INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 2.16]

SYMMETRIES – MORE TRANSFORMATIONS

Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 8] ***[without GDC]***

Determine whether the following functions are **even**, **odd** or **neither**. Prove your claim.

(a) $f(x)=3x^4-5x^2+1$ (b) $f(x)=3x^5-5x^3+1$ (c) $f(x)=3x^5-5x^3+7x$

(d) $f(x)=\dfrac{5x^6+3|x|-1}{x^3+x}$

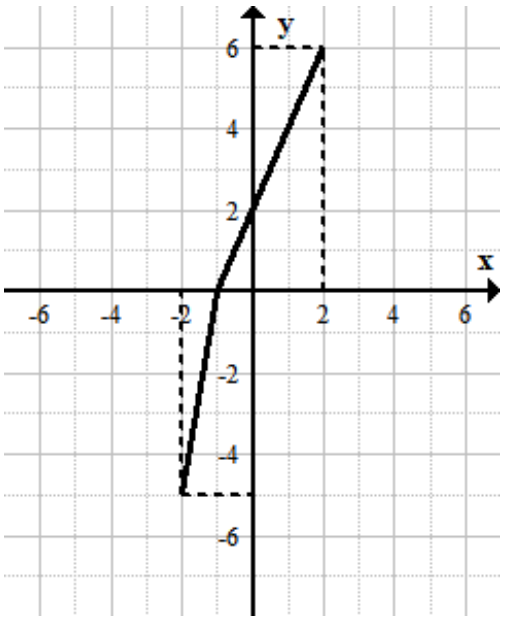
2. [Maximum mark: 36] ***[without GDC]***

The diagrams below show the graph of $y=f(x)$ with endpoints $\mathrm{A}(2,6)$ and $\mathrm{B}(-2,-5)$

For each transformation, write down the images A' and B' of the endpoints and sketch the corresponding graph (on the same diagram).

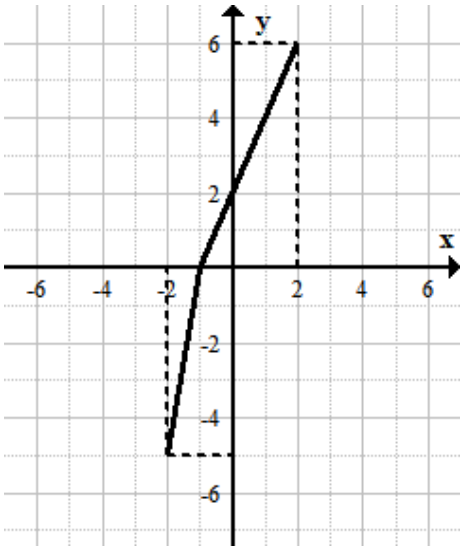
(a) $y=f(x)-2$ [4]

original point	image
$\mathrm{A}(2,6)$	
$\mathrm{B}(-2,-5)$	



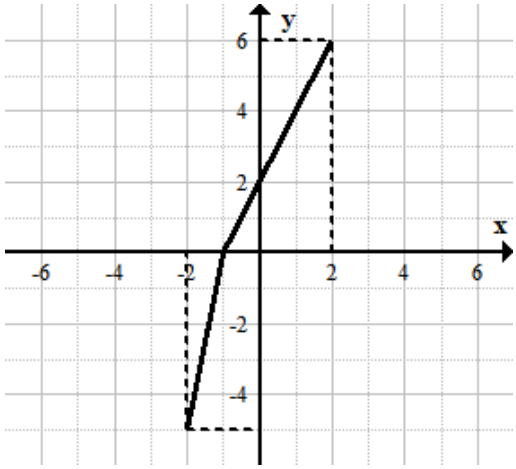
(b) $y = f(x-2)$ [4]

original point	image
$\mathrm{A}(2,6)$	
$\mathrm{B}(-2,-5)$	



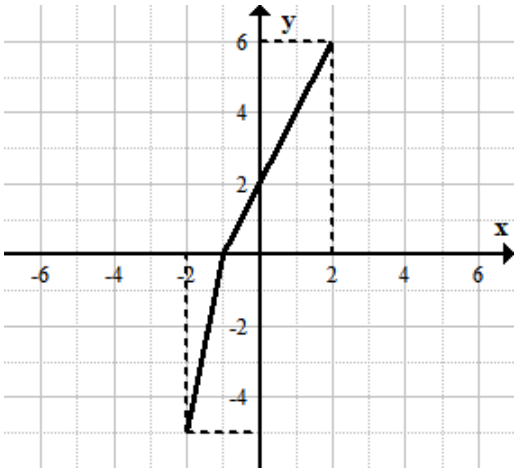
(c) $y = \dfrac{f(x)}{2}$ [4]

original point	image
$\mathrm{A}(2,6)$	
$\mathrm{B}(-2,-5)$	



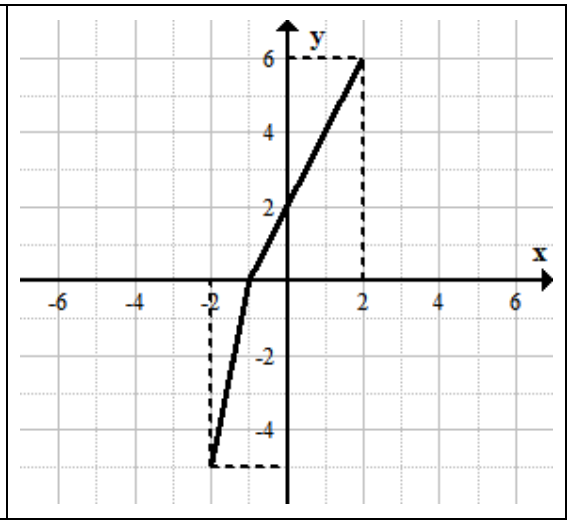
(d) $y = f\left(\dfrac{x}{2}\right)$ [4]

original point	image
$\mathrm{A}(2,6)$	
$\mathrm{B}(-2,-5)$	



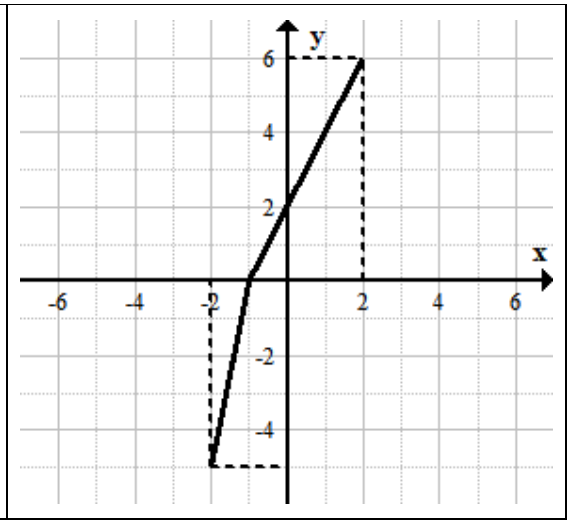
(e) $y = f(2x)$ [4]

original point	image
$A(2,6)$	
$B(-2,-5)$	



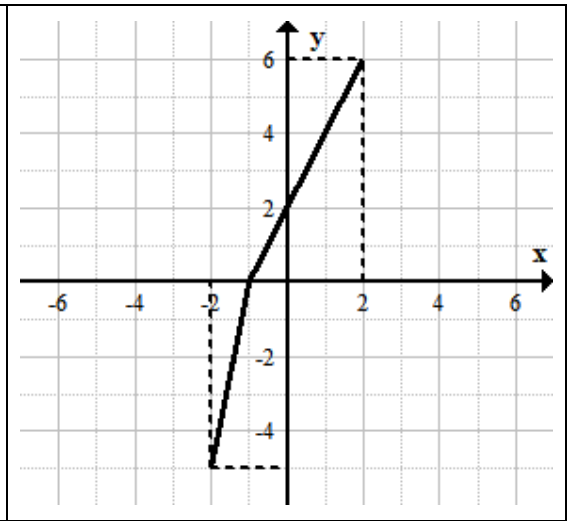
(f) $y = -f(x)$ [4]

original point	image
$A(2,6)$	
$B(-2,-5)$	



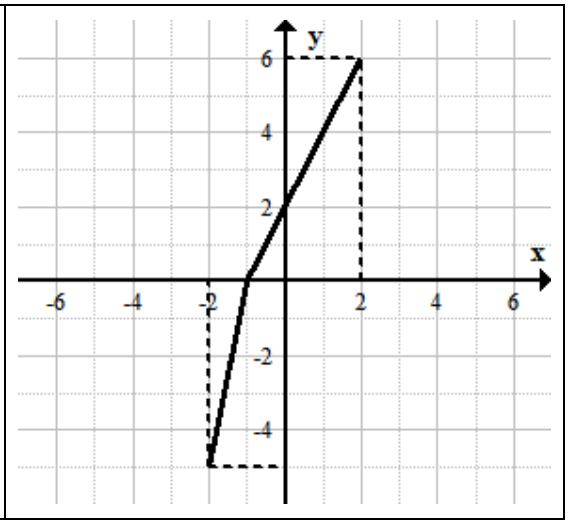
(g) $y = f(-x)$ [4]

