INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 2.15]

MODULUS EQUATIONS AND INEQUALITIES

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O. Practice questions

1. [Maximum mark: 15] ***[without GDC]***

(a) Solve the following equations and inequalities [using the definition of $|x|$] [9]

$\|x\| = 3$	
$\|x\| < 3$	
$\|x\| > 3$	
$\|x\| = -3$	
$\|x\| < -3$	
$\|x\| > -3$	
$\|x\| = 0$	
$\|x\| < 0$	
$\|x\| > 0$	

(b) Solve the following equation and inequalities ` [6]

$2\|x\| - 3 = 0$	
$2\|x\| - 3 < 0$	
$2\|x\| - 3 > 0$	

2. [Maximum mark: 8] ***[without GDC]***

(a) Solve the equation $|x-5|=3$. [3]

(b) Solve the inequalities (i) $|x-5|<3$ (ii) $|x-5|>3$ [5]

METHOD A: using the definition of $|x|$

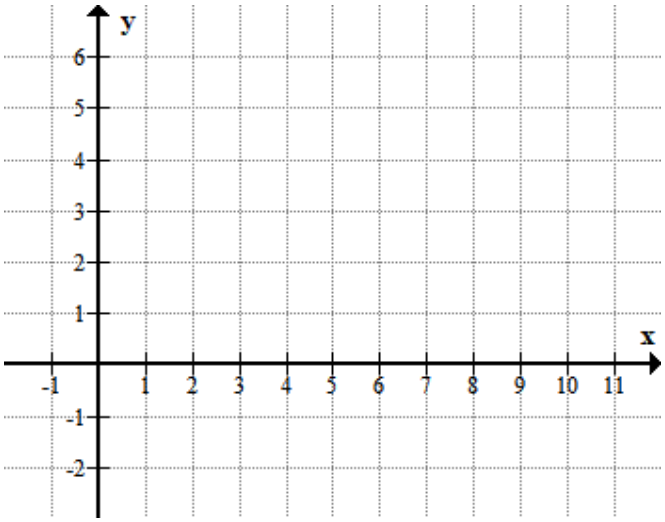
METHOD B: by squaring both sides (since both sides are positive)

3. [Maximum mark: 7] ***[with GDC]***

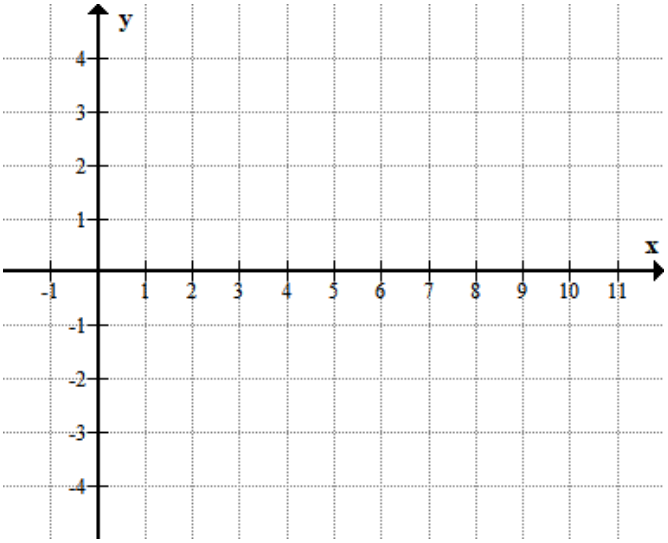
(a) (Using SolveN or the graph mode on your GDC), solve the equation $|x-5|=3$. [2]

(b) (Using the graph mode, solve the inequalities (i) $|x-5|<3$ (ii) $|x-5|>3$ [4]

METHOD A: Draw the graphs of $y_1=|x-5|$ and $y_2=3$



METHOD B: Draw the graph of $y=|x-5|-3$



4. [Maximum mark: 8] ***[with / without GDC]***

(a) Solve the equation $|x-5|=|x-3|$ [4]

(b) Solve the inequality $|x-5|<|x-3|$ [4]

5*. [Maximum mark: 5] ***[with / without GDC]***

Solve the inequality $x^2-3|x|+2>0$

6*. [Maximum mark: 10] ***[with / without GDC]***

(a) Solve the equation $|x-5|=x-3$ [5]

(b) Solve the inequality $|x-5|<x-3$ [5]

7*. [Maximum mark: 12] ***[with / without GDC]***

(a) Solve the equation $|x-5|=|x|-3$ [6]

