INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 3.18-3.19]

INTERSECTIONS AMONG LINES AND PLANES – DISTANCES

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O. Practice questions

1. [Maximum mark: 6] **[without GDC]**

Consider

line $L:\ \boldsymbol{r}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$ and plane $\Pi:\ x+2y+3z=6$.

(a) (i) Explain why the following lines are parallel to L.

(ii) Determine whether each line coincides with line L or is distinct from it.

$L_1:\ \boldsymbol{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$ $L_2:\ \boldsymbol{r}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\4\\6\end{pmatrix}$ $L_3:\ \boldsymbol{r}=\begin{pmatrix}2\\3\\4\end{pmatrix}+\lambda\begin{pmatrix}2\\4\\6\end{pmatrix}$ [3]

(b) (i) Explain why the following planes are parallel to Π.

(ii) Determine whether each plane coincides with plane Π or is distinct from it.

$\Pi_1:\ x+2y+3z=7$, $\Pi_2:\ 2x+4y+6z=6$, $\Pi_3:\ 2x+4y+6z=12$ [3]

2. [Maximum mark: 21] **[with GDC]**

Consider the lines $L_1:\ \boldsymbol{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\1\end{pmatrix}$ $L_2:\ \boldsymbol{r}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\mu\begin{pmatrix}4\\7\\4\end{pmatrix}$

and the planes $\Pi_1:\ 2x+3y+z=7$ $\Pi_2:\ 4x+7y+4z=19$

Find the following.

(a) The acute angle between the lines L_1 and L_2. [4]

(b) The acute angle between the planes Π_1 and Π_2. [1]

(c) The acute angle between the line L_1 and the plane Π_2. [2]

(d) The acute angle between the y-axis and the plane Π_2. [3]

(e) The point of intersection of the lines L_1 and L_2. [5]

(f) The point of intersection of the line L_1 and plane Π_1. [3]

(g) The line of intersection of the planes Π_1 and Π_2. [3]

3. [Maximum mark: 10] **[without GDC]**

Consider the plane Π: $x+2y+3z=6$

and the lines L_1: $\boldsymbol{r}=\begin{pmatrix}2\\3\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$, L_2: $\boldsymbol{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$, L_3: $\boldsymbol{r}=\begin{pmatrix}1\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\-1\end{pmatrix}$.

By attempting to find the intersection of each line with the plane show that

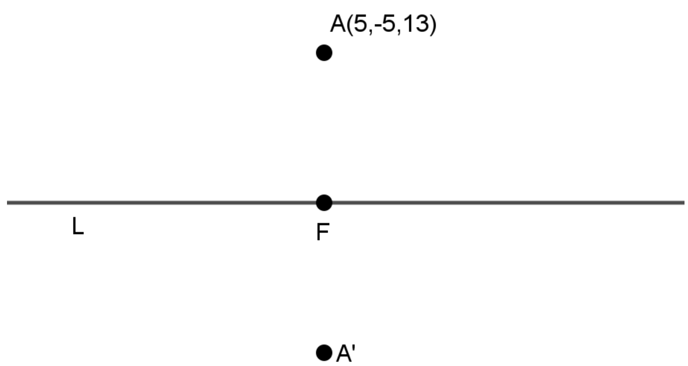
(a) the line L_1 intersects the plane Π at some point P; find the coordinates of P. [4]

(b) the line L_2 is parallel to the plane Π. [3]

(c) the line L_3 lies in the plane Π. [3]

4. [Maximum mark: 8] **[with / without GDC]**

Consider the point A(5,-5,13) and the line L: $\boldsymbol{r}=\begin{pmatrix}2\\3\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$