original point	image
$A(2,6)$	
$B(-2,-5)$	



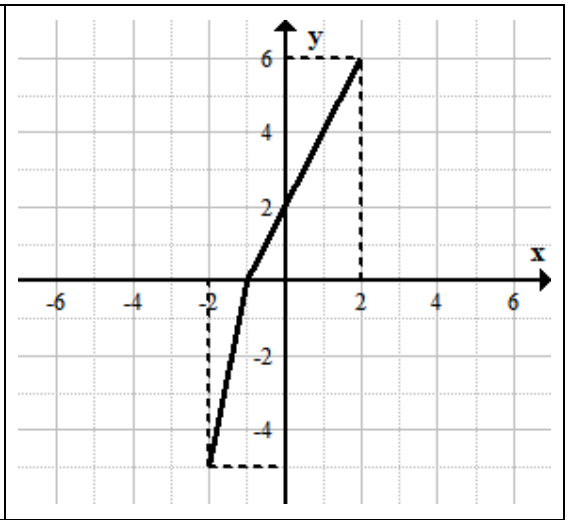
(h) $y = |f(x)|$ [4]

original point	image
$A(2,6)$	
$B(-2,-5)$	



(i) $y = f(|x|)$ [4]

original point	images
$A(2,6)$	
$B(-2,-5)$	no image



3. [Maximum mark: 10] ***[without GDC]***

The sequence of transformations from $y = f(x)$ to $y = 2|f(x)| + 4$ is given below

$f(x)$
$\|f(x)\|$
$2\|f(x)\|$
$2\|f(x)\| + 4$

Give the sequence of transformations, **in the correct order**, from $f(x)$ to the following functions

$y=\lvert f(x)\rvert+3$	$y=\lvert f(x)+3\rvert$	$y=f(\lvert x\rvert+3)$	$y=f(\lvert x+3\rvert)$
$f(x)$	$f(x)$	$f(x)$	$f(x)$

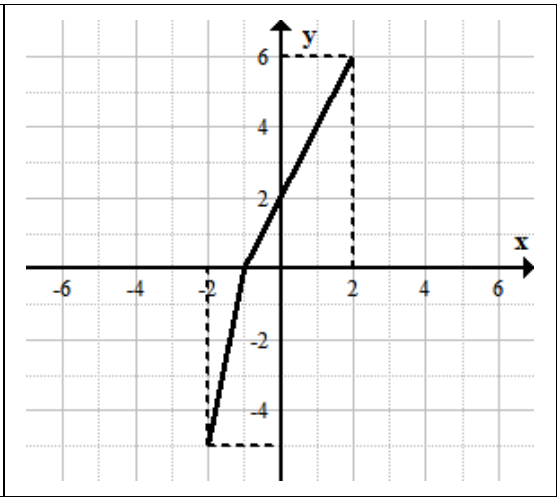
$y=-\lvert f(x)\rvert+1$	$y=f(-\lvert x\rvert+3)$	$y=f(\lvert x+1\rvert+3)$
$f(x)$	$f(x)$	$f(x)$

4. [Maximum mark: 14] ***[without GDC]***

The diagrams show the graph of $y=f(x)$. Sketch the graphs of the functions below

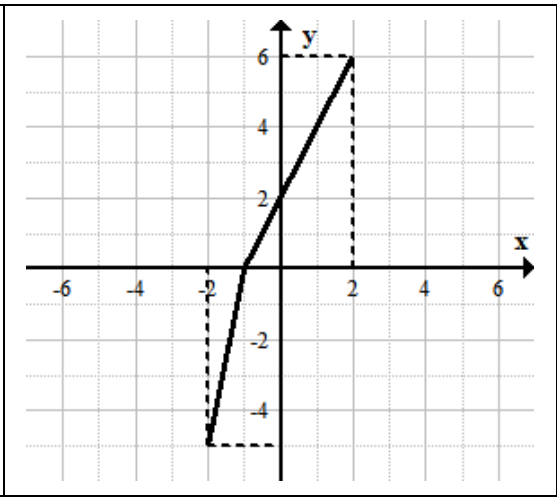
(a) $y=\lvert f(x)\rvert+1$ [4]

original point	image
$\mathrm{A}(2,6)$	



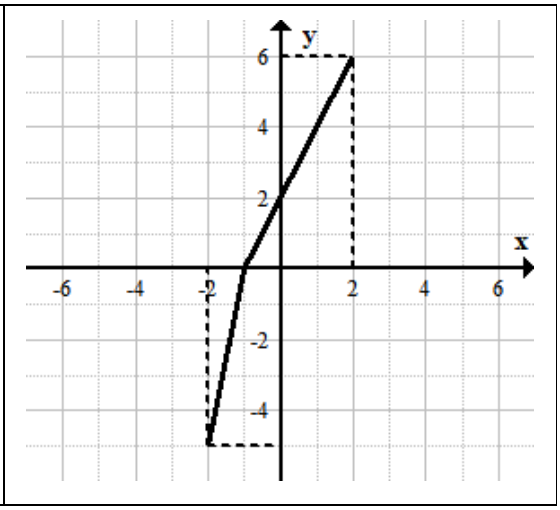
(b) $y = f(|x| - 1)$ [5]

original point	images
$A(2,6)$	



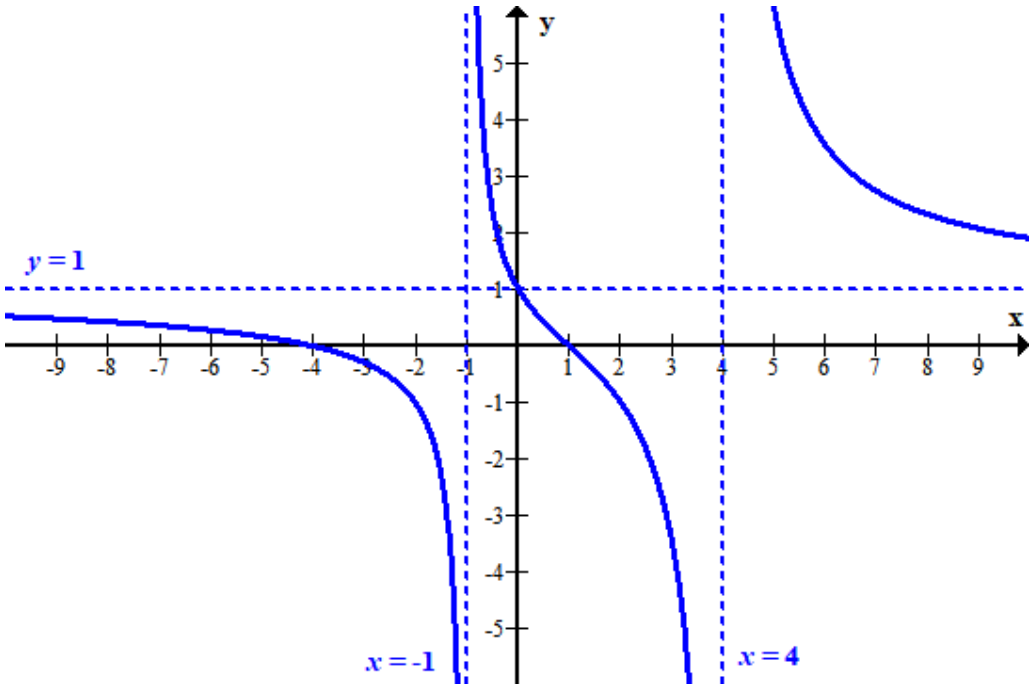
(c) $y = f(|x - 1|)$ [5]

