# *Math AA*

EXERCISES [Math-AA 3.13-3.14]

VECTOR EQUATION OF A LINE

*Compiled by Christos Nikolaidis*

## O. Practice questions

**1.** [Maximum mark: 7]     *[without GDC]*

Complete the following table

| Passing through | Parallel to | Equation of line |
|---|---|---|
| $A(3,5)$ | $\vec{b} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ | $\vec{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ |
| the origin | $\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ | |
| $A(3,5)$ | $x$-axis | |
| $A(3,5)$ | $y$-axis | |
| $A(1,3,5)$ | $\vec{b} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$ | $\vec{r} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$ |
| the origin | $\vec{b} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$ | |
| $A(1,3,5)$ | $x$-axis | |
| $A(1,3,5)$ | $y$-axis | |
| $A(1,3,5)$ | $z$-axis | |

**2.** [Maximum mark: 8]    *[without GDC]*

Complete the following table

| Passing through | Equation of line |
|---|---|
| $A(3,5)$ and $B(4,12)$ | $\vec{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ |
| $A(1,-4)$ and $B(7,6)$ | |
| the origin and $B(7,6)$ | |
| $A(1,3,5)$ and $B(2,10,7)$ | $\vec{r} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$ |
| $A(1,3,5)$ and $B(0,5,3)$ | |
| the origin and $B(2,10,7)$ | |

**3.** [Maximum mark: 3]    *[without GDC]*

Express the following 3D lines in the form $r = a + \lambda b$

(i)  $x$-axis              (ii)  $y$-axis              (iii)  $z$-axis

**4.** [Maximum mark: 5]    *[without GDC]*

The line L has vector equation $\vec{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

(a)   Write down the parametric equations of the line.                                                    [2]

(b)   Find the Cartesian equation of the line in the form $y = mx + c$                    [3]

**5.** [Maximum mark: 4]    *[without GDC]*

The line L has vector equation $\vec{r} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

(a)   Write down the parametric equations of the line.                                                    [2]

(b)   Find the Cartesian equations of the line.                                                             [2]

**6.** [Maximum mark: 4]    *[without GDC]*

Determine whether the points A(5,19) and B(5,20) lie on the line L: $\vec{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \end{pmatrix}$

**7.** [Maximum mark: 4]    *[without GDC]*

Determine whether the points A(5,7,4) and B(9,12, 4) lie on the line $L: \vec{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$

**8.** [Maximum mark: 6]    *[without GDC]*

Express the following 3D lines in the form $r = a + \lambda b$

$L_1$:  $\dfrac{x-2}{3} = \dfrac{y+4}{7} = \dfrac{z-1}{5}$ 

$L_2$:  $\dfrac{x+1}{3} = 5 - y = \dfrac{2z-1}{5}$

$L_3$:  $\dfrac{x-2}{3} = \dfrac{y+4}{7}, z = 10$

$L_4$:  $\dfrac{x-2}{3} = \dfrac{y+4}{0} = \dfrac{z+1}{1}$

**9.** [Maximum mark: 10]    *[with / without GDC]*

Consider the lines $L_1$: $\vec{r} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \end{pmatrix}$ and $L_2$: $\vec{r} = \begin{pmatrix} -2 \\ 20 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ -1 \end{pmatrix}$

    (a)   Show that the lines are perpendicular. [2]

    (b)   Find the point of intersection of the two lines. [3]

    (c)   Find the Cartesian equations of the two lines in the form $ax + by = c$ [4]

    (d)   Solve the simultaneous equations in (c) [1]

**10.** [Maximum mark: 7]    *[with GDC]*

Consider the lines $L_1$: $\vec{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ and $L_2$: $\vec{r} = \begin{pmatrix} 2 \\ 9 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$

    (a)   Show that the two lines intersect. [4]

    (b)   Find the point of intersection. [1]

    (c)   Find the angle between the two lines. [2]

**11.** [Maximum mark: 13]    *[with GDC]*

The line *L* has vector equation $r = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$.

Any point $P$ on *L* has the form $P(2+2\lambda, \lambda, 3+2\lambda)$.