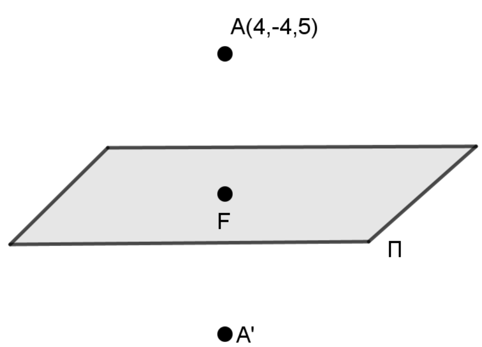
(a) Find the closest point to A on line L (that is the **foot** of the perpendicular line). [4]

(b) Find the **minimum distance** from A to L. [2]

(c) Point A is reflected in L. Find the coordinates of the image A´. [2]

5. [Maximum mark: 8] **[without GDC]**

Consider the point A(4,-4,5) and the plane Π: $x-5y+3z=4$.



(a) Find the closest point to A on plane Π (that is the **foot** of the perpendicular line). [4]

(b) Find the **minimum distance** from A to Π. [2]

(c) Point A is reflected in Π. Find the coordinates of the image A´. [2]

6. [Maximum mark: 10] **[without GDC]**

(a) Find the (minimum) **distance** between the parallel lines

$$L_1: \ \boldsymbol{r} = \begin{pmatrix} 5 \\ -5 \\ 13 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad L_2: \ \boldsymbol{r} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$ [5]

(b) Find the (minimum) **distance** between the parallel planes

$$\Pi_1: \ x + 2y + 3z = 6$$

$$\Pi_2: \ x + 2y + 3z = 20.$$ [5]

7. [Maximum mark: 9] **[without GDC]**

Consider the two planes below

$$\Pi_1: \ x + 2y + 3z = 6$$

$$\Pi_2: \ 3x + 8y + 11z = 30$$

Find the line of intersection L of the two planes by following three different methods:

(a) By finding one common point A, letting $z = 0$, and the direction vector of L obtained by the cross product of the two normal vectors

(b) By finding two common points, A as above, B by letting $y = 0$, and then the line passing through A and B.

(c) By using Gaussian elimination (where the third line is (0 0 0 0)

8. [Maximum mark: 9] **[with GDC]**

(a) Solve the following systems of simultaneous equations

(i)
$$\begin{aligned} 3x + 7y + z &= 20 \\ 2x + y + 5z &= 19 \\ 5x + 2y + 3z &= 18 \end{aligned}$$

(ii)
$$\begin{aligned} 3x + 7y + z &= 20 \\ 2x + y + 5z &= 19 \\ 5x + 8y + 6z &= 40 \end{aligned}$$

(iii)
$$\begin{aligned} 3x + 7y + z &= 20 \\ 2x + y + 5z &= 19 \\ 5x + 8y + 6z &= 39 \end{aligned}$$

(iv)
$$\begin{aligned} 3x + 7y + z &= 20 \\ 3x + 7y + z &= 21 \\ 3x + 7y + z &= 25 \end{aligned}$$ [4]

(b) Correspond the following geometric descriptions to the systems above:

Description	System
The 3 planes are **parallel**	
The 3 planes form a **triangular prism**	
The 3 planes intersect at a **unique point**	
The 3 planes intersect **in a line**	

[3]

(c) Write down the vector equation of the line in the last case of question (b). [2]

9. [Maximum mark: 9] **[without GDC]**

Consider the three planes below (where a,b are real parameters)

$$\begin{aligned} x+2y+5z&=7 \\ 2x+5y+3z&=7 \\ 3x+8y+az&=b \end{aligned} \qquad \text{equivalent augmented matrix: } \left(\begin{array}{ccc|c} 1 & 2 & 5 & 7 \\ 0 & 1 & -7 & 7 \\ 0 & 0 & a-1 & b-7 \end{array}\right)$$

(a) Show that the augmented matrix of the system (on the left), results to the equivalent augmented matrix on the right.

(b) Hence, determine for which values of a and b the three planes

(i) intersect at a unique point

(ii) form a triangular prism

(iii) intersect in a line.

(c) Find the unique solution in the case where $a=2$, $b=7$

(d) In the case of question (b)(iii), find the vector equation of the line of intersection.