original point	images
$A(2,6)$	

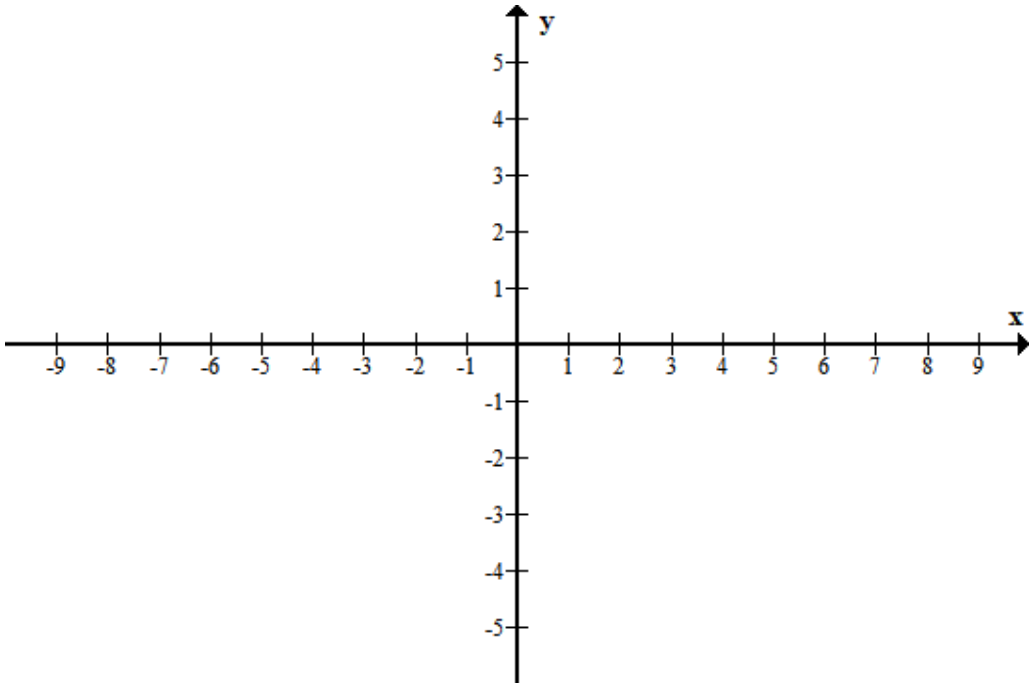


5. [Maximum mark: 5] ***[without GDC]***

The graph of $y = f(x)$ is shown below.

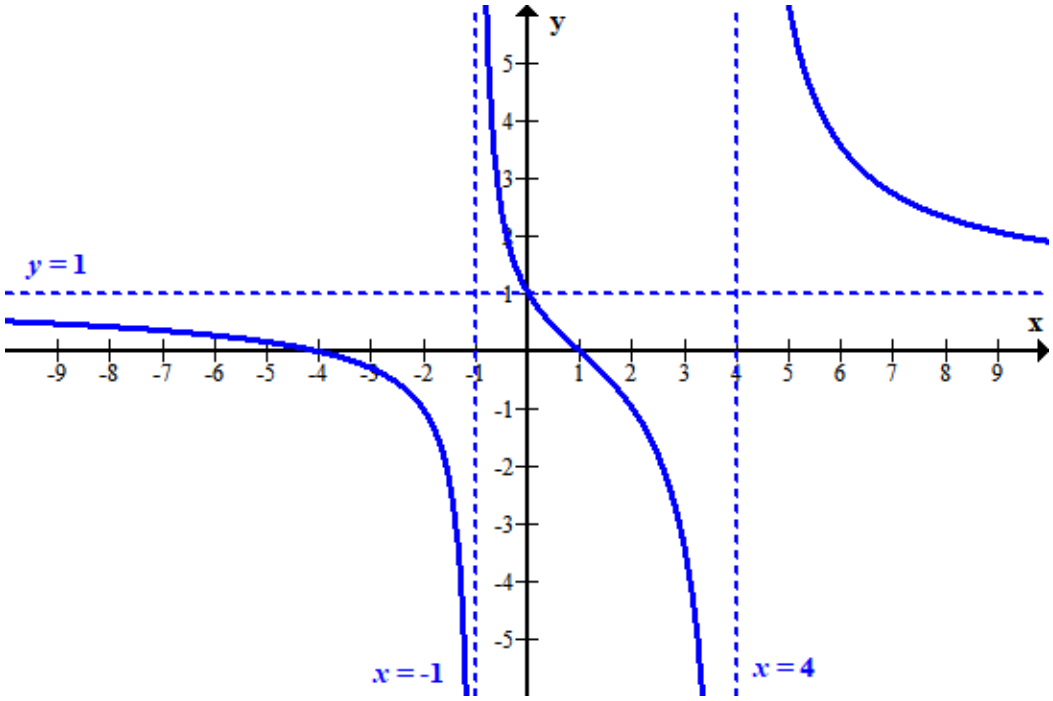


On a new diagram sketch the graph of $y = \frac{1}{f(x)}$. Indicate any asymptotes.

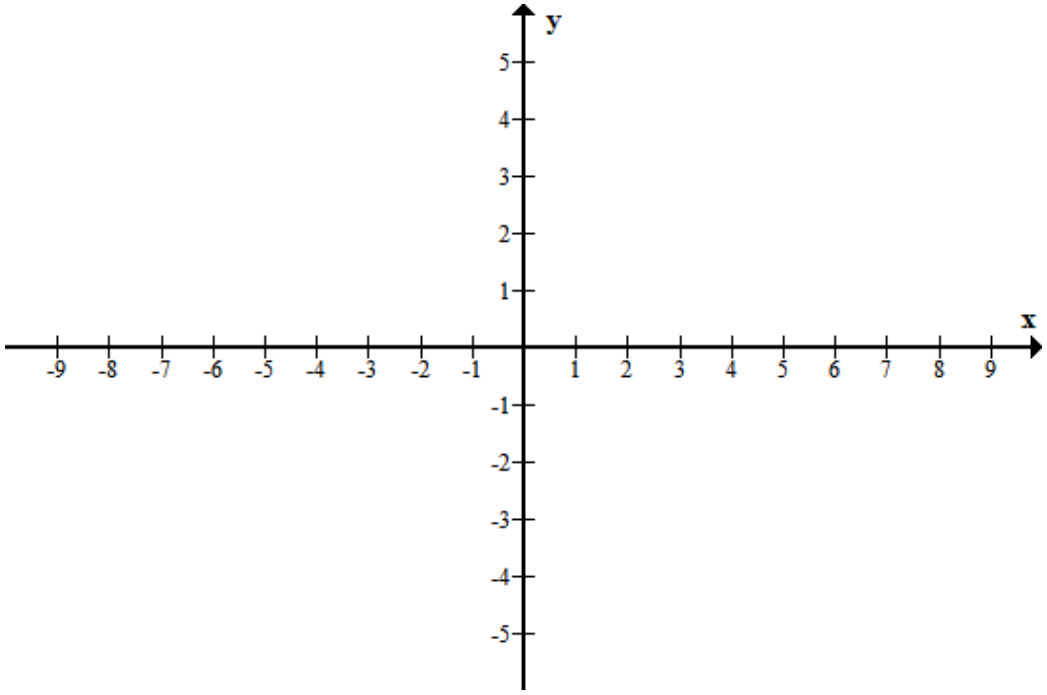


6. [Maximum mark: 6] ***[without GDC]***

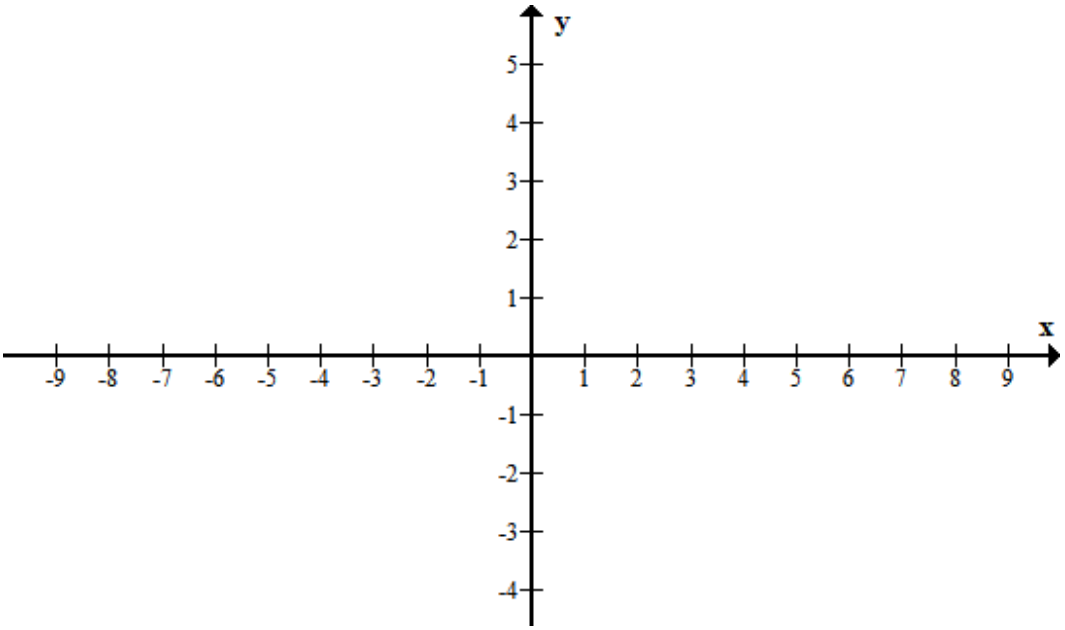
The graph of $y = f(x)$ is shown below.