B(1,0,2)
●

    *L*                   A(8,3,9)
━━━━━━━━━━━━━━━━━━●━━━━━━━━━

    (a)   Confirm that point $A$ (8,3,9) lies on *L* [1]

    (b)   Find two points on *L*, 6 units far from $A$. [4]

    (c)   Find two points on *L*, $\sqrt{54}$ units far from $B$ (1,0,2). [4]

    (d)   Find two points on *L*, $\sqrt{89}$ units far from the origin. [4]

**12.** [Maximum mark: 7]     *[without GDC]*

The line $L$ has vector equation $r = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$. The point $D\,(0,1,0)$ does not lie on $L$.

Any point $P$ on $L$ has the form $P\,(2+2\lambda,\ \lambda,\ 3+2\lambda)$.

(a)   Find the point on $L$ which is closest to the point $D$.                                    [4]

(b)   Find the distance between the line $L$ and the point $D$.                              [1]

(c)   The point $D'$ is the reflection of $D$ in the line $L$. Find the coordinates of $D'$.    [2]

**13.** [Maximum mark: 8]     *[without GDC]*

Consider the lines $L_1$: $\vec{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $L_2$: $\vec{r} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

(a)   Show that the two lines are skew.                                                             [3]

(b)   Find minimum distance between the two lines.                                          [5]

**14\*.** [Maximum mark: 7]     *[without GDC]*

Consider the lines $L_1$: $\vec{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ and $L_2$: $\vec{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$

(a)   Find the equations of the two lines that bisect the angles of the lines $L_1$ and $L_2$.    [5]

(b)   Show that the two bisector lines found in (a) are perpendicular, using an appropriate scalar product.                                                                          [2]

**15\*\*.** [Maximum mark: 8]     *[without GDC]*

Consider the lines $L_1$: $\vec{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ and $L_2$: $\vec{r} = \begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

Find the equations of the two lines that bisect the angles of the lines $L_1$ and $L_2$.
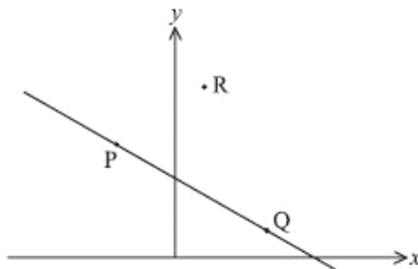
## A.   Exam style questions (SHORT)

**16.** [Maximum mark: 4]     *[without GDC]*

Find a vector equation of the line passing through $(-1, 4)$ and $(3, -1)$. Give your answer in the form $r = p + td$, where $t \in \mathbb{R}$.

**17.** [Maximum mark: 4]    *[without GDC]*

The points P(−2, 4), Q (3, 1) and R (1, 6) are shown in the diagram below.



(a)    Find the vector $\overrightarrow{PQ}$ .                                                         [2]

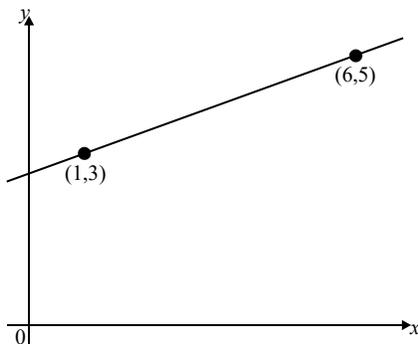(b)    Find a vector equation for the line through R parallel to the line (PQ).        [2]


**18.** [Maximum mark: 4]    *[without GDC]*

The line *L* passes through the origin and is parallel to the vector $2\mathbf{i} + 3\mathbf{j}$.

(a)    Write down a vector equation for *L*.

(b)    Find the Cartesian equation of the line in the form $y=mx$


**19.** [Maximum mark: 4]    *[without GDC]*

The diagram below shows a line passing through the points (1,3) and (6,5).



Find a vector equation for the line in the form $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + t \begin{pmatrix} c \\ d \end{pmatrix}$, where $t \in \mathbb{R}$

**20.** [Maximum mark: 5]    *[without GDC]*

A vector equation of a line is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ , $t \in \mathbb{R}$.

Find the equation of this line in the form $ax + by = c$, where $a, b,$ and $c \in \mathbb{Z}$.


**21.** [Maximum mark: 5]    *[without GDC]*

A line passes through the point (4,−1). Its direction is perpendicular to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

Find the equation of the line in the form $ax + by = p$, where $a, b$ and $p$ are integers.

**22.** [Maximum mark: 4]    *[without GDC]*

Find a vector equation of the line *L* which passes through A (3, 2, 1) and B (1, 5, 3).

**23.** [Maximum mark: 4]    *[without GDC]*

A vector equation for the line *L* is $r = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Which of the following are also vector equations for the same line *L*?

