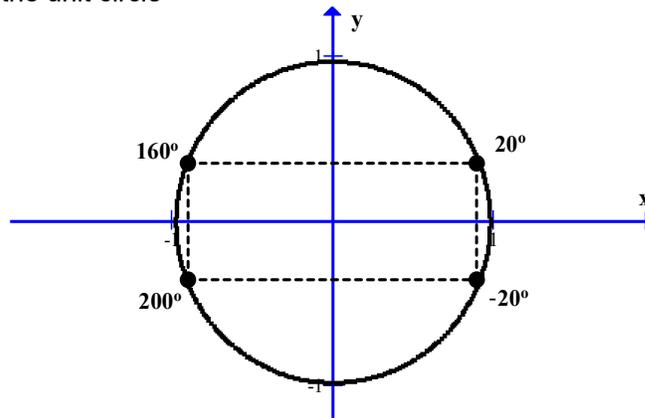> ## ***EXERCISES [Math-AA 3.5]***
> ### ***SIN, COS, TAN ON THE UNIT CIRCLE – IDENTITIES***
> *Compiled by Christos Nikolaidis*

O.   Practice questions

1.   [Maximum mark: 12]   *[without GDC]*

Let $\sin 20° = p$, and $\cos 20° = q$ (so that $p^2 + q^2 = 1$).

By observing the unit circle

express the following in terms of p **and/or** q

sin160°		sin200°	
cos160°		cos200°	
tan160°		tan200°	

sin340°		sin(−20°)	
cos340°		cos(−20°)	
tan340°		tan(−20°)	

2.   [Maximum mark: 5]   *[without GDC]*

Let $\sin 20° = p$, and $\cos 20° = q$ (so that $p^2 + q^2 = 1$)

Write down expressions for the following

$\tan 20° =$		(in terms of p and q)
$\sin 40° =$		(in terms of p and q)
$\cos 40° =$		(in terms of p and q)
		(in terms of p **only**)
		(in terms of q **only**)

3.   [Maximum mark: 7]   *[without GDC]*

Given that $\sin x = p$, where x is an **acute** angle

(a)   Find the value of $\cos x$ in terms of p ;   [2]

(b)   **Hence**, express the following in terms of p :

   (i) $\tan x$ (ii) $\cos 2x$ (iii) $\sin 2x$ (iv) $\tan 2x$ (v) $\sin 4x$ [5]

....

....

	Formula	**Expression in terms of** p
$\tan x$		
$\cos 2x$		
$\sin 2x$		
$\tan 2x$		
$\sin 4x$		

4.   [Maximum mark: 15]   *[without GDC]*

Let θ be an angle in the first quadrant, i.e. $0° < \theta < 90°$.

(a)   Given that $\sin \theta = \dfrac{3}{4}$, find   (i) $\cos \theta$ (ii) $\tan \theta$.   [5]

(b)   Given that $\cos \theta = \dfrac{3}{4}$, find   (i) $\sin \theta$ (ii) $\tan \theta$ [5]

(c)   Given that $\tan \theta = \dfrac{3}{4}$, find   (i) $\sin \theta$ (ii) $\cos \theta$ [5]

5.   [Maximum mark: 15]   *[without GDC]*

Let θ be an angle in the **second** quadrant, i.e. $90° < \theta < 180°$

(a)   Given that $\sin \theta = \dfrac{3}{4}$, find   (i) $\cos \theta$ (ii) $\tan \theta$.   [5]

(b)   Given that $\cos \theta = -\dfrac{1}{4}$, find   (i) $\sin \theta$ (ii) $\tan \theta$.   [5]

(c)   Given that $\tan \theta = -4$, find   (i) $\sin \theta$ (ii) $\cos \theta$.   [5]

6.   [Maximum mark: 6]   *[without GDC]*

Prove the identities

(a) $\dfrac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$.   (b) $\dfrac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan \theta$ [3+3]

7*.   [Maximum mark: 15]   *[without GDC]*

Let θ be an angle for which $\cos \theta + \sin \theta = a$ and $\cos \theta > \sin \theta$.

(a)   By taking squares on both sides, find the value of $\sin 2\theta$ in terms of a.   [3]

(b)   Expand $(\cos \theta - \sin \theta)^2$ and hence find the value of $\cos \theta - \sin \theta$ in terms of a.   [3]

8**.   [Maximum mark: 6]   *[without GDC]*

Let θ be an angle for which $\cos \theta + \sin \theta = \dfrac{4}{3}$.

(a)   Find $\sin 2\theta$.   [3]

(b)   Find $\cos 4\theta$.   [3]

9**.   [Maximum mark: 6]   *[without GDC]*

Let θ be an angle for which $\cos \theta - \sin \theta = \dfrac{1}{2}$.