(a) On a new diagram sketch the graph of $y = f(|x|)$. Indicate any asymptotes. [3]

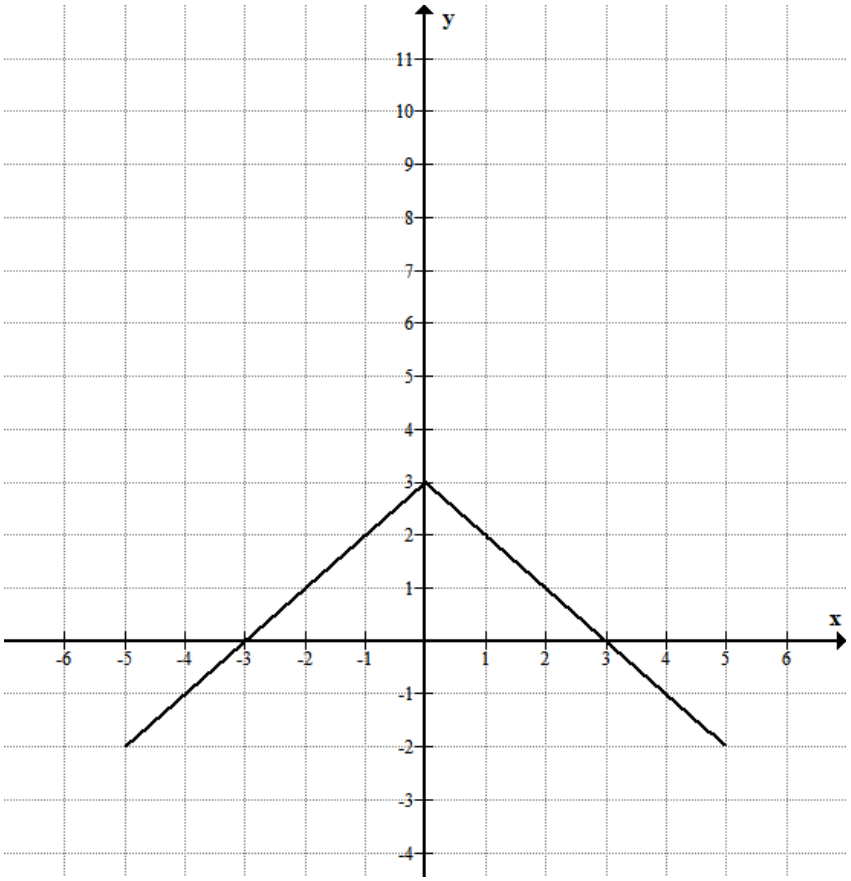


(b) On a new diagram sketch the graph of $y = |f(x)|$. Indicate any asymptotes. [3]



7. [Maximum mark: 5] ***[without GDC]***

The graph of $y = f(x)$ is shown below. On the same diagram, sketch the graph of $y = f(x)^2$



8. [Maximum mark: 21] ***[without GDC]***

Let $f(x)=\dfrac{2x-4}{x+2}$

(a) Complete the following table [6]

Function	$y=f(x)$	$y=\dfrac{1}{f(x)}$	$y=f^{-1}(x)$
Domain			
Range			

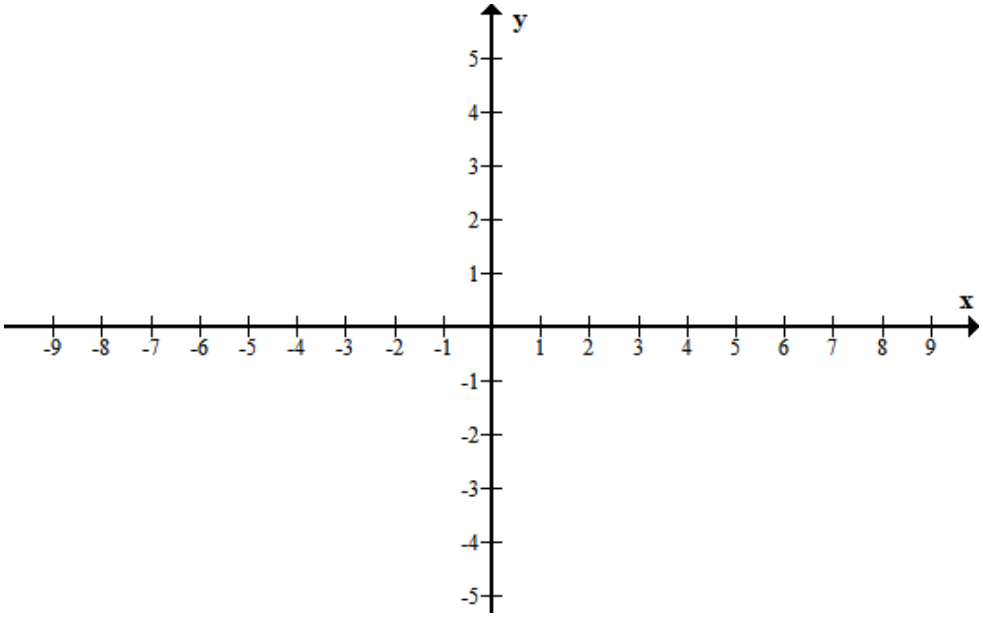
(b) The line $y=2$ is a horizontal asymptote of the graph of $f(x)$. Find the image of this line under the following transformations: [4]

Transformation	$2f(x)$	$f(x)+2$	$f(x-7)$	$-f(x)$	$\dfrac{1}{f(x)}$
Horizontal asymptote					

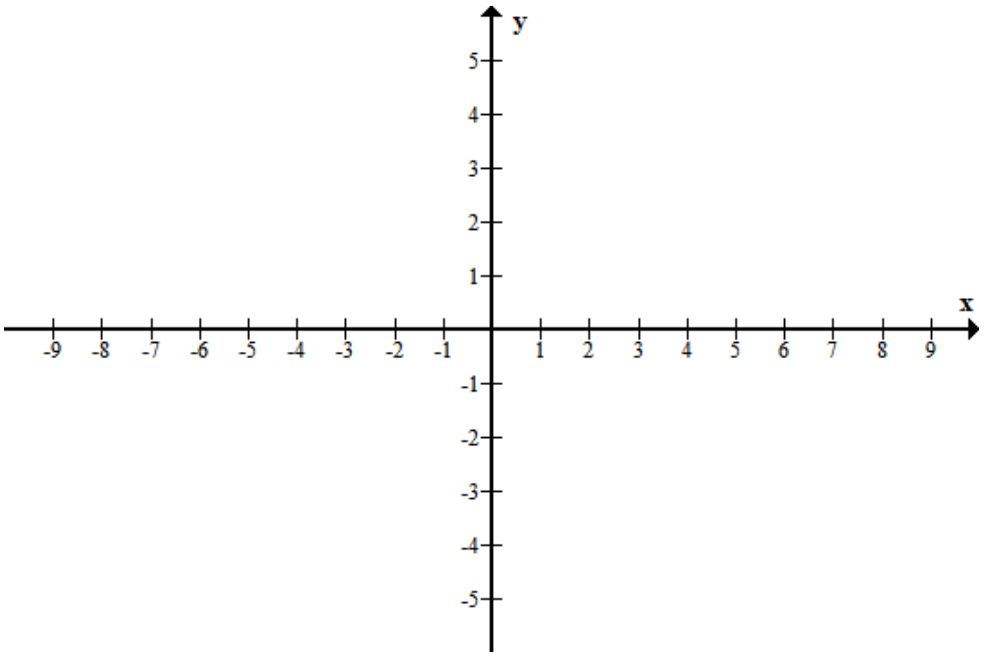
(c) Point $\mathrm{A}(3,0.4)$ lies on $y=f(x)$, Find the image of point A under the transformation $y=2f(3x)+5$. [2]

