INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 3.4]

ARCS AND SECTORS

Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 6] ***[without GDC]***

(a) Express the following angles in radians; give your answer in terms of π.

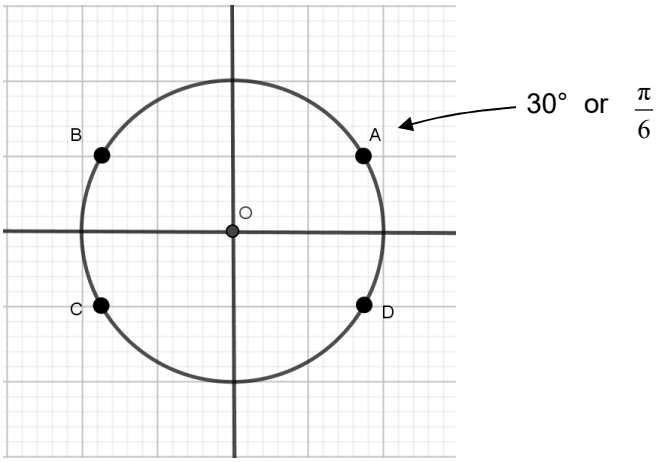
(i) 20° (ii) 18° (iii) 540° [3]

(b) Express the following angles in degrees

(i) $\frac{\pi}{18}$ rad (ii) $\frac{\pi}{5}$ rad (iii) 2.5π rad. [3]

2. [Maximum mark: 12] ***[without GDC]***

In the diagram below, the point A represents the angle of 30°, or otherwise $\frac{\pi}{6}$ rad, on the unit circle.



(a) Complete the table below with the angle values corresponding to the symmetric points B, C and D,

(i) in degrees within the interval $[0°, 360°]$

(ii) in radians within the interval $[0, 2\pi]$ [6]

	A	B	C	D
in degrees	30°			
in radians	$\frac{\pi}{6}$			

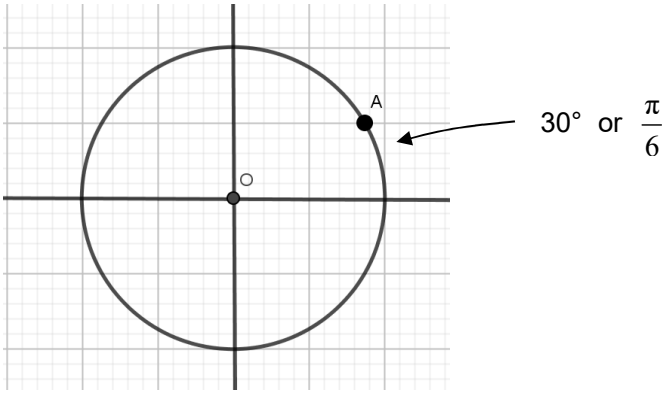
Suppose now that C represents the angle of 220°, or otherwise $\frac{11\pi}{9}$ rad, in the third quadrant. Let A, B and D be the symmetric points of C on the unit circle, as shown in the diagram above.

(b) Complete the table below in a similar way as in (a). [6]

	A	B	C	D
in degrees			220°	
in radians			$\frac{11\pi}{9}$	

3. [Maximum mark: 8] ***[without GDC]***

In the diagram below, the point A represents the angle of 30°, or otherwise $\frac{\pi}{6}$ rad, on the unit circle.



The general formula for the angles corresponding to point A is

in **degrees**: $30° + 360°k$ $\quad k \in Z$

in **radians**: $\frac{\pi}{6} + 2k\pi$ $\quad k \in Z$

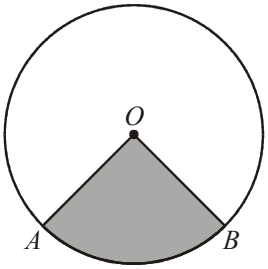
Determine the values of the angle at point A within the following intervals:

	In degrees		**in radians**	
2nd period backwards	$-720° \le \theta < -360°$		$-4\pi \le \theta < -2\pi$	
1st period backwards	$-360° \le \theta < 0°$		$-2\pi \le \theta < 0$	
1st period	$0° \le \theta < 360°$	30°	$0 \le \theta < 2\pi$	$\frac{\pi}{6}$
2nd period	$360° \le \theta < 720°$		$2\pi \le \theta < 4\pi$	
3rd period	$720° \le \theta < 1080°$		$4\pi \le \theta < 6\pi$	

[8]

4. [Maximum mark: 20] ***[without GDC]***

The diagram shows a circle center O radius of $10\ \mathrm{cm}$.