10. [Maximum mark: 22] ***[without GDC]***

Please revisit the exercises [**Math-AA 1.10] SYSTEMS OF LINEAR EQUATIONS**

Exercise	Determine the geometric interpretation of each result
1	If $a \neq 0$, The lines intersect at some unique point. If $a=0$, $b \neq 0$ No point of intersection. The three planes form a prism If $a=0$, $b=0$ The planes intersect in the line: $\boldsymbol{r}=\begin{pmatrix}-4\\7\\0\end{pmatrix}+\lambda\begin{pmatrix}7\\-5\\1\end{pmatrix}$
2	(i) (ii) (iii)
3	
4	
5	
6	
7	
8	If $k \neq 3$: If $k=3$:
9	If $k \neq 5$: If $k=5$:
10	If $k \neq 1$: If $k=1$:
11	If $a \neq -1$: If $a=-1$:
12	If $k \neq 3, \frac{1}{3}$: If $k=\frac{1}{3}$: If $k=3$:
13	If $a \neq 7$, If $a=7$, $b \neq 2$ If $a=7$, $b=2$

A. Exam style questions (SHORT)

11. [Maximum mark: 6] ***[with GDC]***

A ray of light coming from the point $(-1, 3, 2)$ is travelling in the direction of vector $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ and meets the plane $\pi:\ x+3y+2z-24=0$. Find the angle that the ray of light makes with the plane.

12. [Maximum mark: 6] ***[with GDC]***

The line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+1}{2}$ is reflected in the plane $x+y+z=1$. Calculate the angle between the line and its reflection. Give your answer in **radians**.

13. [Maximum mark: 6] ***[with GDC]***

Find the cosine of the angle θ between the planes π_1 and π_2, where

π_1 has equation $-2x+y-z=2$ and π_2 has equation $x+2y-z=6$.

14. [Maximum mark: 6] ***[with GDC]***

Find the angle between the plane $3x-2y+4z=12$ and the z-axis. Give your answer to the nearest degree.

15. [Maximum mark: 6] ***[without GDC]***

A plane Π has equation $\boldsymbol{r}\bullet\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}=16$ and a line l has equations $\frac{x-4}{-1}=\frac{y+2}{2}=\frac{z-6}{4}$

Show that the line l lies in the plane.

16. [Maximum mark: 5] ***[without GDC]***

The plane $6x-2y+z=11$ contains the line $x-1=\frac{y+1}{2}=\frac{z-3}{l}$. Find l.

17. [Maximum mark: 5] ***[without GDC]***

Find the coordinates of the point where the line given by the parametric equations $x=2\lambda+4$, $y=-\lambda-2$, $z=3\lambda+2$, intersects the plane with equation $2x+3y-z=2$.

18. [Maximum mark: 5] ***[without GDC]***

Find the coordinates of the point of intersection of the line L with the plane P where:

$$L:\frac{x+3}{2}=\frac{y-1}{-1}=\frac{z-1}{2}$$
$$P:2x+3y-z=-5$$

19. [Maximum mark: 6] ***[without GDC]***

The line $\boldsymbol{r} = \boldsymbol{i} + \boldsymbol{k} + \mu(\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k})$ and the plane $2x - y + z + 2 = 0$ intersect at the point P.

Find the coordinates of P.

20. [Maximum mark: 6] ***[without GDC]***

The line $\frac{x-3}{2} = y+1 = \frac{5-z}{3}$ and the plane $2x - y + 3z = 10$ intersect at the point P.

Find the coordinates of P.

21. [Maximum mark: 5] ***[with / without GDC]***

Find the equation of the line of intersection of the two planes

$$-4x + y + z = -2 \text{ and } 3x - y + 2z = -1.$$

22. [Maximum mark: 5] ***[with / without GDC]***

Find an equation for the line of intersection of the following two planes.

$$x + 2y - 3z = 2$$
$$2x + 3y - 5z = 3$$

23. [Maximum mark: 6] ***[without GDC]***

The point A is the foot of the perpendicular from the point $(1, 1, 9)$ to the plane $2x + y - z = 6$. Find the coordinates of A.