A.    $r = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

B.    $r = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t \begin{pmatrix} 6 \\ 2 \end{pmatrix}$.

C.    $r = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

D.    $r = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

**24.** [Maximum mark: 10]    *[without GDC]*

Given the points $A(1,1,1)$ $B(2,2,2)$ $C(5,5,5)$ $D(4,5,6)$

(a)    Find the vectors AB and BC and hence explain why *A, B* and *C* are collinear.    [4]

(b)    Find the vector equation of the line *AD*.    [3]

(c)    Find the cosine of the angle *BAD*.    [3]

**25.** [Maximum mark: 5]    *[with GDC]*

Calculate the acute angle between the lines with equations

$$r = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \end{pmatrix} \text{ and } r = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

**26.** [Maximum mark: 5]    *[without GDC]*

Two lines $L_1$ and $L_2$ have these vector equations.

$$L_1 : r = 2i + 3j + t(i - 3j) \qquad L_2 : r = i + 2j + s(i - j)$$

The angle between $L_1$ and $L_2$ is $\theta$. Find the cosine of the angle $\theta$.

**27.** [Maximum mark: 6]    *[with GDC]*

The lines $L_1$ and $L_2$ have parametric equations

$$L_1 : x = 1 + 2\lambda, \ y = 1 + 3\lambda, \ z = 1 - \lambda \qquad L_2 : x = 2 - \mu, \ y = 3 + 4\mu, \ z = 4 + 2\mu$$

Find the angle between $L_1$ and $L_2$.

**28.** [Maximum mark: 6]    *[without GDC]*

The vector equations of two lines are given below.

$$r_1 = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + s \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad r_2 = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

The lines intersect at the point P. Find the position vector of P.

**29.** [Maximum mark: 6]     *[without GDC]*

The lines $l_1$ and $l_2$ have equations

$$r_1 = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} \text{ and } r_2 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$$

respectively, where $\lambda$ and $\mu$ are parameters.

(a)    Show that $l_1$ passes through the point (2, –7 , 4).

(b)    Determine whether the lines $l_1$ and $l_2$ intersect.

**30.** [Maximum mark: 6]     *[without GDC]*

The line $L_1$ is represented by $r_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line $L_2$ by $r_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines $L_1$ and $L_2$ intersect at point T. Find the coordinates of T.

**31.** [Maximum mark: 6]     *[without GDC]*

The vector equations of the lines $L_1$ and $L_2$ are given by

$$L_1: r = i + j + k + \lambda(i + 2j + 3k); \qquad L_2: r = i + 4j + 5k + \mu(2i + j + 2k).$$

The two lines intersect at the point P. Find the position vector of P.

**32.** [Maximum mark: 6]     *[without GDC]*

Consider the four points A(1, 4, –1), B(2, 5, –2), C(5, 6, 3) and D(8, 8, 4). Find the point of intersection of the lines (AB) and (CD).

**33.** [Maximum mark: 5]     *[without GDC]*

Point A(3, 0, –2) lies on the line $r = 3i - 2k + \lambda(2i - 2j + k)$, where $\lambda$ is a real parameter.

Find the coordinates of **one** point which is 6 units from A, and on the line.

**34.** [Maximum mark: 6]     *[without GDC]*

The line $L$ passes through A (0, 3) and B (1, 0). The origin is at O. The point R($x$, $3-3x$) is on $L$, and (OR) is perpendicular to $L$.

(a)    Write down the vectors $\overrightarrow{AB}$ and $\overrightarrow{OR}$ .                                      [3]

(b)    Use the scalar product to find the coordinates of R.                      [3]

**35.** [Maximum mark: 7]     *[without GDC]*

Consider the points A (1, 3, –17) and B (6, – 7, 8) which lie on the line $l$.

(a)    Find an equation of line $l$, giving the answer in parametric form.          [4]

(b)    The point $P$ is on $l$ such that $\overrightarrow{OP}$ is perpendicular to $l$. Find the coordinates of P.     [3]

**36.** [Maximum mark: 6+6]   *[without GDC]*

The line *L* is given by the parametric equations $x = 1 - \lambda, y = 2 - 3\lambda, z = 2$.

Find the coordinates of the point on *L* that is nearest to the origin.

Use two different methods.