The minor sector is shaded while the major sector is unshaded.

(a) Given that the size of $\mathrm{A\hat{O}B}$ is $1.5\ \mathrm{rad}$, complete the table. [10]

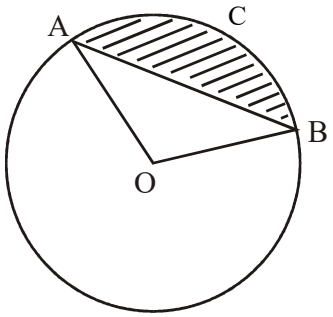
Length of	the **minor** arc AB	
	the **major** arc AB	
Area of	the **minor** sector	
	the **major** sector.	
Perimeter of	the **minor** sector	
	the **major** sector	

(b) Given that the size of $\mathrm{A\hat{O}B}$ is $\frac{\pi}{2}$ rad, complete the table. [10]

Length of	the **minor** arc AB	
	the **major** arc AB	
Area of	the **minor** sector	
	the **major** sector.	
Perimeter of	the **minor** sector	
	the **major** sector	

5. [Maximum mark: 11] ***[with GDC]***

The diagram shows a circle centre O and radius $r = 8$. The angle $\mathrm{A\hat{O}B}$ is $\theta = 1.3$ radians.



(a) Find the length of the arc ACB. [2]

(b) Find the area of the sector OACB. [2]

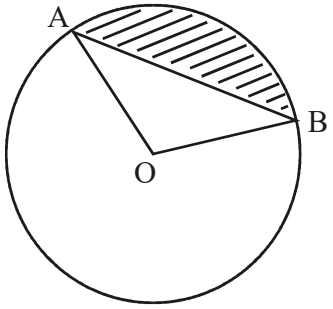
(c) Find the length of the chord AB. [3]

(d) Find the area of the triangle OAB. [2]

(e) Find the area of the shaded segment. [2]

6*. [Maximum mark: 14] ***[with GDC]***

The diagram shows a circle centre O and radius r. The angle $\mathrm{A\hat{O}B}$ is θ radians.



Find the value of θ, in each of the following three cases

(a) Given that the area of the triangle OAB is equal to the shaded area. [4]

(b) Given that ratio of the lengths of the two arcs (minor AB to major AB) is 1:5. [4]

(c) Given that the ratio of the shaded area to the non-shaded area is 1:5. [5]

A. Exam style questions (SHORT)

7. [Maximum mark: 6] ***[with GDC]***

The following diagram shows a circle of centre O, and radius 15 cm. The arc ACB subtends an angle of 2 radians at the centre O.

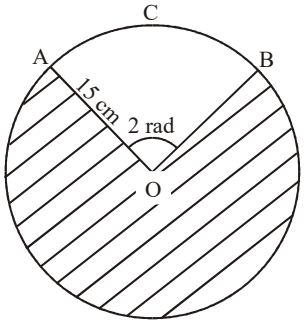


Diagram not to scale

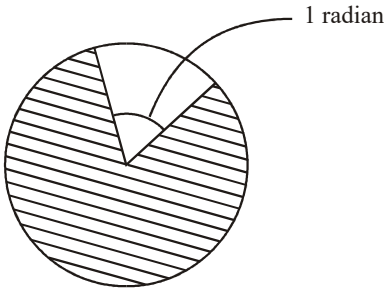
$A\hat{O}B = 2$ radians
OA = 15 cm

(a) Find the length of the arc ACB; [2]

(b) Find the area of the shaded region. [4]

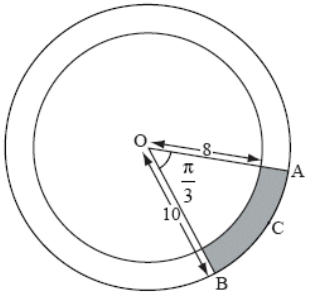
8. [Maximum mark: 4] ***[with / without GDC]***

The diagram shows a circle of radius 5 cm. Find the perimeter of the shaded region.