(a)   Find $\sin 2\theta$.   [3]

(b)   Find $\cos 4\theta$.   [3]

A. Exam style questions (SHORT)

10. [Maximum mark: 4]   *[without GDC]*

Given that $\sin\theta = \dfrac{1}{2}$, $\cos\theta = -\dfrac{\sqrt{3}}{2}$ and $0° < \theta < 360°$,

(a) find the value of θ; [2]

(b) write down the **exact** value of $\tan\theta$. [2]
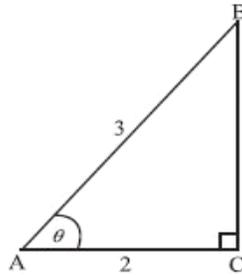
11. [Maximum mark: 5]   *[without GDC]*

Given that $\sin x = \dfrac{1}{3}$, where x is an acute angle, find the **exact** value of

(a) $\cos x$; [3]

(b) $\cos 2x$. [2]

12. [Maximum mark: 6]   *[without GDC]*

The following diagram shows a triangle ABC, where $\hat{ACB}$ is 90°, $AB = 3$, $AC = 2$ and $\hat{BAC}$ is θ.

(a) Show that $\sin 2\theta = \dfrac{4\sqrt{5}}{9}$. [3]

(b) Find the **exact** value of $\cos 2\theta$. [3]

13. [Maximum mark: 4]   *[without GDC]*

If A is an obtuse angle in a triangle and $\sin A = \dfrac{5}{13}$, calculate the value of $\sin 2A$.

14. [Maximum mark: 6]   *[without GDC]*

Let $p = \sin 40°$, $q = \cos 110°$. Give your answers to the following in terms of p and/or q

(a) Write down an expression for (i) $\sin 140°$;   (ii) $\cos 70°$. [2]

(b) Find an expression for $\cos 140°$. [3]

(c) Find an expression for $\tan 140°$. [1]

15. [Maximum mark: 6]   *[without GDC]*

(a) Given that $\cos A = \dfrac{1}{3}$ and $0 \le A \le \dfrac{\pi}{2}$, find $\cos 2A$. [3]

(b) Given that $\sin B = \dfrac{2}{3}$ and $\dfrac{\pi}{2} \le B \le \pi$, find $\cos B$. [3]

16*. [Maximum mark: 7]   *[without GDC]*

The straight line with equation $y = \dfrac{3}{4}x$ makes an acute angle θ with the x-axis.

(a)   Write down the value of $\tan\theta$ [1]

(b)   Find the value of   (i) $\sin 2\theta$;   (ii) $\cos 2\theta$.   [6]

17*. [Maximum mark: 7]   *[without GDC]*

Let $f(x) = \sin^3 x + \cos^3 x \tan x$, $\dfrac{\pi}{2} < x < \pi$.

(a)   Show that $f(x) = \sin x$.   [2]

(b)   Let $\sin x = \dfrac{2}{3}$. Show that $f(2x) = -\dfrac{4\sqrt{5}}{9}$.   [5]

18*. [Maximum mark: 6]   *[without GDC]*

By using the double angle identities for $\cos 2\theta$,

(a)   show that $\sin 15^o = \dfrac{\sqrt{2-\sqrt{3}}}{2}$;   [3]

(b)   find a similar expression for $\cos 15^o$.   [3]

19*. [Maximum mark: 5]   *[without GDC]*

Prove that $\dfrac{\sin 4\theta (1 - \cos 2\theta)}{\cos 2\theta (1 - \cos 4\theta)} = \tan\theta$, for $0 < \theta < \dfrac{\pi}{2}$, and $\theta \neq \dfrac{\pi}{4}$.

20. [Maximum mark: 6]   *[without GDC]*

Given that $2\sin 4x - 3\sin 2x = 0$, and $\sin 2x \neq 0$, find the value of $\cos^2 x$.

21*. [Maximum mark: 6]   *[without GDC]*

Given that $a\sin 4x + b\sin 2x = 0$, for $0 < x < \dfrac{\pi}{2}$, find an expression for $\cos^2 x$ in terms of a and b.

22*. [Maximum mark: 5]   *[without GDC]*

In triangle ABC, AB = 9 cm, AC = 12 cm, and $\hat{B}$ is twice the size of $\hat{C}$.

Find the cosine of $\hat{C}$.