**METHOD A** (use the fact that the vector OP is perpendicular to the line).       [6]

**METHOD B** (use the formula of the distance *d* and then differentiation).       [6]

**37.** [Maximum mark: 9]     *[without GDC]*

The line *L* has vector equation

$$\vec{r} = 5\vec{i} + 9\vec{j} + 6\vec{k} + t(\vec{i} + 2\vec{j} + 2\vec{k})    \text{ where } t \text{ is a scalar.}$$

(a)    Find the coordinates of the point on line *L* which is nearest to point A(0, 2, 2).       [5]

(b)    Calculate the shortest distance from the point *A*(0, 2, 2) to the line.       [2]

(c)    The point A is reflected in line *L*. Find the coordinates of the image A´.       [2]

**38.** [Maximum mark: 8]     *[without GDC]*

Two lines $L_1$ and $L_2$ are given by $r_1 = \begin{pmatrix} 9 \\ 4 \\ -6 \end{pmatrix} + s \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $r_2 = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$.

(a)    Let $\theta$ be the acute angle between $L_1$ and $L_2$. Show that $\cos\theta = \dfrac{52}{140}$.       [3]

(b)    (i)    P is the point on $L_1$ when $s = 1$. Find the position vector of P.

    (ii)    Show that P is also on $L_2$.       [3]

(c)    A third line $L_3$ has direction vector $\begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$. If $L_1$, $L_3$ are parallel, find the value of $x$.       [2]

**39*.** [Maximum mark: 22]    *[with GDC]*

The lines $l_1$ and $l_2$ have vector equations

$$r_1 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \text{ and } r_2 = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

respectively, where $\lambda$ and $\mu$ are parameters.

Find the equations of the two lines that bisect the angles of the lines $l_1$ and $l_2$.

**40* .** [Maximum mark: 8]     *[with GDC]*

Consider the skew lines $L_1: \vec{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $L_2: \vec{r} = \begin{pmatrix} -3 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

(a)    Show that the lines $L_1$ and $L_2$ are skew.

(b)    Find minimum distance between $L_1$ and $L_2$.

## B.   Exam style questions (LONG)

**41.**   [Maximum mark: 14]   *[without GDC]*

Consider the points A(1, −1, 4), B(2, − 2, 5) and O(0, 0, 0).

(a)   Calculate the cosine of the angle between $\overrightarrow{OA}$ and $\overrightarrow{AB}$.   [5]

(b)   Find a vector equation of the line $L_1$ which passes through A and B.   [2]

The line $L_2$ has equation $r = 2i + 4j + 7k + t(2i + j + 3k)$, where $t \in \mathbb{R}$.

(c)   Show that the lines $L_1$ and $L_2$ intersect and find the coordinates of their point of

intersection.   [7]

**42.**   [Maximum mark: 18]   *[with GDC]*

The following diagram shows a solid figure ABCDEFGH. Each of the six faces is a
parallelogram.
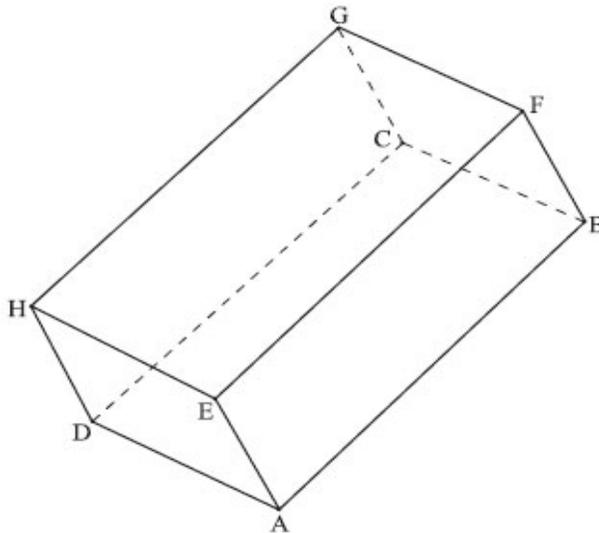


The coordinates of A and B are A (7, –3, –5), B(17, 2, 5).