9. [Maximum mark: 6] ***[without GDC]***

The diagram shows two concentric circles with centre O.



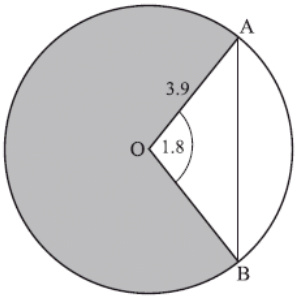
The radius of the smaller circle is 8 cm, the radius of the larger circle is 10 cm. Points A, B and C are on the circumference of the larger circle such that $A\hat{O}B$ is $\frac{\pi}{3}$ radians

(a) Find the length of the arc ACB. [2]

(b) Find the area of the shaded region. [4]

10. [Maximum mark: 7] ***[with GDC]***

The circle shown has centre O and radius 3.9 cm. Points A and B lie on the circle and angle AOB is 1.8 radians.

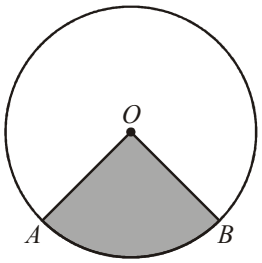


(a) Find AB. [3]

(b) Find the area of the shaded region. [4]

11. [Maximum mark: 4] ***[with GDC]***

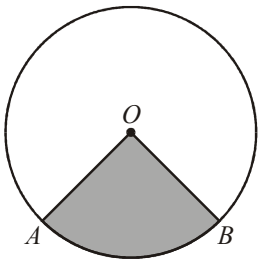
O is the centre of the circle which has a radius of $5.4\ \text{cm}$.



The area of the shaded sector OAB is $21.6\ \text{cm}^2$. Find the length of the minor arc AB.

12. [Maximum mark: 6] ***[with GDC]*** [diagram as above]

The diagram below shows a circle of centre O, and radius r.



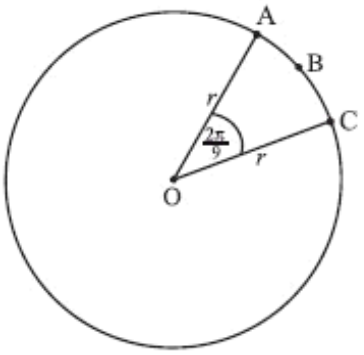
The shaded sector OAB has an area of $27\ \text{cm}^2$. Angle $\text{A}\hat{\text{O}}\text{B} = \theta = 1.5$ radians.

(a) Find the radius r. [4]

(b) Calculate the length of the minor arc AB. [2]

13. [Maximum mark: 6] ***[with GDC]***

The diagram below shows a circle centre O, with radius r.



The length of arc ABC is 3π cm and $\text{A}\hat{\text{O}}\text{C} = \frac{2\pi}{9}$.

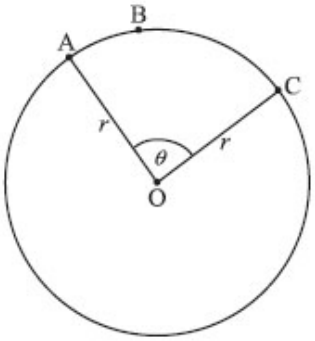
(a) Find the value of r. [2]

(b) Find the perimeter of sector OABC. [2]

(c) Find the area of sector OABC. [2]

14. [Maximum mark: 6] ***[without GDC]***

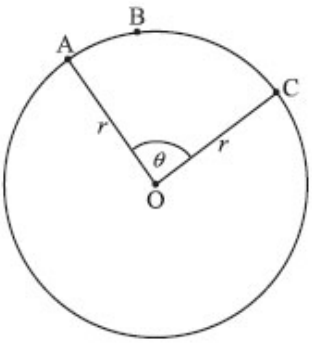
The following diagram shows a circle with radius r and centre O. The points A, B and C are on the circle and $\text{A}\hat{\text{O}}\text{C} = \theta$.



The area of sector OABC is $\frac{4}{3}\pi$, the length of arc ABC is $\frac{2}{3}\pi$.