24. [Maximum mark: 6] ***[without GDC]***

Let P be the point $(1, 0, -2)$ and Π be the plane $x + y - 2z + 3 = 0$. Let P′ be the reflection of P in the plane Π. Find the coordinates of the point P′.

25. [Maximum mark: 6] ***[without GDC]***

(a) If $\boldsymbol{u} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{v} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, show that $\boldsymbol{u} \times \boldsymbol{v} = 7\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$. [2]

(b) Let $\boldsymbol{w} = \lambda\boldsymbol{u} + \mu\boldsymbol{v}$ where λ and μ are scalars. Show that *w* is perpendicular to the line of intersection of the planes $x + 2y + 3z = 5$ and $2x - y + 2z = 7$ for all values of λ and μ. [4]

B. Exam style questions (LONG)

26. [Maximum mark: 12] ***[with GDC]***

(a) The point $\mathrm{P}(1, 2, 11)$ lies in the plane π_1. The vector $3\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to π_1. Find the Cartesian equation of π_1. [2]

(b) The plane π_2 has equation $x + 3y - z = -4$.

(i) Show that the point P also lies in the plane π_2.

(ii) Find a vector equation of the line of intersection of π_1 and π_2. [5]

(c) Find the acute angle between π_1 and π_2. [5]

27. [Maximum mark: 11] ***[without GDC]***

The plane π contains the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-5}{6}$ and the point $(1, -2, 3)$.

(a) Show that the equation of π is $6x + 2y - 3z = -7$. [7]

(b) Calculate the distance of the plane π from the origin. [4]

28. [Maximum mark: 13] ***[with GDC]***

A plane $\boldsymbol{\pi_1}$ has equation $\boldsymbol{r} \bullet \begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix} = 15$.

(a) A point $\mathrm{P}(p, -p, p)$ lies on plane $\boldsymbol{\pi_1}$ and Q is the point where the plane $\boldsymbol{\pi_1}$ meets the y-axis.

(i) Find the coordinates of P and of Q.

(ii) Show that $\overrightarrow{PQ}$ is parallel to the vector $\boldsymbol{u}$, where $\boldsymbol{u} = \boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$. [5]

(b) Another plane $\boldsymbol{\pi_2}$ intersects $\boldsymbol{\pi_1}$ in the line (PQ). The point $\mathrm{T}(1, 0, -1)$ lies on $\boldsymbol{\pi_2}$.

(i) Find the equation of $\boldsymbol{\pi_2}$, giving your answer in the form $Ax + By + Cz = D$.

(ii) Find the angle between $\boldsymbol{\pi_1}$ and $\boldsymbol{\pi_2}$. [8]

29. [Maximum mark: 13] ***[with / without GDC]***

(a) Solve the following system of linear equations

$$\begin{aligned} x + 3y - 2z &= -6 \\ 2x + y + 3z &= 7 \\ 3x - y + z &= 6 \end{aligned}$$

[3]

(b) Find the vector $\boldsymbol{v} = (\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}) \times (2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$. [3]

(c) If $\boldsymbol{a} = \boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{u} = m\boldsymbol{a} + n\boldsymbol{b}$ where m, n are scalars, and $\boldsymbol{u} \neq 0$, show that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{u}$ for all m and n. [3]

(d) The line l lies in the plane $3x - y + z = 6$, passes through the point $(1, -1, 2)$ and is perpendicular to v. Find the equation of l. [4]

30. [Maximum mark: 10] ***[with GDC]***

The coordinates of the points P, Q, R and S are (4,1,–1), (3,3,5), (1,0, 2), and (1,1,2), respectively.

(a) Find an equation of the line l which passes through point Q and is parallel to $\overrightarrow{PR}$.

(b) Find an equation of the plane π which contains the line l and passes through S.