$f(x)$	$\mathrm{A}(3,0.4)$
$f(3x)$	
$2f(3x)$	
$y=2f(3x)+5$	

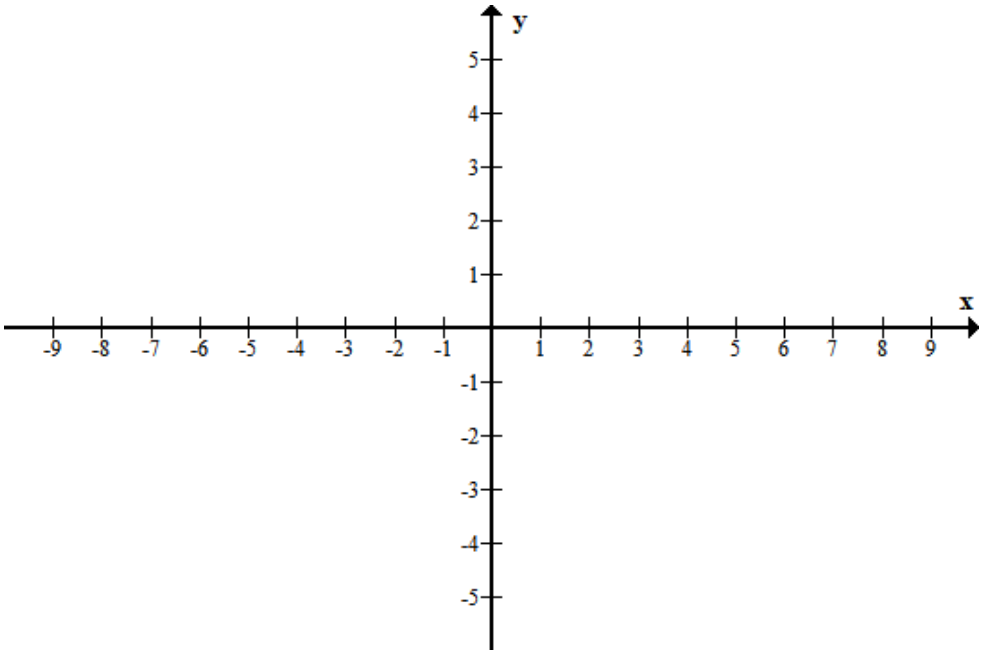
(d) Sketch the graph of $f(x)$; indicate any asymptotes and intersections with axes. [3]



(e) Sketch the graph of $\frac{1}{f(x)}$; indicate any asymptotes and intersections with axes. [3]



(f) Sketch the graph of $f^{-1}(x)$; indicate any asymptotes and intersections with axes. [3]



A. Exam style questions (SHORT)

9. [Maximum mark: 8] ***[without GDC]***

Determine whether the following functions are **even**, **odd** or **neither**. Prove your claim.

(a) $f(x) = |x| - x$

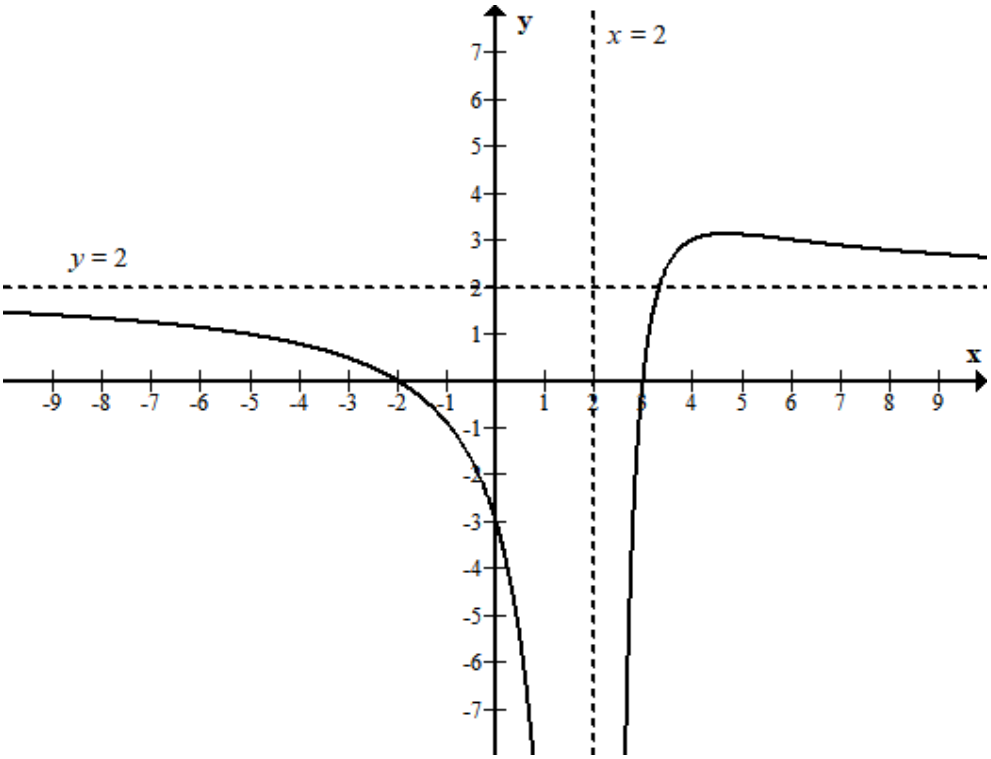
(b) $f(x) = |x| - 3$

(c) $f(x) = |x - 3|$ [2]

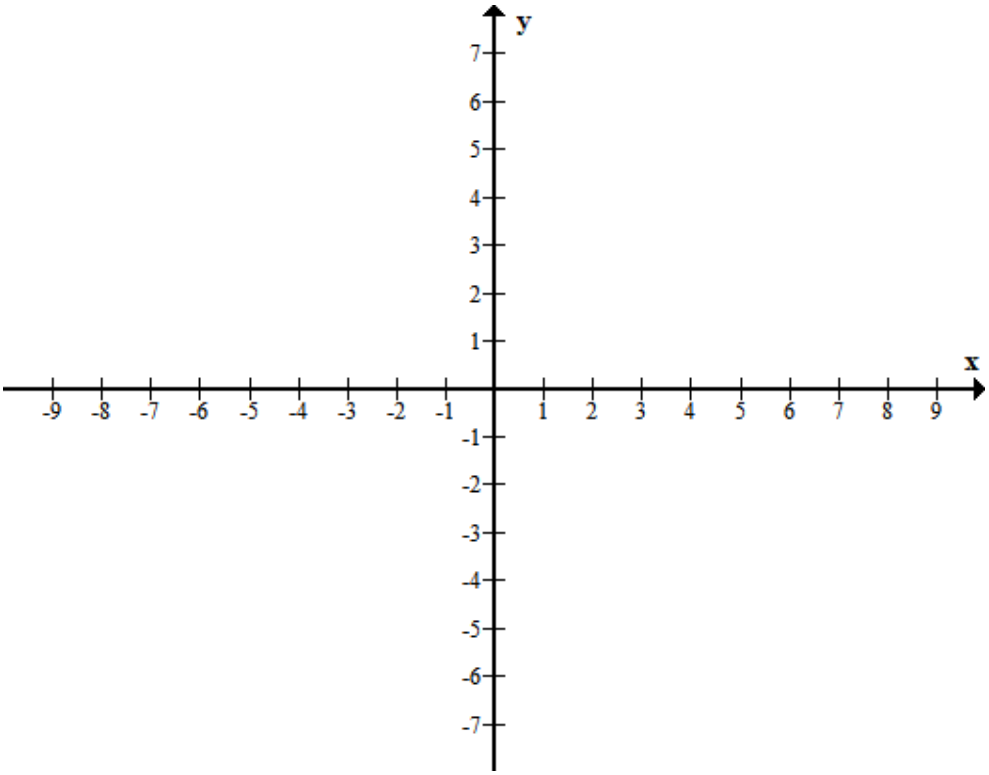
(d) $f(x) = 3x|x| + \frac{1}{x}$

10. [Maximum mark: 5] ***[without GDC]***

The graph of $y = f(x)$ is shown below.



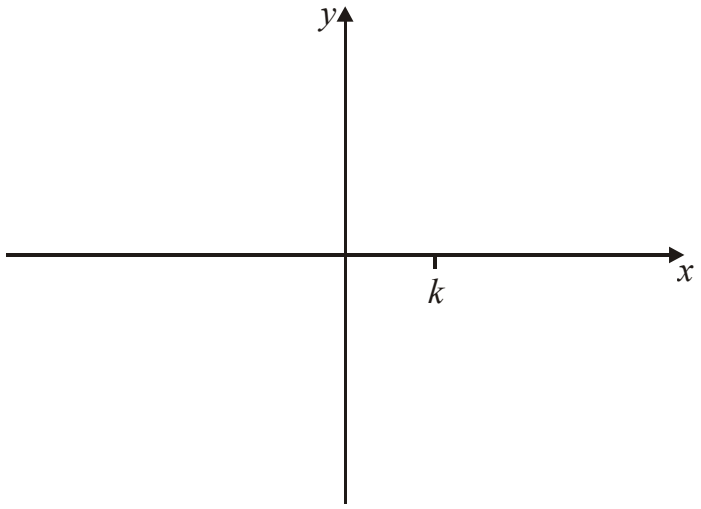
On a new diagram sketch the graph of $y = \frac{6}{f(x)}$. Indicate any asymptotes.