15. [Maximum mark: 6] ***[with GDC]***

The area of the sector OAC is 180 cm^2, the length of the arc AC is 24 cm.



Find the value of r and of θ.

16. [Maximum mark: 5] ***[with GDC]***

The diagram below shows a sector AOB of a circle of radius 15 cm and centre O. The angle θ at the centre of the circle is 2 radians.

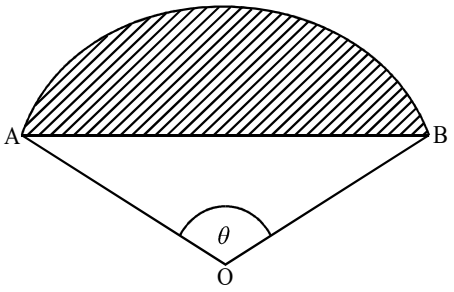


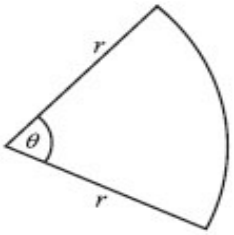
Diagram not to scale

(a) Calculate the area of the sector AOB. [2]

(b) Calculate the area of the shaded region. [3]

17. [Maximum mark: 6] ***[with GDC]***

The following diagram shows a sector of a circle of radius r cm, and angle θ at the centre. The perimeter of the sector is 20 cm.

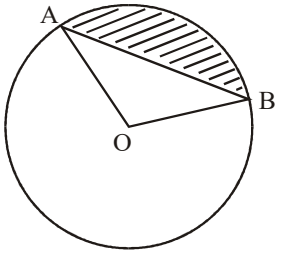


(a) Show that $\theta = \frac{20-2r}{r}$. [2]

(b) The area of the sector is 25 cm^2. Find the value of r. [4]

18. [Maximum mark: 6] ***[with GDC]***

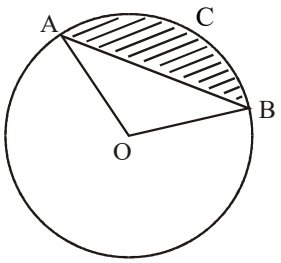
The diagram shows a circle centre O and radius $\mathrm{OA}=5\,\mathrm{cm}$. The angle $\mathrm{AOB}=135°$.



Find the area of the shaded region.

19. [Maximum mark: 6] ***[with GDC]***

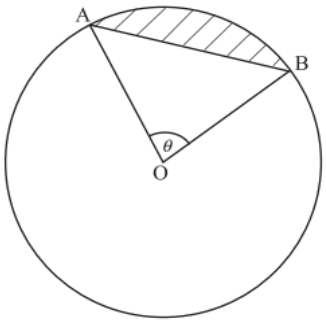
The diagram shows a circle centre O and radius r. The length of the arc ACB is $2r$.



The area of the shaded segment may be expressed as kr^2. Find the value of k.

20*. [Maximum mark: 6] ***[with GDC]***

The diagram shows a circle centre O and radius 1, with $\mathrm{A\hat{O}B} = \theta$, $\theta \neq 0$. The area of $\Delta\,\mathrm{AOB}$ is three times the shaded area.



(a) Show that $3\theta = 4\sin\theta$. [4]

(b) Find the value of θ. [2]

21*. [Maximum mark: 6] ***[without GDC]***

The diagram above [Ex. 20] shows a circle centre O and radius r, with $\mathrm{A\hat{O}B} = \theta$.

The ratio of the shaded area to the area of $\Delta\,\mathrm{AOB}$ is 2:5. Show that $5\theta = 7\sin\theta$.

22*. [Maximum mark: 6] ***[with / without GDC]***

In the following diagram, O is the centre of the circle and (AT) is the tangent to the circle at T.

