INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 1.10]

SYSTEMS OF LINEAR EQUATIONS

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O. Practice questions

1. [Maximum mark: 11] ***[without GDC]***

A system of simultaneous equations and the corresponding augmented matrix are given below (a, b are constant real numbers).

$\begin{aligned} x+2y+3z&=10 \\ y+5z&=7 \\ az&=b \end{aligned}$	$\left(\begin{array}{ccc\|c} 1 & 2 & 3 & 10 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & a & b \end{array}\right)$

(a) Complete the following table [5]

The system has		Possible values of a	Possible values of b
(i)	a unique solution		
(ii)	no solutions		
(iii)	∞ – ly many solutions		

(b) Find the unique solution when $a=1$ and $b=1$. [3]

(c) In the case (a) (iii), express the set of solutions in terms of a parameter $\lambda \in \mathbb{R}$. [3]

[Confirm the results (b) and (c) with your GDC]

2. [Maximum mark: 13] ***[with / without GDC]***

Consider the following systems of simultaneous equations.

(i) $\begin{aligned} x+y+z&=3 \\ x+2y+3z&=6 \\ x+2y+4z&=7 \end{aligned}$ (ii) $\begin{aligned} x+y+z&=3 \\ x+2y+3z&=6 \\ 2x+3y+4z&=10 \end{aligned}$ (iii) $\begin{aligned} x+y+z&=3 \\ x+2y+3z&=6 \\ 2x+3y+4z&=9 \end{aligned}$

(a) Use your GDC to state the solution for each system.

For (iii), express the solution in terms of a parameter $\lambda \in \mathbb{R}$. [3]

(b) Use Gaussian elimination to find the solution for each system. [10]

(c) State two particular solutions (x, y, z) for system (iii). [2]

A. Exam style questions (SHORT)

3. [Maximum mark: 5] ***[without GDC]***

Solve the following system of equations:

$$x + 3y - 2z = -6$$
$$2x + y + 3z = 7$$
$$3x - y + z = 6.$$

[Confirm the result with your GDC]

4. [Maximum mark: 5] ***[without GDC]***

Solve the following system of equations:

$$3x - 2y + z = -4$$
$$x + y - z = -2$$
$$2x + 3y = 4$$

[Confirm the result with your GDC]

5. [Maximum mark: 5] ***[without GDC]***

Solve the following system of equations:

$$x - 3y + z = 1 \qquad 2x + 2y - z = 2 \qquad x - 5y + 3z = 3$$

[Confirm the result with your GDC]

6. [Maximum mark: 6] ***[without GDC]***

Find the non-unique solution for the following system of simultaneous equations

$$x - y - z = 3$$
$$x - 2y + z = 2$$
$$2x - y - 4z = 7$$

[Confirm the result with your GDC]

7. [Maximum mark: 6] ***[with / without GDC]***

Find the general solution for the following system of equations

$$2x - 7y + 5z = 1$$
$$6x + 3y - z = -1$$
$$-14x - 23y + 13z = 5$$

8. [Maximum mark: 7] ***[with / without GDC]***

Consider the system of equations

$$x + 2y - 3z = k$$
$$3x + y + 2z = 4$$
$$5x + 7z = 5$$

(a) Find the set of values of k for which the system of equations has no solution. [4]

(b) Find the value of k for which the system is consistent; for this k, find the solution. [3]

9. [Maximum mark: 7] ***[without GDC]***

The following system of equations has an infinite number of solutions.

$$2x - y - 9z = 7$$
$$x + 2y + 3z = 1$$
$$2x + y - 3z = k$$

(a) Find the value of k. [4]

(b) Find the general solution [3]

[Confirm the result (b) with your GDC]

10. [Maximum mark: 9] ***[without GDC]***

The variables x, y, z satisfy the simultaneous equations (where k is a constant)

$$x + 2y + z = k$$
$$2x + y + 4z = 6$$
$$x - 4y + 5z = 9$$

(a) (i) Show that these equations do **not** have a unique solution.

(ii) Find the value of k for which the system is consistent (there is a solution). [6]

(b) For this value of k, find the general solution of these equations. [3]

B. Exam style questions (LONG)

11*. [Maximum mark: 14] ***[without GDC]***

(a) Show that the following system of equations has a unique solution when $a \neq -1$.

$$x + 3y - z = 0,$$
$$3x + 5y - z = 0,$$
$$x - 5y + (2 - a)z = 9 - a^2$$

[4]

(b) State the unique solution in terms of a. [6]

(c) Hence, solve the system

$$x + 3y - z = 0,$$
$$3x + 5y - z = 0,$$
$$x - 5y + z = 8$$

[2]

(d) Investigate the case $a = -1$. [2]

12*. [Maximum mark: 13] ***[with / without GDC]***

Consider the system of equations

$$x + 2y + kz = 0$$
$$x + 3y + z = 3$$
$$kx + 8y + 5z = 6$$

(a) Find the set of values of k for which this system of equations has a **unique** solution. [6]

(b) Find the solution if $k = 0$. [2]

(c) For each value of k that results in a **non-unique** solution, find the solution set. [5]

13. [Maximum mark: 11] ***[with / without GDC]***

Consider the system of equations

$$x + 2y + z = 0$$
$$2x + 5y + z = b$$
$$5x + 8y + az = -4$$

(a) Find the possible values of a and b for which the system has

(i) a unique solution (ii) no solution (iii) infinitely many solutions. [6]

(b) In case (a) (i), express only the value of z in terms of a and b. [1]

(c) In case (a) (iii), find the general solution. [4]