11. [Maximum mark: 6] ***[without GDC]***

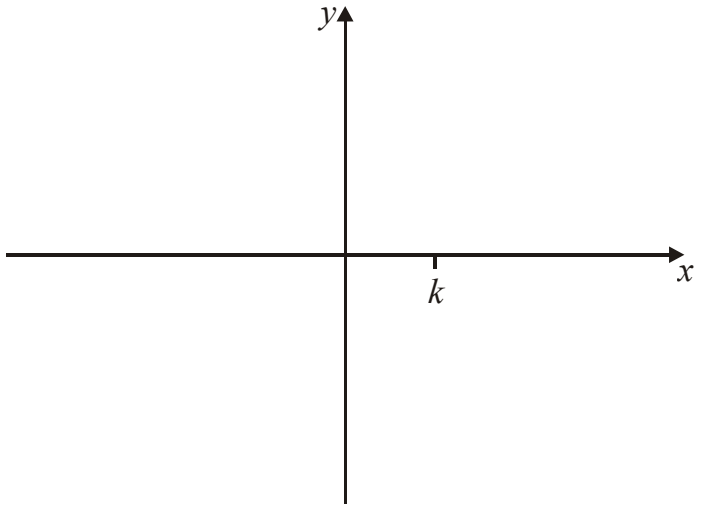
Let $f(x) = \frac{k}{x-k}$, $x \neq k$, $k > 0$

(a) On the diagram below, sketch the graph of f Label clearly any points of intersection with the axes, and any asymptotes.



[3]

(b) On the diagram below, sketch the graph of $\frac{1}{f}$. Label clearly any points of intersection with the axes.

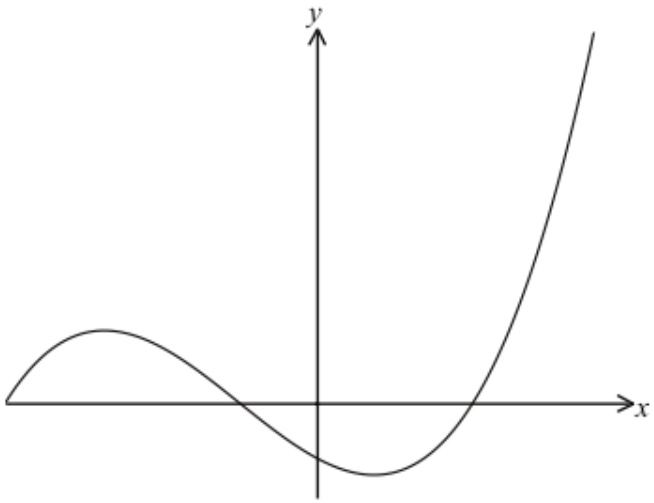


[3]

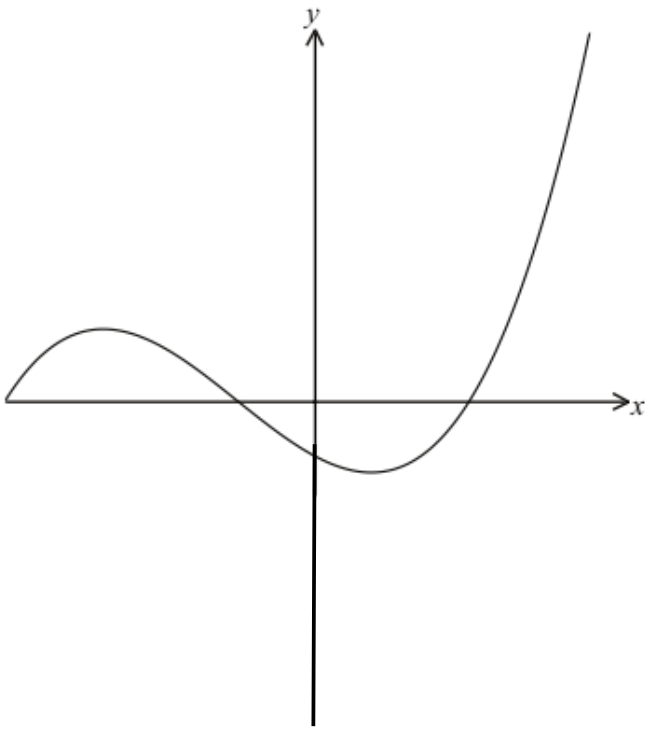
12. [Maximum mark: 6] ***[without GDC]***

Each of the diagrams below shows the graph of a function f. Sketch on the given axes the graph of

(a) $|f(-x)|$; [3]

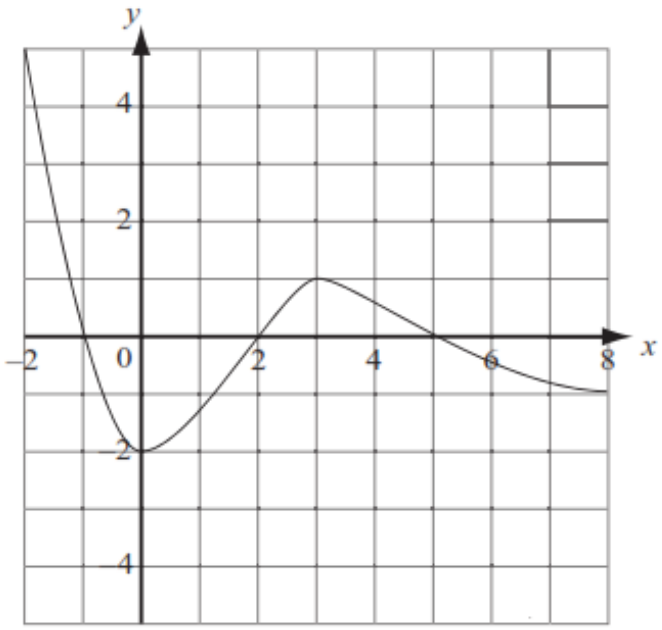


(b) $\frac{1}{f(x)}$; [3]

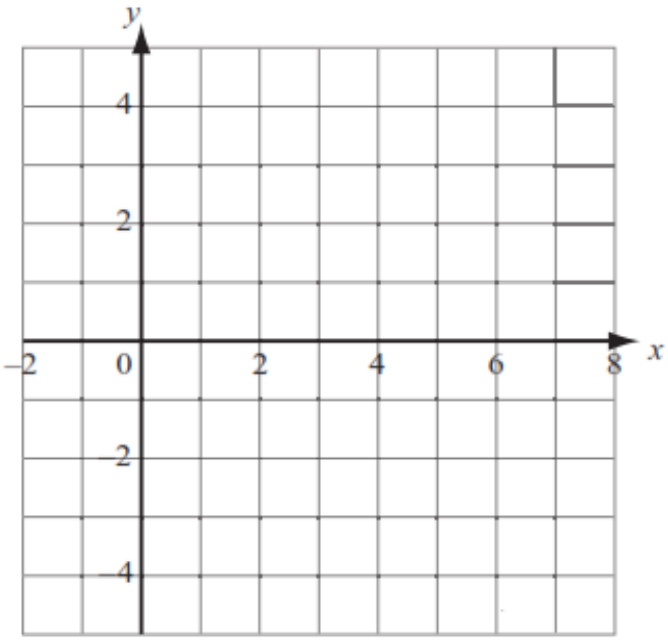


13. [Maximum mark: 6] ***[without GDC]***

The graph of $y = f(x)$ for $-2 \le x \le 8$ is shown.

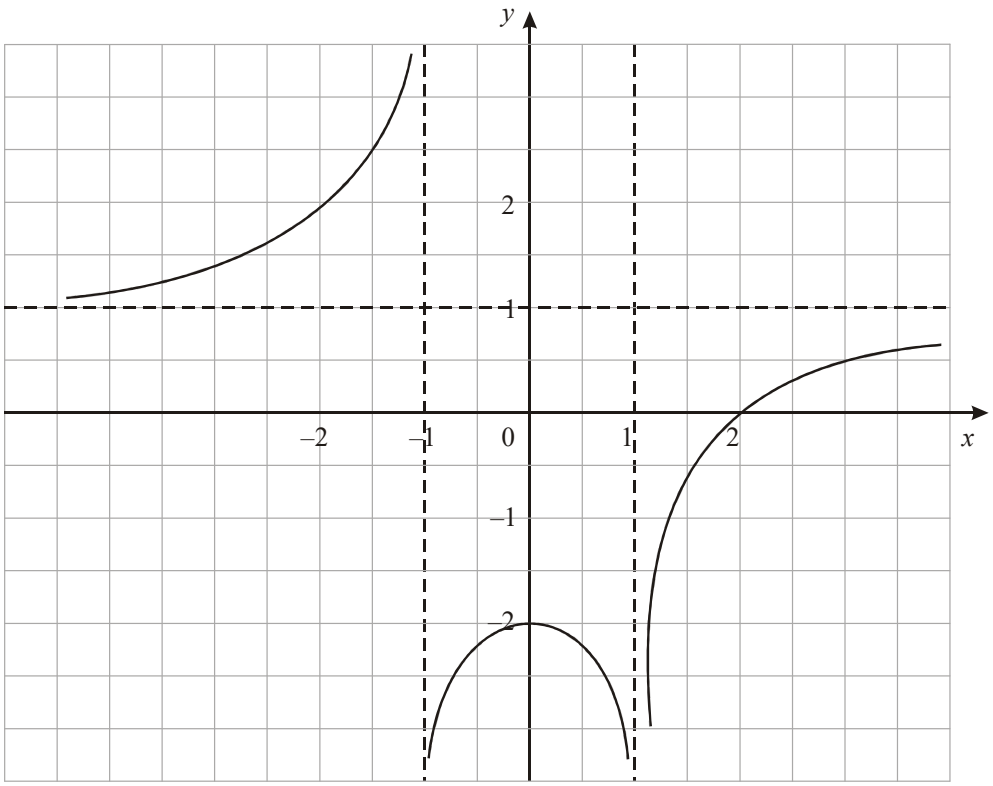


On the set of axes provided, sketch the graph of $y = \frac{1}{f(x)}$, clearly showing any asymptotes and indicating the coordinates of any local maxima or minima.