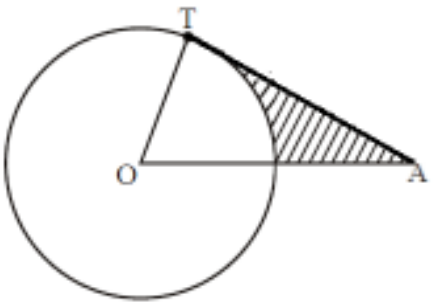
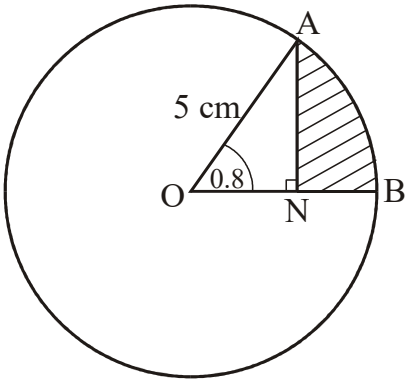


Diagram not to scale

If OA = 12 cm, and the circle has a radius of 6 cm, find the area of the shaded region.

23*. [Maximum mark: 6] ***[with GDC]***

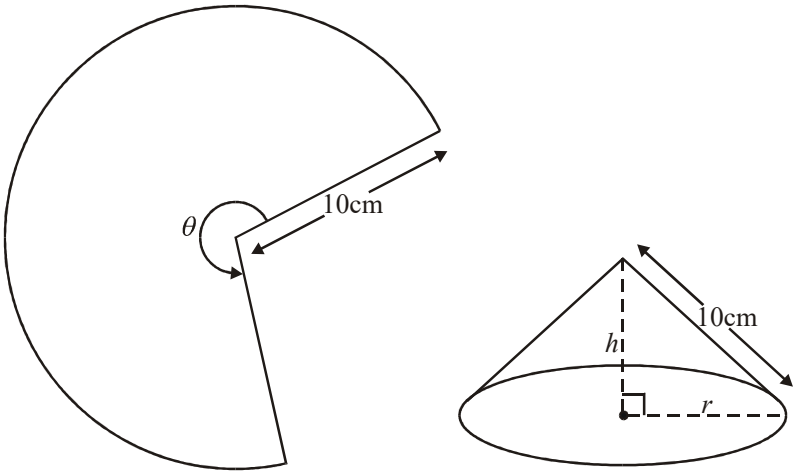
The diagram below shows a circle of radius 5 cm with centre O. Points A and B are on the circle, and $\mathrm{A\hat{O}B}$ is 0.8 radians. The point N is on [OB] such that [AN] is perpendicular to [OB]..



Find the area of the shaded region.

24*. [Maximum mark: 6] ***[without GDC]***

The diagrams show a circular sector of radius $10\ \mathrm{cm}$ and angle θ radians which is formed into a cone of slant height $10\ \mathrm{cm}$. The vertical height h of the cone is equal to the radius r of its base.



(a) Find the value of r. [2]

(b) Find the angle θ radians. [4]

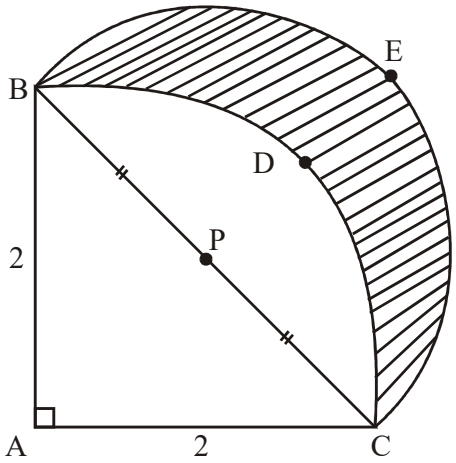
25. [Maximum mark: 6] ***[without GDC]***

The diagram below shows a triangle and two arcs of circles.

The triangle ABC is a right-angled isosceles triangle, with $AB = AC = 2$. The point P is the midpoint of $[BC]$.

The arc BDC is part of a circle with centre A.

The arc BEC is part of a circle with centre P.