(a)   Find   (i) $\overrightarrow{AB}$;   (ii) $\left|\overrightarrow{AB}\right|$.   [4]

The following information is given.   $\overrightarrow{AD} = \begin{pmatrix} -6 \\ 6 \\ 3 \end{pmatrix}$, $\left|\overrightarrow{AD}\right| = 9$, $\overrightarrow{AE} = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix}$, $\left|\overrightarrow{AE}\right| = 6$

(b)   (i)   Calculate   (a) $\overrightarrow{AD} \bullet \overrightarrow{AE}$   (b) $\overrightarrow{AB} \bullet \overrightarrow{AD}$   (c) $\overrightarrow{AB} \bullet \overrightarrow{AE}$

      (ii)   Hence, write down the size of the angle between any two intersecting edges   [5]

(c)   Calculate the volume of the solid ABCDEFGH.   [2]

(d)   The coordinates of G are (9, 14, 12). Find the coordinates of H.   [3]

(e)   The lines (AG), (HB) intersect at point P. Find the acute angle at P.   [4]

**43.** [Maximum mark: 13]    *[with GDC]*

The diagram below shows a cuboid (rectangular solid) OJKLMNPQ.

The vertex O is (0, 0, 0), J is (6, 0, 0), K is (6, 0, 10), M is (0, 7, 0) and Q is (0, 7, 10).
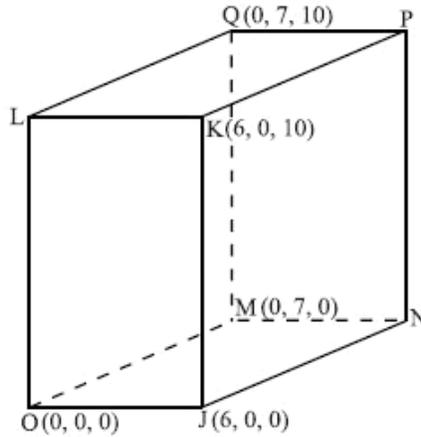


(a)   Find in the form $r = a + tb$  the equations of lines:   (i) (JQ)      (ii)  (MK)          [6]

(b)   Find the acute angle between (JQ) and (MK).          [2]

(c)   The lines (JQ) and (MK) intersect at D. Find the position vector of D.          [5]


**44.** [Maximum mark: 13]    *[with GDC]*

Points A, B, and C have position vectors $4i + 2j$, $i - 3j$ and $-5i - 5j$. Let D be a point on the $x$-axis such that ABCD forms a parallelogram.

(a)   (i)    Find $\overrightarrow{BC}$.   (ii)    Find the position vector of D.          [4]

(b)   Find the angle between $\overrightarrow{BD}$ and $\overrightarrow{AC}$.          [4]

The line $L_1$ passes through A and is parallel to $i + 4j$.

The line $L_2$ passes through B and is parallel to $2i + 7j$.

A vector equation of $L_1$ is $r = (4i + 2j) + s(i + 4j)$.

(c)   Write down a vector equation of $L_2$ in the form $r = b + tq$.          [1]

(d)   The lines $L_1$ and $L_2$ intersect at the point P. Find the position vector of P.          [4]


**45.** [Maximum mark: 14]    *[with GDC]*

The point O has coordinates (0, 0, 0), point A has coordinates (1, –2, 3) and point B has coordinates (–3, 4, 2).

(a)   (i)    Find $\overrightarrow{AB}$          (ii)    Find $B\hat{A}O$.          [5]

(b)   The line $L_1$ has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on $L_1$.          [2]

(c)    The line $L_2$ passes through A and is parallel to $\overrightarrow{OB}$.

   (i)    Find a vector equation for $L_2$, giving your answer in the form $r = a + tb$.

   (ii)   Point C $(k, -k, 5)$ is on $L_2$. Find the coordinates of C.                    [5]

(d)    The line $L_3$ has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

   Find the value of $p$ at C.                                                          [2]


**46.**   [Maximum mark: 15]    ***[with GDC]***

Three of the coordinates of the parallelogram STUV are S(–2, –2), T(7, 7), U(5, 15).

(a)    Find the vector $\overrightarrow{ST}$ and hence the coordinates of V.               [4]

(b)    Find a vector equation of the line (UV) in the form $r = p + td$ where $t \in \mathbb{R}$.   [2]

(c)    Show that the point E with position vector $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$ is on the line (UV), and find the

   value of $t$ for this point.                                                        [2]

The point W has position vector $\begin{pmatrix} a \\ 17 \end{pmatrix}$, $a \in \mathbb{R}$.

(d)    (i)    If $\left| \overrightarrow{EW} \right| = 2\sqrt{13}$, show that one value of $a$ is –3 and find the other possible
                value of $a$.