14. [Maximum mark: 8] ***[without GDC]***

The diagram shows the graph of $f(x)$.



(b) The equation $f(x) = k$ has exactly one solution. Write down the possible values of k. [2]

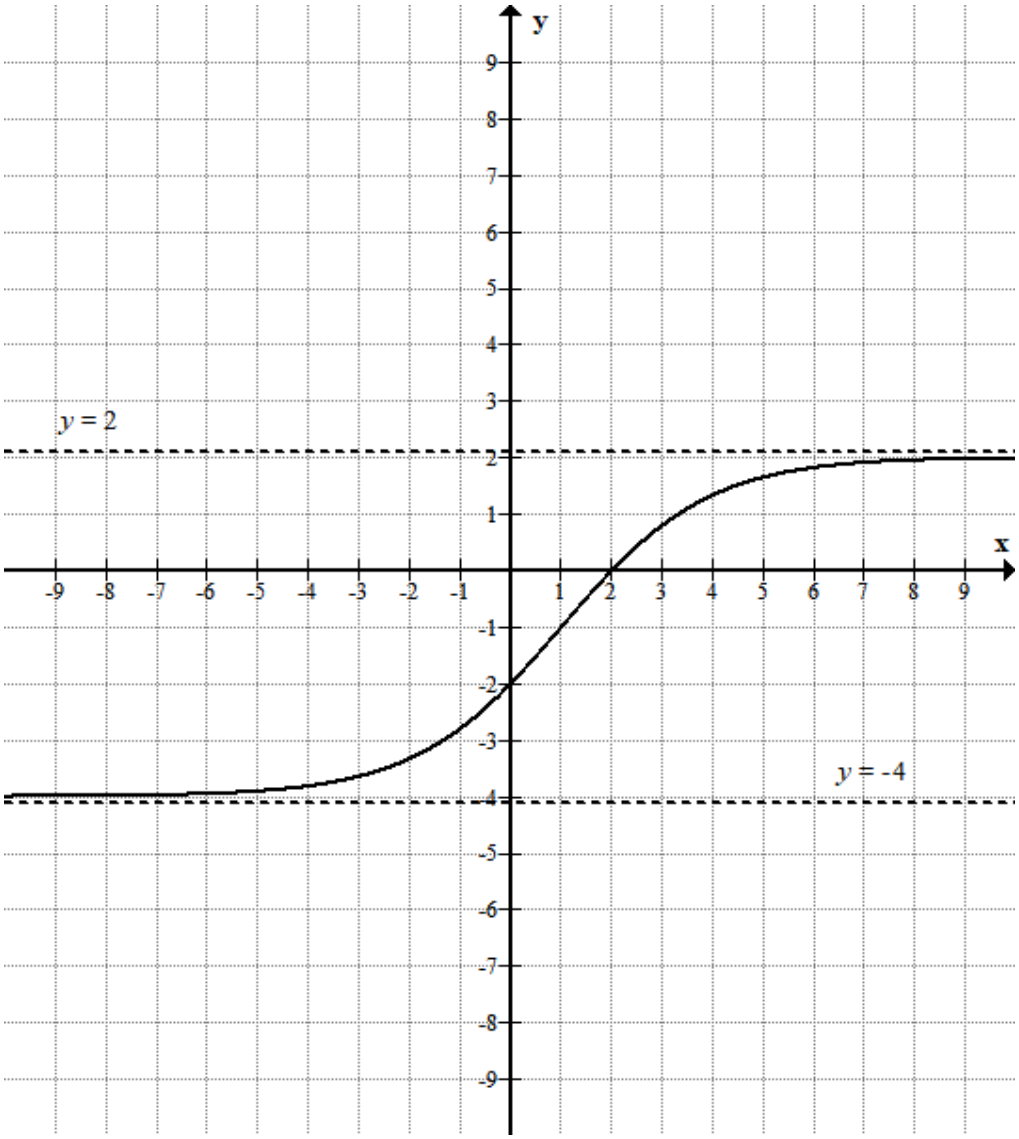
(b) On the same diagram, sketch the graph of $\frac{1}{f(x)}$, indicating any asymptotes. [3]

(c) Write down the coordinates of the local maximum point, the local minimum point, the x-intercepts and the y-intercept of $\frac{1}{f(x)}$. [3]

15. [Maximum mark: 6] ***[without GDC]***

The diagram shows the graph of $f(x)$, with horizontal asymptotes $y = 2$ and $y = -4$.

(a) On the same diagram, sketch the graph of $y = \frac{8}{f(x)}$ indicating any asymptotes.



(b) Write down the domain and the range

(i) of $y = f(x)$ (ii) of $y = \frac{8}{f(x)}$ [4]

16. [Maximum mark: 7] ***[without GDC]***

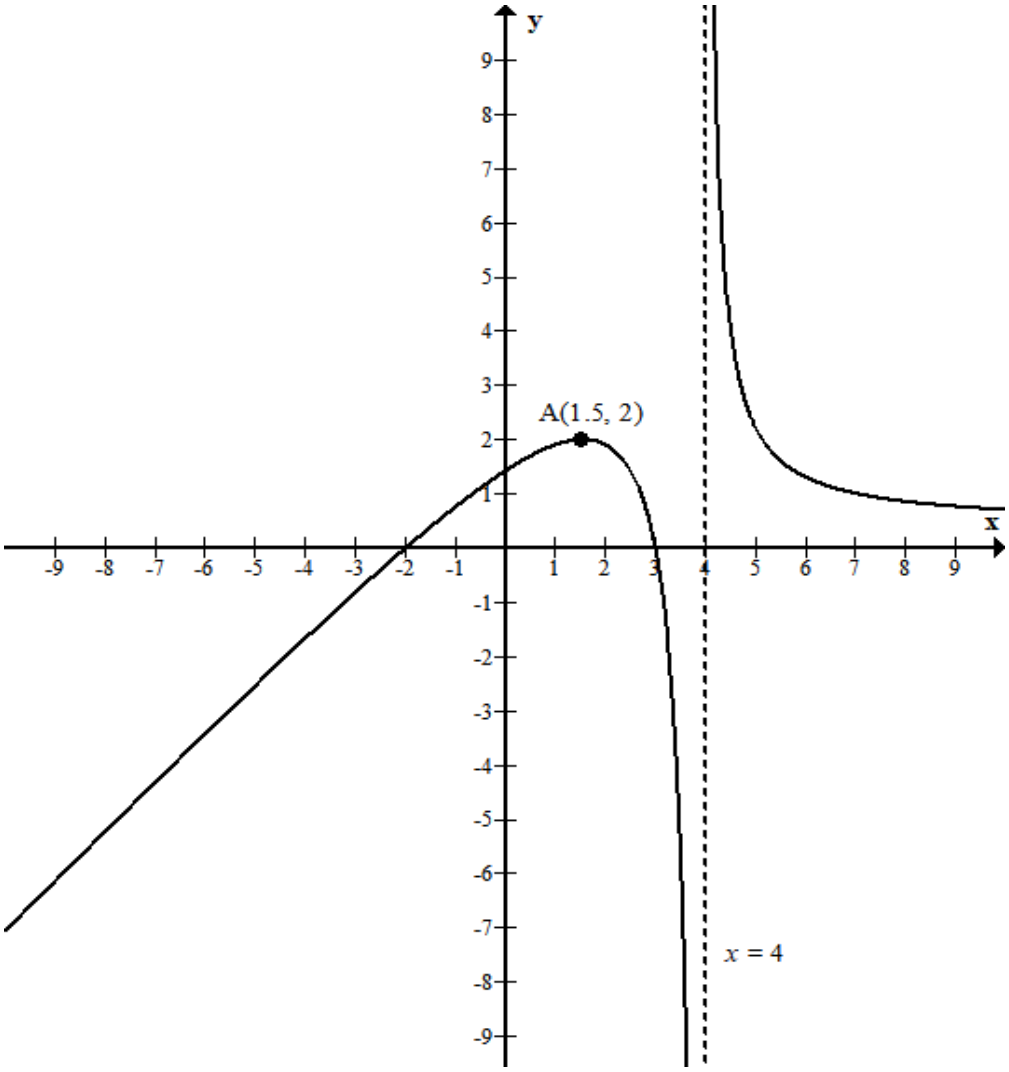
The diagram shows the graph of $f(x)$. It has a local maximum at $\mathrm{A}\left(\frac{3}{2},2\right)$, a vertical asymptote at $x=4$ and the y-intercept is at $\left(0,\frac{3}{2}\right)$.