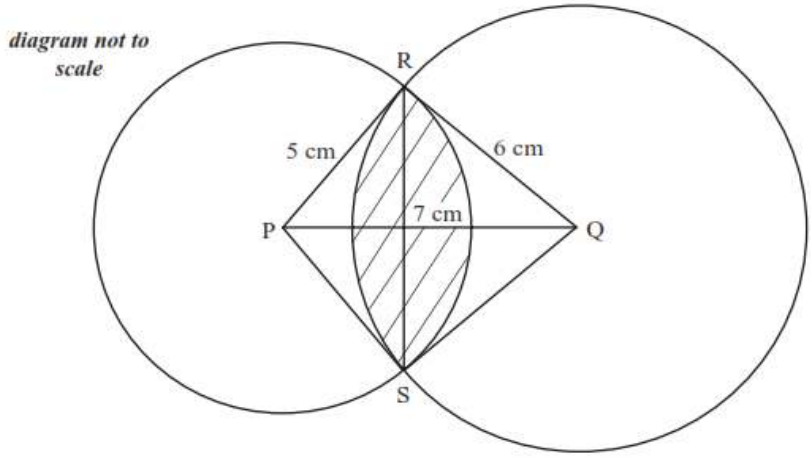


(a) Calculate the area of the segment $BDCP$. [3]

(b) Calculate the area of the shaded region $BECD$. [3]

26*. [Maximum mark: 7] ***[with GDC]***

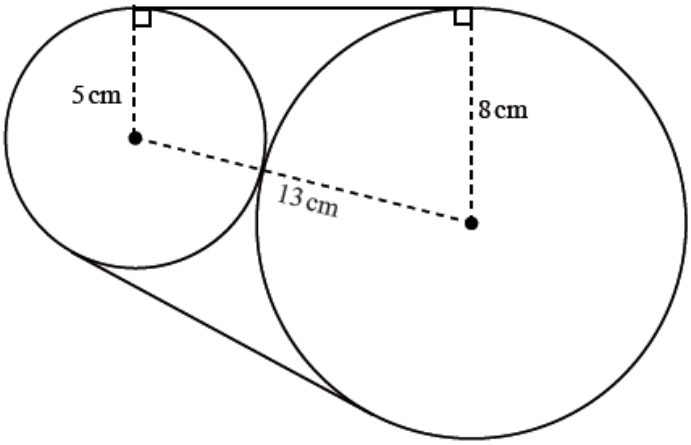
The diagram below shows a pair of intersecting circles with centres at P and Q with radii of 5 cm and 6 cm respectively. RS is the common chord of both circles and PQ is 7 cm.



Find the area of the shaded region.

27*. [Maximum mark: 8] ***[with GDC]***

Two discs, one of radius 8 cm and one of radius 5 cm, are placed such that they touch each other. A piece of string is wrapped around the discs. This is shown in the diagram below.



Calculate the length of the string needed to go around the discs.

28*. [Maximum mark: 5] ***[without GDC]***

A circular disc is cut into twelve sectors whose areas are in an arithmetic sequence.

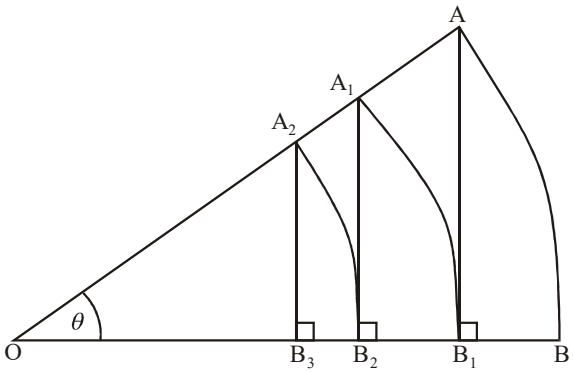
(a) Explain why the angles of the sectors are also in an arithmetic sequence. [2]

(b) The angle of the largest sector is twice the angle of the smallest sector. Find the size of the angle of the smallest sector. [5]

29*. [Maximum mark: 7] ***[with GDC]***

The diagram shows a sector AOB of a circle of radius 1 and centre O, where $\mathrm{A\hat{O}B} = \theta$

The lines (AB_1), (A_1B_2), (A_2B_3) are perpendicular to OB. A_1B_1, A_2B_2 are all arcs of circles with centre O.



(a) Calculate, in terms of θ, the sum to infinity of the arc lengths

$$AB + A_1B_1 + A_2B_2 + A_3B_3 + \ldots$$ [5]

(b) Given that the sum to infinity of the arc lengths is $\frac{2\pi}{3}$, find the value of θ. [2]

B. Exam style questions (LONG)

30. [Maximum mark: 13] ***[with GDC]***

The following diagram shows a circle with centre O and radius 4 cm.