   (ii)   For $a = -3$, calculate the angle between $\overrightarrow{EW}$ and $\overrightarrow{ET}$.         [7]


**47.**   [Maximum mark: 16]    ***[with GDC]***

Points P and Q have position vectors $-5i + 11j - 8k$ and $-4i + 9j - 5k$ respectively, and
both lie on a line $L_1$.

(a)    (i)    Find $\overrightarrow{PQ}$.

   (ii)   Hence show that the equation of $L_1$ can be written as

            $r = (-5 + s)\,i + (11 - 2s)\,j + (-8 + 3s)\,k.$                             [4]

The point R $(2, y_1, z_1)$ also lies on $L_1$.

(b)    Find the value of $y_1$ and of $z_1$.                                             [3]

The line $L_2$ has equation $r = 2i + 9j + 13k + t\,(i + 2j + 3k)$.

(c)    The lines $L_1$ and $L_2$ intersect at a point T. Find the position vector of T.    [5]

(d)    Calculate the angle between the lines $L_1$ and $L_2$.                             [4]

**48.** [Maximum mark: 19]   *[with GDC]*

The position vector of point A is $2i+3j+k$ and the position vector of point B is $4i-5j+21k$

(a) (i)   Show that $\overrightarrow{AB} = 2i-8j+20k$.

(ii)  Find the unit vector $u$ in the direction of $\overrightarrow{AB}$.

(iii) Show that $u$ is perpendicular to $\overrightarrow{OA}$. [6]

Let S be the midpoint of [AB]. The line $L_1$ passes through S and is parallel to $\overrightarrow{OA}$.

(b) (i)   Find the position vector of S.

(ii)  Write down the equation of $L_1$. [4]

The line $L_2$ has equation $r = (5i+10j+10k) + s(-2i+5j-3k)$.

(c)  Explain why $L_1$ and $L_2$ are not parallel. [2]

(d)  The lines $L_1$ and $L_2$ intersect at the point P. Find the position vector of P. [7]

**49.** [Maximum mark: 22]   *[with GDC]*

The lines $l_1$ and $l_2$ have vector equations

$$r_1 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \text{ and } r_2 = \begin{pmatrix} 4 \\ 4 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

respectively, where $\lambda$ and $\mu$ are parameters.

(a)  Find the acute angle between $l_1$ and $l_2$. [2]

(b)  Find the acute angle between $l_1$ and $z$-*axis.* [2]

(c)  Find the coordinates of the point of intersection of $l_1$ and $l_2$. [4]

(d)  Find the reflection of point A(1,4,3) in line $l_2$. [5]

(e)  Find the reflection of line $l_1$ in line $l_2$. [2]

Let $d_1$ and $d_2$ be the direction vectors of the two lines.

(f) (i)   Show that $|d_1| = |d_2|$.

(ii)  Write down the vectors $d_1+d_2$ and $d_1-d_2$.

(iii) **Hence** find the equations of the two bisector lines between $l_1$ and $l_2$. [7]

**50.** [Maximum mark: 17]   *[with GDC]*

The triangle ABC has vertices at the points A(−1, 2, 3), B(−l, 3, 5) and C(0, −1, 1).

(a)  Find the size of the angle $\theta$ between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [4]

(b)  Hence, or otherwise, find the area of triangle ABC. [2]

Let $l_1$ be the line parallel to $\overrightarrow{AB}$ which passes through D(2, −1, 0) and $l_2$ be the line parallel to $\overrightarrow{AC}$ which passes through E(−1, 1, 1).

(c) (i)   Find the equations of the lines $l_1$ and $l_2$.

(ii)  Hence show that $l_1$ and $l_2$ do not intersect. [5]

(d)  Find the shortest distance between $l_1$ and $l_2$. [6]

**51.** [Maximum mark: 20]   *[with GDC]*

   (a)   A curve is defined by the implicit equation $2xy + 6x^2 - 3y^2 = 6$

       Show that the tangent at the point A with coordinates $\left(1, \dfrac{2}{3}\right)$ has gradient $\dfrac{20}{3}$.     [6]

   (b)   The line $x = 1$ cuts the curve at point A, with coordinates $\left(1, \dfrac{2}{3}\right)$, and at point B.

       Find, in form $r = \begin{pmatrix} a \\ b \end{pmatrix} + s \begin{pmatrix} c \\ d \end{pmatrix}$

       (i)   the equation of the tangent at A;

       (ii)  the equation of the normal at B;     [10]

   (c)   Find the acute angle between the tangent at A and the normal at B.     [4]