Let $g(x)=\frac{1}{f(x)}$.

(a) Write down the coordinates of

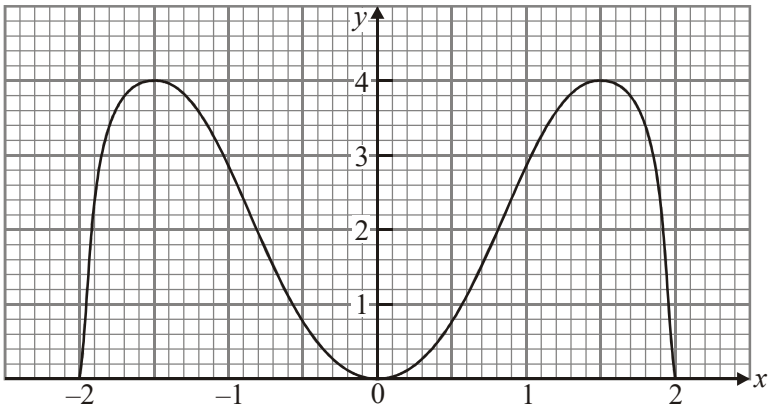
(i) The y-intercept of $y=g(x)$ (ii) The local minimum of $y=g(x)$ [2]

(b) On the same diagram, sketch the graph of $y=g(x)$. [5]



17*. [Maximum mark: 7] ***[without GDC]***

The graph of $y = f(x)$, where $-2 \le x \le 2$ is shown below.

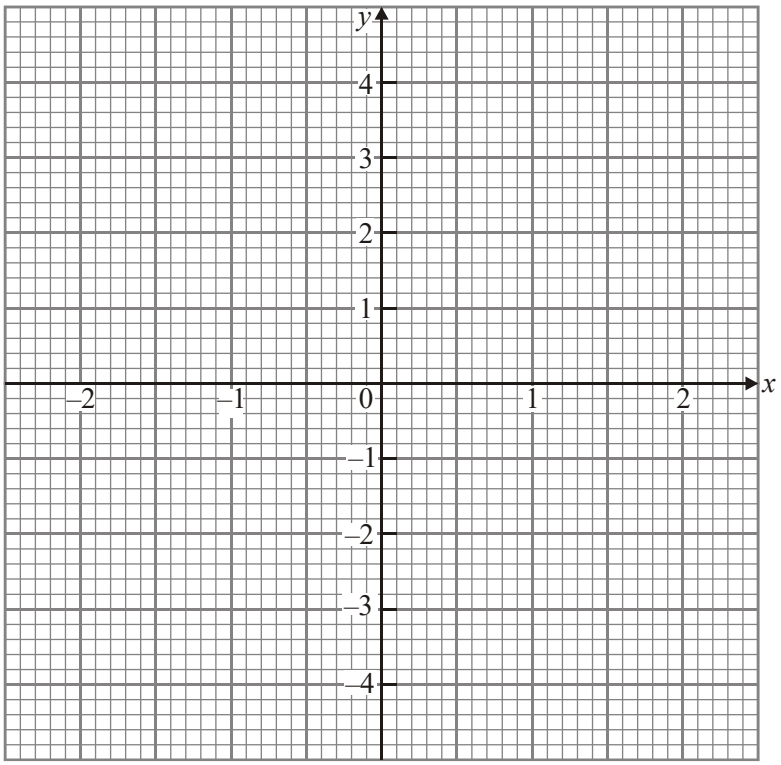


(a) Write down the range of the following functions

Function	Range
$y = f(x)$	
$y = f(x)^2$	
$y = \sqrt{f(x)}$	

[3]

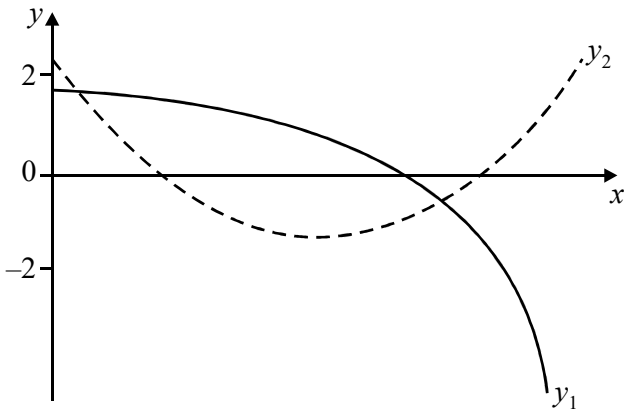
(b) Sketch, on the axes provided below, the graph of $y^2 = f(x)$ for $-2 \le x \le 2$.



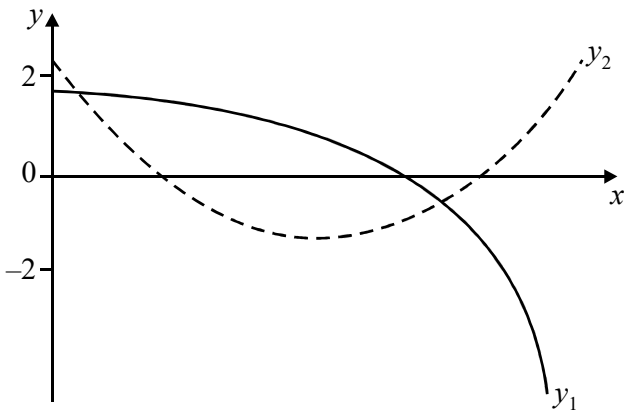
[4]

18*. [Maximum mark: 6] ***[without GDC]***

The diagram shows the graph of the functions y_1 and y_2.

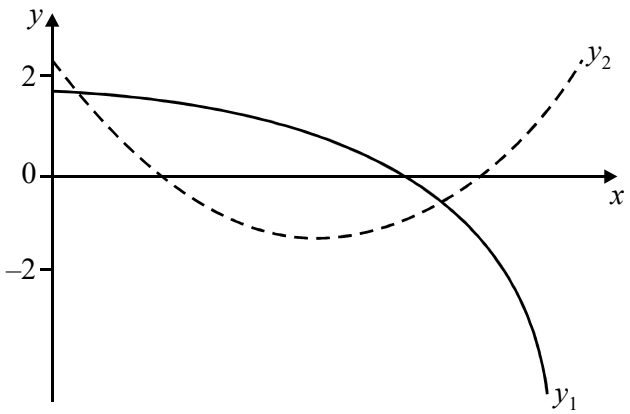


(a) On the following diagram sketch the graph of $\frac{y_1}{y_2}$. Indicate clearly where the x-intercepts and asymptotes occur.



[4]

(b) On the following diagram sketch the graph of $y_1 - y_2$. Indicate clearly where the x-intercepts occur.



[2]

19. [Maximum mark: 7] ***[with GDC]***

Consider the graph of the function, f, defined by

$$f(x) = 3x^4 - 4x^3 - 30x^2 - 36x + 112, \quad -2 \le x \le 4.5$$

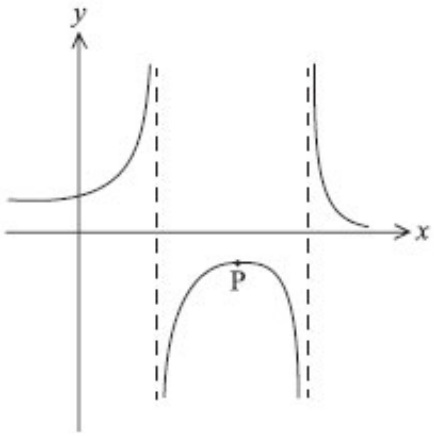
(a) The equation $f(x) = 0$ has one solution at $x = 4$; find the other solution. [1]

(b) Find

(i) the coordinates of the minimum point of the graph of f;

(ii) the range of f. [3]

A sketch of the graph of $\frac{1}{f}$ is given below.



(c) Write down the **equations** of the two vertical asymptotes. [2]

(d) The graph of $\frac{1}{f}$ has a maximum at P. Write down the x-coordinate of P. [1]