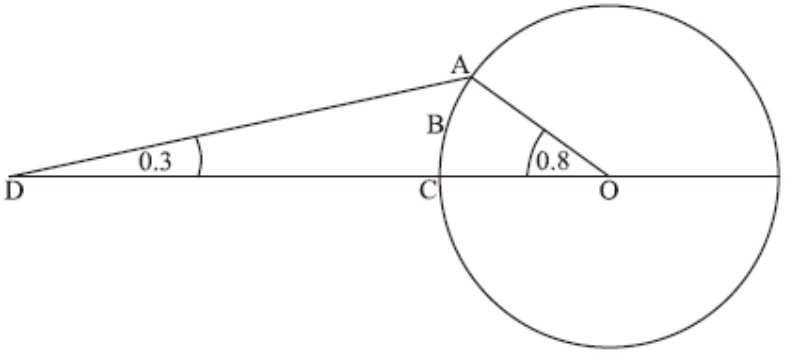


diagram not to scale

The points A, B and C lie on the circle. The point D is outside the circle, on (OC).

Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

(a) Find AD. [3]

(b) Find OD. [4]

(c) Find the area of sector OABC. [2]

(d) Find the area of region ABCD. [4]

31. [Maximum mark: 13] ***[with GDC]***

The diagram shows a circle, centre O, with a radius 12cm. The chord AB subtends at an angle of 75° at the centre. The tangents to the circle at A and at B meet at P.

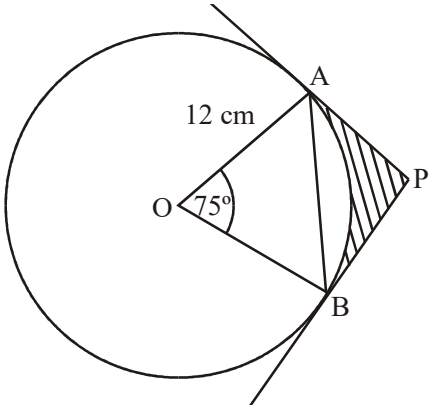


diagram not to scale

(a) Using the cosine rule, show that the length of AB is $12\sqrt{2(1-\cos 75°)}$. [2]

(b) Find the length of BP. [3]

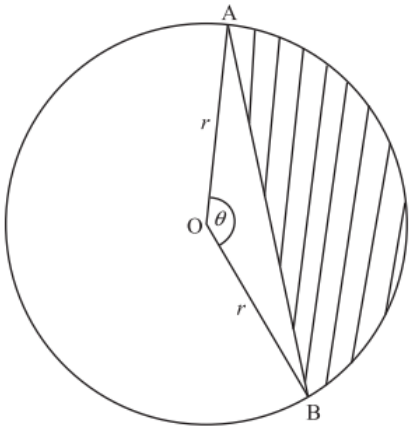
(c) Hence find (i) the area of triangle OBP; (ii) the area of triangle ABP. [4]

(d) Find the area of **sector** OAB. [2]

(e) Find the area of the shaded region. [2]

32. [Maximum mark: 10] ***[with GDC]***

The following diagram shows a circle centre O, radius r. The angle $\mathrm{A\hat{O}B}$ at the centre of the circle is θ radians. The chord AB divides the circle into a minor segment (the shades region) and a major segment.



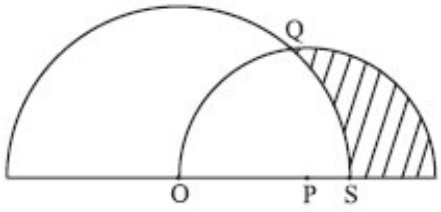
(a) Show that the area of the minor segment is $\frac{1}{2}r^2(\theta - \sin\theta)$. [3]

(b) Given that the ratio of the areas of the two segments is 2:3, show that $\sin\theta = \theta - \frac{4\pi}{5}$. [5]

(c) Hence find the value of θ. [2]

33*. [Maximum mark: 17] ***[with GDC]***

The following diagram shows two semi-circles. The larger one has centre O and radius 4 cm. The smaller one has centre P, radius 3 cm, and passes through O. The line (OP) meets the larger semi-circle at S. The semi-circles intersect at Q.