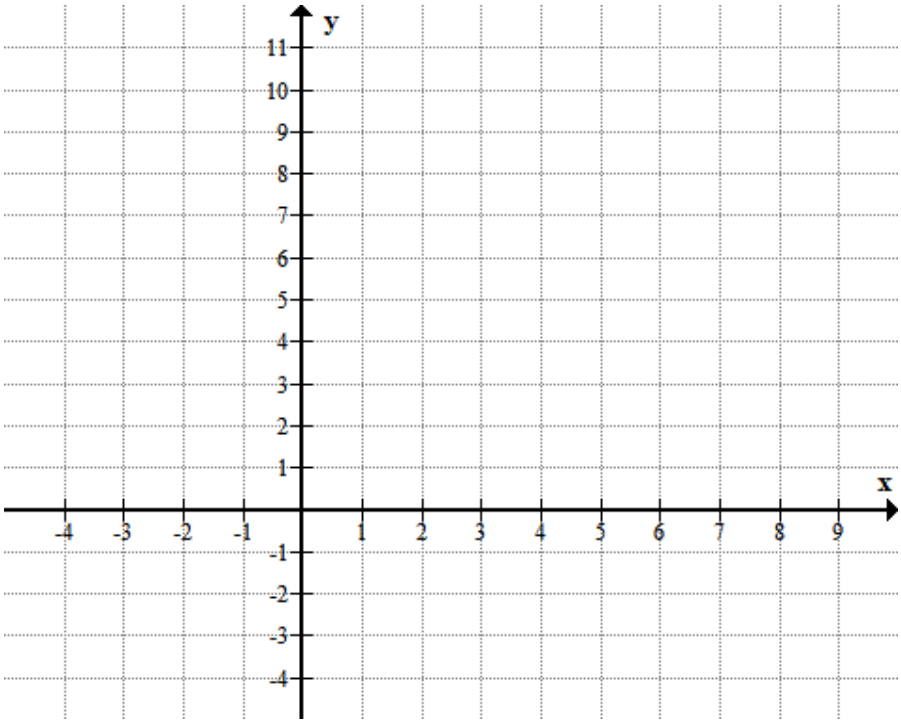
(b) Solve the inequality $|x-5|<|x|-3$ [6]

8*. [Maximum mark: 8] ***[without GDC]***

(a) Draw the graph of $f(x)=|x-5|-|x|+3$ [4]

(b) **Hence**, solve the inequalities

(i) $f(x)>0$ (ii) $f(x)>4$ (iii) $f(x)>-4$ (iv) $f(x)>10$ [4]



A. Exam style questions (SHORT)

9. [Maximum mark: 6] ***[without GDC]***

(a) Solve the equation $\left|\frac{2x-6}{7}\right| = 8$. [3]

(b) Solve the inequality $\left|\frac{2x-6}{7}\right| \leq 8$. [3]

10. [Maximum mark: 4] ***[without GDC]***

Solve the equation $\left|x^2 - 9\right| = 7$.

11. [Maximum mark: 5] ***[with / without GDC]***

Find the values of x for which $\left|5-3x\right| \leq \left|x+1\right|$.

12. [Maximum mark: 5] ***[with / without GDC]***

Solve the inequality $\left|x-2\right| \geq \left|2x+1\right|$.

13. [Maximum mark: 5] ***[with GDC]***

Solve the inequality $\left|\frac{x+9}{x-9}\right| \leq 2$.

14. [Maximum mark: 6] ***[without GDC]***

Solve the inequality $\left|\frac{x+1}{x}\right| \geq 2$.

15. [Maximum mark: 6] ***[without GDC]***

Solve the inequality $\left|x-5\right| - \left|x-10\right| \leq 7$.

Notice: confirm the result by using the graph mode of the GDC

B. Exam style questions (LONG)

16. [Maximum mark: 14] ***[without GDC]***

Let $f(x)=|x-3|+|x+2|$

(a) Draw the graph of the function f in the diagram below. [4]

