$, solve the equation $f(x) = 0$.

41. [Maximum mark: 6] **[without GDC]**

Solve $\tan^2 2\theta = 1$, in the interval $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

42. [Maximum mark: 6] **[without GDC]**

Solve the equation $\sin 2x = 2\sqrt{3} \cos^2 x$, in the interval $0 \leq x \leq 2\pi$.

43. [Maximum mark: 6] **[without GDC]**

(a) Express $2 \cos^2 x + \sin x$ in terms of $\sin x$ only. [2]

(b) Solve the equation $2 \cos^2 x + \sin x = 2$ for x in the interval $0 \leq x \leq \pi$. [4]

44. [Maximum mark: 6] **[without GDC]**

Consider the trigonometric equation $2 \sin^2 x = 1 + \cos x$.

(a) Write this equation in the form $f(x) = 0$, where $f(x) = a \cos^2 x + b \cos x + c$,
and $a, b, c \in \mathbb{Z}$. [2]

(b) Factorize $f(x)$ [1]

(c) Solve $f(x) = 0$ for $0^\circ \leq x \leq 360^\circ$. [3]

45. [Maximum mark: 7] **[with / without GDC]**

Solve $\cos 2x - 3 \cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \leq x \leq 2\pi$.

46. [Maximum mark: 7] **[without GDC]**

(a) Show that $4 - \cos 2\theta + 5 \sin \theta \equiv 2 \sin^2 \theta + 5 \sin \theta + 3$. [2]

(b) **Hence**, solve the equation $4 - \cos 2\theta + 5 \sin \theta = 0$ for $0 \leq \theta \leq 2\pi$. [5]

47. [Maximum mark: 5] **[without GDC]**

It is given that $\tan^{-1} 0.6 = 31^\circ$ to the nearest degree. Solve the equation

$$3 \cos x = 5 \sin x, \text{ for } x \text{ in the interval } 0^\circ \leq x \leq 360^\circ,$$

giving the solutions to the nearest degree.

48. [Maximum mark: 6] **[without GDC]**

Given that $\sin^{-1} \frac{2}{3} = 42^\circ$ to the nearest degree, solve the equation

$$3 \sin^2 x - 11 \sin x + 6 = 0 \quad \text{for } 0^\circ \leq x \leq 180^\circ.$$

giving the solutions to the nearest degree.

49. [Maximum mark: 6] **[with GDC]**

(a) Write the expression $3 \sin^2 x + 4 \cos x$ in the form $a \cos^2 x + b \cos x + c$. [2]

(b) Find the two values of $\cos x$ that satisfy the equation $3 \sin^2 x + 4 \cos x - 4 = 0$. [2]

(c) Hence or otherwise, solve the equation

$$3 \sin^2 x + 4 \cos x - 4 = 0, \quad 0^\circ \leq x \leq 90^\circ \quad [2]$$

50. [Maximum mark: 6] **[with GDC]**

Solve the equations

(a) $\cos 3x = \cos(0.5x)$, for $0 \leq x \leq \pi$. [2]

(b) $3 \sin^2 x - 11 \sin x + 6 = 0$ for $0^\circ \leq x \leq 180^\circ$. [2]

B. Exam style questions (LONG)

51. [Maximum mark: 15] **[without GDC]**

Solve the following equations for x in the interval $[0, \pi]$.

(a) $\sin^3 x = \sin x$. [5]

(b) $2 \sin^3 x = \sin x$. [5]

(c) $4 \cos^3 x = \cos x$. [5]

- 52*. [Maximum mark: 15] **[without GDC]**

Consider the functions $f(x) = \cos 2x + \cos x + 1$ and $g(x) = \sin 2x + \sin x$, both defined

in the interval $0 \leq x \leq \pi$.

(a) Find the zeros of $f(x)$. [5]

(b) Find the zeros of $g(x)$. [4]

(c) Find the coordinates of the points of intersection of $y = f(x)$ and $y = g(x)$. [6]

53. [Maximum mark: 11] **[without GDC]**

Consider the equation $3 \cos 2x + \sin x = 1$.

(a) Write this equation in the form $f(x) = 0$, where $f(x) = p \sin^2 x + q \sin x + r$, and $p, q, r \in \mathbb{Z}$. [2]

(b) Factorize $f(x)$. [3]

(c) Write down the number of solutions of $f(x) = 0$, for $0 \leq x \leq 2\pi$. [2]

(d) Find the two solutions in the 3rd and 4th quadrant, that is for $\pi \leq x \leq 2\pi$ [4]