(c) Find the shortest distance between the point P and the plane π. [4]

31. [Maximum mark: 15] ***[without GDC]***

Consider the plane $P: -x+y+2z=3$ and the point D(2, –1, –6).

(a) Find a set of parametric equations for the line L through the point D and perpendicular to the plane P. [3]

(b) Find the point of intersection E, of the line L and the plane P. [4]

(c) Find the distance from the point D to the plane P. [2]

(d) Find a unit vector that is perpendicular to the plane P. [2]

(e) The point F is a reflection of D in the plane P. Find the coordinates of F. [4]

32. [Maximum mark: 14] ***[without GDC]***

The points A, B, C, D have the following coordinates

A : (1, 3, 1) B : (1, 2, 4) C : (2, 3, 6) D : (5, – 2, 1).

The plane containing the points A,B,C is denoted by Π and the line passing through D perpendicular to Π is denoted by L. Let P be the point of intersection of L and Π.

(a) Find the cartesian equation of Π. [5]

(b) Find the cartesian equations of L. [4]

(c) Determine the coordinates of P. [3]

(d) Find the perpendicular distance of D from Π. [2]

33. [Maximum mark: 16] ***[without GDC]***

A line l_1 has equation $\frac{x+2}{3}=\frac{y}{1}=\frac{z-9}{-2}$. Let M be a point on l_1 with parameter μ.

(a) Express the coordinates of M in terms of μ. [1]

(b) The line l_2 is parallel to l_1 and passes through P(4, 0, –3).

(i) Write down an equation for l_2. (ii) Express $\overrightarrow{PM}$ in terms of μ. [4]

(c) The vector $\overrightarrow{PM}$ is perpendicular to l_1.

(i) Find the value of μ. (ii) Find the distance between l_1 and l_2. [5]

(d) The plane π_1 contains l_1 and l_2. Find an equation for π_1, giving your answer in the form $Ax + By + Cz = D$. [4]

(e) The plane π_2 has equation $x-5y-z=-11$. Verify that l_1 is the line of intersection of the planes π_1 and π_2. [2]

34. [Maximum mark: 29] ***[without GDC]***

(a) Show that lines $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$ intersect and find the coordinates of P, the point of intersection. [8]

(b) Find the Cartesian equation of the plane Π that contains the two lines. [6]

(c) The point $Q\,(3, 4, 3)$ lies on Π. The line L passes through the midpoint of [PQ]. Point S is on L such that $|\overrightarrow{PS}|=|\overrightarrow{QS}|=3$, and the triangle PQS is normal to the plane Π. Given that there are two possible positions for S, find their coordinates. [15]

35. [Maximum mark: 12] ***[with / without GDC]***

(a) Consider the planes $\pi_1 : r=\begin{pmatrix}2\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\8\end{pmatrix}+\mu\begin{pmatrix}1\\-3\\-9\end{pmatrix}$ and $\pi_2 : r=\begin{pmatrix}2\\0\\1\end{pmatrix}+s\begin{pmatrix}1\\2\\1\end{pmatrix}+t\begin{pmatrix}1\\1\\1\end{pmatrix}$.

(i) For points which lie in π_1 and π_2, show that, $\lambda=\mu$.

(ii) Hence, find a vector equation of the line of intersection of π_1 and π_2. [5]

(b) The plane π_3 contains the line $\frac{2-x}{3}=\frac{y}{-4}=z+1$ and is perpendicular to $3\boldsymbol{i}-2\boldsymbol{j}+\boldsymbol{k}$. Find the Cartesian equation of π_3. [4]

(c) Find the intersection of π_1, π_2 and π_3. [3]

36. [Maximum mark: 14] ***[without GDC]***

(a) The line l_1 passes through the point $\mathrm{A}(0, 1, 2)$ and is perpendicular to the plane $x-4y-3z=0$. Find the Cartesian equation of l_1. [2]

(b) The line l_2 is parallel to l_1 and passes through the point $\mathrm{P}(3, -8, -11)$. Find the vector equation of the line l_2. [2]

(c) (i) The point Q is on the line l_1 such that $\overrightarrow{PQ}$ is perpendicular to l_1 and l_2. Find the coordinates of Q.