23*. [Maximum mark: 6]   *[without GDC]*

The triangle ABC has an obtuse angle at B, BC = 10, $\hat{A} = x$ and $\hat{B} = 2x$.
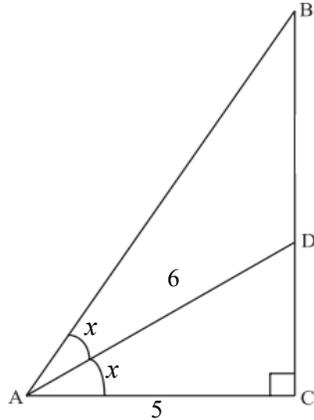
(a)   Find AC, in terms of $\cos x$.

(b)   Given that the area of triangle ABC is $50\cos x$, find angle $\hat{C}$.

B.   Exam style questions (LONG)

24.   [Maximum mark: 13]   *[without GDC]*

Let ABC be the right-angled triangle, where $\hat{\mathrm{C}} = 90$. The line (AD) bisects $\mathrm{B\hat{A}C}$.
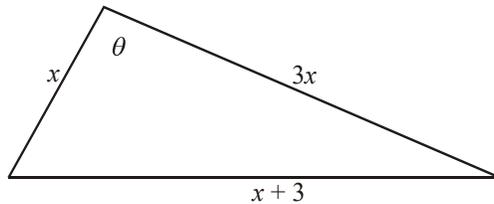
$\mathrm{AC} = 5$ and $\mathrm{AD} = 6$. as shown in the diagram.

(a)   Write down the value of $\cos x$.   [1]

(b)   Find $\cos \mathrm{B\hat{A}C}$.   [4]

(c)   **Hence** find AB.   [3]

(d)   Find $\sin \mathrm{B}$.   [2]

(e)   Find $\tan \mathrm{B\hat{A}D}$.   [3]

25*.   [Maximum mark: 10]   *[with GDC]*

The area of the triangle shown below is $2.21\,\mathrm{cm}^2$. The length of the shortest side is

$x\,\mathrm{cm}$ and the other two sides are $3x\,\mathrm{cm}$ and $(x+3)\,\mathrm{cm}$.


(a)   Using the formula for the area of the triangle, write down an expression for $\sin\theta$

in terms of x.   [2]

(b)   Using the cosine rule, write down an expression for $\cos\theta$ in terms of x [2]

(c)   (i)   Using your answers to parts (a) and (b), show that,

$$\left(\frac{3x^2 - 2x - 3}{2x^2}\right)^2 = 1 - \left(\frac{4.42}{3x^2}\right)^2$$

(ii)   Hence find

(a)   the possible values of x;

(b)   the corresponding values of θ, **in radians,** using your answer to

part (b) above.   [6]

26*. [Maximum mark: 20]   *[with GDC]*

(a)   Let $y = -16x^2 + 160x - 256$. Given that y has a maximum value, find

  (i)   the value of x giving the maximum value of y;

  (ii)   this maximum value of y.

The triangle XYZ has XZ = 6, YZ = x, XY = z as shown below. The perimeter of

triangle XYZ is 16.   [4]
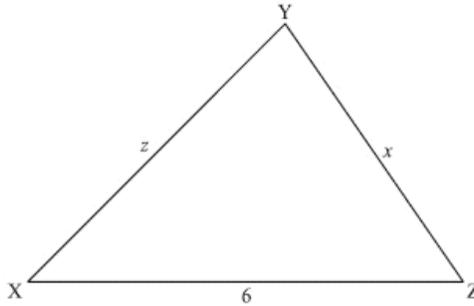
(b)   (i)   Express z in terms of x.

  (ii)   Using the cosine rule, express z^2 in terms of x and $\cos Z$.

  (iii) Hence, show that $\cos Z = \dfrac{5x - 16}{3x}$.   [7]

Let the area of triangle XYZ be A.

(c)   Show that $A^2 = 9x^2 \sin^2 Z$.   [2]

(d)   Hence, show that $A^2 = -16x^2 + 160x - 256$.   [4]

(e)   (i)   Hence, write down the maximum area for triangle XYZ.

  (ii)   What type of triangle is the triangle with maximum area?   [3]

27.   [Maximum mark: 10]   *[with GDC]*

The diagram below shows a plan for a window in the shape of a trapezium.
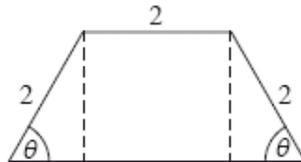
Three sides of the window are 2 m long. The angle between the sloping sides of the

window and the base is θ, where $0 < \theta < \dfrac{\pi}{2}$.

(a)   Show that the area of the window is given by $y = 4\sin\theta + 2\sin 2\theta$.   [5]

(b)   Zoe wants a window to have an area of 5 m^2. Find the two possible values of θ.   [3]

(c)   John wants two windows which have the same area A but different values of θ.

  Find all possible values for A.   [2]