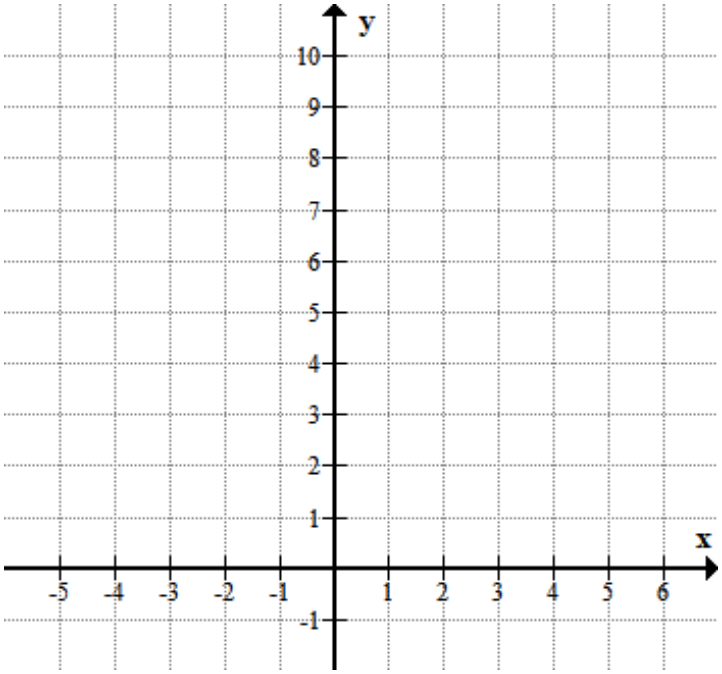
(b) Write down the range of f. [2]

(c) Express the function in the form $f(x)=\begin{cases} ax+b & x<-2 \\ c & -2\le x<3 \\ dx+e & x\ge 3 \end{cases}$ [4]

(d) Hence, or otherwise,

(i) solve the equation $f(x)=9$;

(ii) solve the inequality $f(x)\le 9$. [4]



17. [Maximum mark: 14] ***[without GDC]***

Let $f(x)=|x-3|-|x+2|$

(a) Draw the graph of the function f in the diagram below. [4]

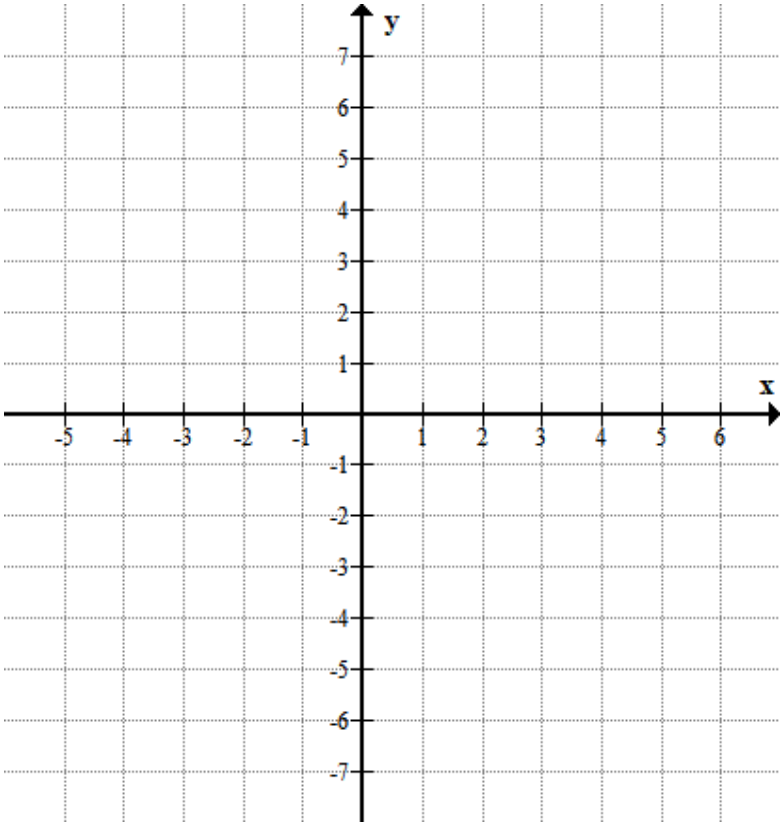
(b) Write down the range of f. [2]

(c) Express the function in the form $f(x)=\begin{cases} a & x<-2 \\ bx+c & -2\le x<3 \\ d & x\ge 3 \end{cases}$ [4]

(d) Hence, or otherwise,

(i) solve the equation $f(x)=0$;

(ii) solve the inequality $f(x)>1$. [4]



18. [Maximum mark: 12] ***[without GDC]***

Let $f(x) = |x|$, $g(x) = |2x - 5|$ and $h(x) = \frac{x+4}{2}$

(a) Solve $f(x) > g(x)$ [5]

(b) Solve $f(x) < h(x)$ [5]

(c) **Hence**, solve $g(x) < f(x) < h(x)$. [2]