NTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 1.7]

DEDUCTIVE PROOF

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O. Practice questions

1. [Maximum mark: 8] ***[without GDC]***

The binomial theorem gives the expansion of $(a+b)^n$.

Use a LHS to RHS proof to show the following expansions for $n = 2, 3, 4$.

(a) $(a+b)^2 \equiv a^2 + 2ab + b^2$. [use the fact $x^2 = x \cdot x$] [2]

(b) $(a+b)^3 \equiv a^3 + 3a^2b + 3ab^2 + b^3$. [use the fact $x^3 = x^2 \cdot x$] [3]

(c) $(a+b)^4 \equiv a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. [use the fact $x^4 = x^3 \cdot x$] [3]

2. [Maximum mark: 6] ***[without GDC]***

By expanding the appropriate side of each relation below, show that

(a) $2(x-3)^2 + 1 \equiv 2x^2 - 12x + 19$ [3]

(b) $3x^2 + 6x + 5 \equiv 3(x+1)^2 + 2$ [3]

3. [Maximum mark: 6] ***[without GDC]***

(a) Verify that $x^2 + 1 = 2x + 4$ is true for $x = 3$ and $x = -1$ [4]

(b) Explain why $x^2 + 1 \equiv 2x + 4$ **is not** an identity. [2]

4. [Maximum mark: 8] ***[with GDC]***

(a) By expanding each side of the following relation independently, show that

$$(2x-3y)(3x-2y) \equiv 6(x+y)^2 - 25xy$$ [4]

(b) Verify the result for $x = 53$ and $y = 2$. [2]

(c) Verify the result whenever $x = y$. [2]

5*. [Maximum mark: 6] ***[without GDC]***

By consecutive simplifications of the following relation (using the symbol $\Leftrightarrow$), show that

$$\frac{2a^2+ab-b^2}{4a^2-b^2} \equiv \frac{a+b}{2a+b}$$

given that $b \neq \pm 2a$.

6*. [Maximum mark: 6] ***[without GDC]***

Show that

$$\frac{4x^2-1}{2x^2-x-1} \equiv \frac{2x-1}{x-1}, \quad \text{where } x \neq 1, x \neq 1/2 .$$

7*. [Maximum mark: 6] ***[without GDC]***

Any integer has one of the forms

$2n$ (even)

$2n+1$ (odd)

(a) Prove that the sum of two even numbers is an even number. [3]

(b) Prove that the sum of two odd numbers is an even number. [3]

8*. [Maximum mark: 6] ***[without GDC]***

Given that $2x^2+bx+5 \equiv (x+5)(ax+c)$ find the values of a, b, c .

A. Exam style questions (SHORT)

9. [Maximum mark: 7] ***[without GDC]***

(a) By using the identity $(x+1)^2 \equiv x^2+2x+1$, show that

$$(x+1)^3 \equiv x^3+3x^2+3x+1$$ [3]

(b) Verify the result

(i) for $x=1$ (ii) for $x=2$ [2]

(c) Use the result in question (a) with $x=100$ to find 101^3 . [2]

10. [Maximum mark: 8] ***[without GDC]***

(a) By using a LHS to RHS (or vice versa) proof, show that

(i) $\dfrac{2}{x-1}-\dfrac{2}{x-2} \equiv \dfrac{-2}{x^2-3x+2}$, $x \neq 1, 2$

(ii) $\dfrac{6}{x^2-9} \equiv \dfrac{1}{x-3}-\dfrac{1}{x+3}$, $x \neq \pm 3$ [6]

(b) Verify the identities above for $x=4$ [2]

11. [Maximum mark: 4] ***[without GDC]***

Show that

(a) $(a+2b)(2a+b) \equiv 2(a^2+b^2)+5ab$ [2]

(b) $a^2-b^2 \equiv (a-b)(a+b)$ [2]

12. [Maximum mark: 5] ***[without GDC]***

Show that

(a) $a^3-b^3 \equiv (a-b)(a^2+ab+b^2)$ [3]

(b) $a^3+b^3 \equiv (a+b)(a^2-ab+b^2)$ [2]

13. [Maximum mark: 4] ***[without GDC]***

Show that $\dfrac{5x+1}{4x^2+2x} \equiv \dfrac{1}{2x}+\dfrac{3}{4x+2}$

14. [Maximum mark: 6] ***[without GDC]***

Consider the equation $(x+3)^2-9=2x^2+3x+2$.

(a) Check the validity of the equation

(i) for $x=1$ (ii) for $x=2$ (iii) $x=3$ [4]

(b) Justify if we can use "≡" instead of "=" to obtain

$$(x+3)^2-9 \equiv 2x^2+3x+2\,.$$ [2]

15.** [Maximum mark: 6] ***[without GDC]***

(a) Prove that $\dfrac{x^2+y^2}{2} \geq xy$ for any $x, y \in R$ [3]

(b) Use (a) and two similar properties to show that

$x^2+y^2+z^2 \geq xy+yz+zx$ for any $x, y, z \in R$ [3]