(a) (i) Explain why OPQ is an isosceles triangle.

(ii) Use the cosine rule to show that $\cos \mathrm{O\hat{P}Q} = \frac{1}{9}$.

(iii) Hence show that $\sin \mathrm{O\hat{P}Q} = \frac{\sqrt{80}}{9}$.

(iv) Find the area of the triangle OPQ. [7]

(b) Consider the smaller semi-circle, with centre P.

(i) Write down the size of $\mathrm{O\hat{P}Q}$.

(ii) Calculate the area of the sector OPQ. [3]

(c) Consider the semi-circle, with centre O. Calculate the area of the sector QOS.

(d) Hence calculate the area of the shaded region. [4]

34*. [Maximum mark: 20] ***[with GDC]***

The following diagram shows the triangle AOP, where $OP = 2\text{cm}$, $AP = 4\text{cm}$ and $AO = 3\text{cm}$

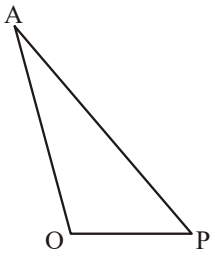


diagram not to scale

(a) Calculate $A\hat{O}P$, giving your answer in radians. [3]

The following diagram shows two circles which intersect at the points A and B. The smaller circle C_1 has centre O and radius 3 cm, the larger circle C_2 has centre P and radius 4 cm, and OP = 2 cm. The point D lies on the circumference of C_1 and E on the circumference of C_2. Triangle AOP is the same as triangle AOP in the diagram above.

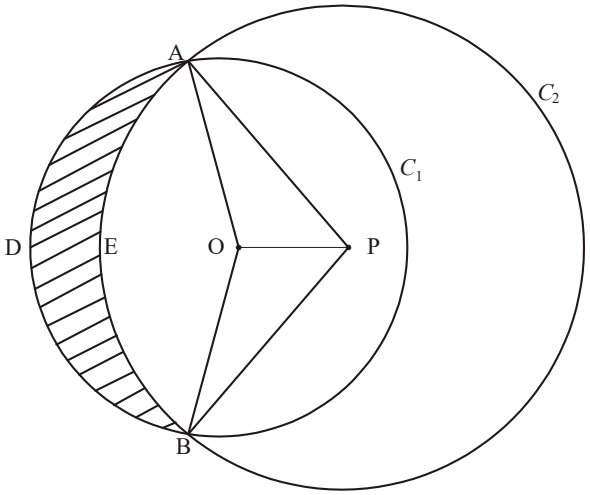


diagram not to scale

(b) Find $A\hat{O}B$, giving your answer in radians. [2]

(c) Show that $A\hat{P}B \cong 1.63$ radians. [3]

(d) Calculate the area of

(i) sector PAEB; (ii) sector OADB. [5]

(e) Show that the area of the quadrilateral AOBP $\cong$ 5.81 cm^2. [3]

(f) Find the areas of

(i) AOBE. (ii) the shaded region AEBD. [4]

35. [Maximum mark: 15] ***[with GDC]***

The diagram below shows a circle with centre O and radius 8 cm.

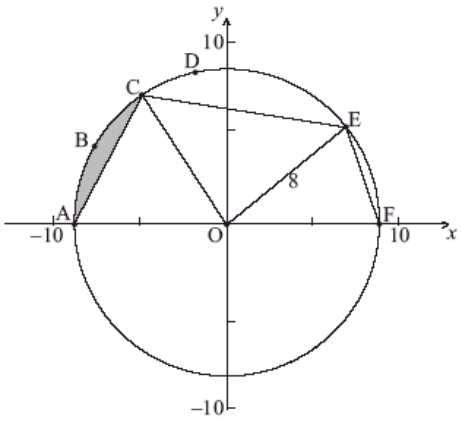


diagram not to scale

The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length of arc ABC is 6 cm.

(a) Find the size of angle AOC. [2]

(b) Hence find the area of the shaded region. [6]

The area of sector OCDE is 45 cm^2.

(c) Find the size of angle COE. [2]

(d) Find EF. [5]