(ii) Hence find the distance between l_1 and l_2. [10]

37. [Maximum mark: 23] ***[without GDC]***

Two planes π_1 and π_2 are represented by the equations

$$\pi_1 : \boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\pi_2 : 2x - y - 2z = 4$$

(a) (i) Find $\begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

(ii) Show that the equation of π_1 can be written as $x - 2y + 2z = 11$. [4]

(b) Show that π_1 is perpendicular to π_2. [4]

(c) The line l_1 is the line of intersection of π_1 and π_2.

Find the vector equation of l_1, giving the answer in parametric form. [5]

(d) The line l_2 is parallel to **both** π_1 and π_2, and passes through $\mathrm{P}(3, -5, -1)$.

Find an equation for l_2 in Cartesian form. [3]

(e) Let Q be the foot of the perpendicular from P to the plane π_2.

(i) Find the coordinates of Q.

(ii) Find PQ. [7]

38. [Maximum mark: 27] ***[with GDC]***

Consider the vectors $\boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{c} = 2\boldsymbol{i} - 5\boldsymbol{j} - \boldsymbol{k}$.

(a) Given that $\boldsymbol{c} = m\boldsymbol{a} + n\boldsymbol{b}$ where $m, n \in \mathbb{Z}$, find the value of m and of n. [5]

(b) Find a unit vector, $\boldsymbol{u}$, normal to both $\boldsymbol{a}$ and $\boldsymbol{b}$. [5]

(c) The plane π_1 contains the point $\mathrm{A}(1, -1, 1)$ and is normal to $\boldsymbol{b}$. The plane intersects the x, y and z axes at the points L, M and N respectively.

(i) Find a Cartesian equation of π_1.

(ii) Write down the coordinates of L, M and N. [5]

(d) The line through the origin, O, normal to π_1 meets π_1 at the point P.

(i) Find the coordinates of P.

(ii) **Hence** find the distance of π_1 from the origin. [7]

(e) The plane π_2 has equation $x + 2y + 4z = 4$. Calculate the angle between π_2 and a line parallel to $\boldsymbol{a}$. [5]

39. [Maximum mark: 18] ***[with / without GDC]***

(a) Write the vector equations of the following lines in parametric form.

$$\boldsymbol{r}_1 = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + m \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \qquad \boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + n \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$ [2]

(b) Hence show that these two lines intersect and find the point of intersection, A. [5]

(c) Find the Cartesian equation of the plane Π that contains these two lines. [4]

(d) Let B be the point of intersection of the plane Π and the line $\boldsymbol{r} = \begin{pmatrix} -8 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}$.

Find the coordinates of B. [4]

(e) If C is the mid-point of AB, find the vector equation of the line perpendicular to the plane Π and passing through C. [3]

40. [Maximum mark: 20] ***[without GDC]***

The points A, B, C have position vectors $\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, $3\boldsymbol{i} + \boldsymbol{k}$ respectively and lie in the plane π.

(a) Find the area of the triangle ABC; [6]

(b) Hence, or otherwise, find the shortest distance from C to the line AB; [4]

(c) Find the Cartesian equation of the plane π. [4]

Line L passes through the origin and is normal to the plane π, it intersects π at D.

(d) Find the coordinates of the point D; [5]

(e) Find the distance of π from the origin. [1]

41. [Maximum mark: 12] ***[with GDC]***

The point A $(2, 5, -1)$ is on the line L, which is perpendicular to the plane with equation

$$\Pi : x + y + z = 1$$

(a) Find the Cartesian equation of the line L. [3]

(b) Find the point of intersection of the line L and the plane (the **foot** of L on Π). [4]

(c) The point A is reflected in the plane. Find the coordinates of the image of A. [2]

(d) The point $B\left(\frac{4}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ lies in Π. Find the distance from point B to line L. [3]