INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 1.6]

BINOMIAL THEOREM

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O. Practice questions

1. [Maximum mark: 9] ***[without GDC]***

Write down the expansions of

(a) $(1+x)^3$ and $(1-x)^3$. [3]

(b) $(1+x)^4$ and $(1-x)^4$. [3]

(c) $(1+x)^5$ and $(1-x)^5$. [3]

2. [Maximum mark: 9] ***[with GDC]***

Write down the first four terms in ascending powers of x

(a) in the expansion of $(1+x)^{10}$ [3]

(b) in the expansion of $(1+2x)^{10}$ [3]

(c) in the expansion of $(2-x)^{10}$ [3]

3. [Maximum mark: 5] ***[with GDC]***

Find the term in x^7 in the expansion of $(1-x)^{10}$

4. [Maximum mark: 5] ***[with GDC]***

Find the term in x^{16} in the expansion of $(1-x^2)^{10}$

5. [Maximum mark: 10] ***[with GDC]***

(a) Expand (i) $\left(x-\frac{1}{x}\right)^3$ (ii) $\left(x-\frac{1}{x}\right)^4$ [5]

(b) In the expansion of $\left(x-\frac{1}{x}\right)^{10}$

(i) find the constant term

(ii) find the term in x^2

(iii) find the term in x^{-2}. [5]

6*. [Maximum mark: 10] ***[with GDC]***

(a) Expand $(2x+1)^4$ [2]

Hence, find

(b) the term in x^2 in the expansion of $x(2x+1)^4$ [2]

(c) the term in x^2 in the expansion of $(x+1)(2x+1)^4$ [3]

(d) the term in x^2 in the expansion of $(3-x^2)(2x+1)^4$. [3]

7. [Maximum mark: 6] ***[without GDC]***

(a) Show that

(i) ${}^8C_7 = 8$, (ii) ${}^8C_6 = 28$, (iii) ${}^8C_5 = 56$ [3]

(b) Find

(i) ${}^{10}C_9$, (ii) ${}^{10}C_8$, (iii) ${}^{10}C_7$ [3]

8*. [Maximum mark: 9] ***[without GDC]***

(a) Show that

(i) ${}^nC_0 = 1$, (ii) ${}^nC_1 = n$, (iii) ${}^nC_2 = \frac{n(n-1)}{2}$ [5]

(b) Find similar expressions for

(i) ${}^nC_3$ (ii) ${}^nC_4$ [4]

9*. [Maximum mark: 9] ***[with GDC]***

In the expansion of $\left(3x - \frac{2}{x}\right)^{12}$ find

(a) the constant term; [3]

(b) the coefficient of x^4; [3]

(c) the first three terms in descending powers of x. [3]

10*. [Maximum mark: 9] ***[with GDC]***

In the expansion of $\left(3x^2 - \frac{2}{x}\right)^{12}$ find

(a) the constant term; [3]

(b) the coefficient of x^6; [3]

(c) the first three terms in descending powers of x. [3]

11*. [Maximum mark: 9] ***[with GDC]***

In the expansion of $\left(3x^3 - \frac{2}{x}\right)^{12}$ find

(a) the constant term; [3]

(b) the coefficient of x^4; [3]

(c) the first three terms in descending powers of x. [3]

A. Exam style questions (SHORT)

12. [Maximum mark: 4] ***[without GDC]***

Express $\left(\sqrt{3}+\sqrt{2}\right)^3$ in the form $a\sqrt{2}+b\sqrt{3}$, where $a,b\in Z$.

13. [Maximum mark: 4] ***[without GDC]***

Express $\left(\sqrt{3}+\sqrt{2}\right)^4$ in the form $a+b\sqrt{6}$, where $a,b\in Z$.

14. [Maximum mark: 6] ***[without GDC]***

Express $\left(\sqrt{3}-2\right)^3$ in the form $a\sqrt{3}+b$, where $a,b\in Z$.

15. [Maximum mark: 6] ***[with / without GDC]***

Given that $\left(3+\sqrt{7}\right)^3 = p+q\sqrt{7}$ where p and q are integers, find

(a) p; (b) q.

16. [Maximum mark: 6] ***[without GDC]***

(a) Expand $\left(\mathrm{e}+\frac{1}{\mathrm{e}}\right)^4$ in terms of e. [4]

(b) Express $\left(\mathrm{e}+\frac{1}{\mathrm{e}}\right)^4 + \left(\mathrm{e}-\frac{1}{\mathrm{e}}\right)^4$ as the sum of three terms. [2]

17. [Maximum mark: 6] ***[with / without GDC]***

Complete the following expansion.

$$(2+ax)^4 = 16+32ax+\ldots$$

18. [Maximum mark: 4] ***[with GDC]***

Use the binomial theorem to complete this expansion.

$$(3x+2y)^4 = 81x^4+216x^3y+\ldots$$

19. [Maximum mark: 8] ***[without GDC]***

Find (i) ${}^7C_3$ (ii) ${}^{200}C_2$

20. [Maximum mark: 8] ***[without GDC]***

(a) Verify that ${}^5C_2+{}^5C_3={}^6C_3$ [3]

(b) Prove that ${}^{19}C_9+{}^{19}C_{10}={}^{20}C_{10}$ [5]

21. [Maximum mark: 4] ***[with / without GDC]***

Find the coefficient of a^5b^7 in the expansion of $(a+b)^{12}$.

22. [Maximum mark: 4] ***[with / without GDC]***

Find the coefficient of a^3b^4 in the expansion of $(5a+b)^7$.

23. [Maximum mark: 6] ***[without GDC]***

The fifth term in the expansion of the binomial $(a+b)^n$ is given by ${}^{10}C_4p^6(2q)^4$.

(a) Write down the value of n. [1]

(b) Write down a and b, in terms of p and/or q. [2]

(c) Write down an expression for the sixth term in the expansion. [3]

24. [Maximum mark: 5] ***[with GDC]***

Find the coefficient of x^7 in the expansion of $(2+3x)^{10}$, giving your answer as a whole number.

25. [Maximum mark: 5] ***[with / without GDC]***

Consider the expansion of $(x+2)^{11}$.

(a) Write down the number of terms in this expansion. [1]

[b) Find the term containing x^2. [4]

26. [Maximum mark: 6] ***[with / without GDC]***

Find the coefficient of x^3 in the expansion of $(2-x)^5$.

27. [Maximum mark: 4] ***[with GDC]***

Find the coefficient of x^5 in the expansion of $(3x-2)^8$

28. [Maximum mark: 6] ***[with GDC]***

Find the term containing x^3 in the expansion of $(2-3x)^8$.

29. [Maximum mark: 6] ***[with GDC]***

Find the coefficient of x^3 in the binomial expansion of $\left(1-\frac{1}{2}x\right)^8$.

30. [Maximum mark: 6] ***[with GDC]***

Find the coefficient of the x^3 term in the expansion of $\left(2-\frac{3x}{2}\right)^6$.

31. [Maximum mark: 5] ***[with GDC]***

Find the term in x^3 in the expansion of $\left(\frac{2}{3}x-3\right)^8$.

32. [Maximum mark: 6] ***[with / without GDC]***

One of the terms of the expansion of $(x + 2y)^{10}$ is $ax^8 y^2$. Find the value of a.

33. [Maximum mark: 6] ***[with GDC]***

Find the term containing x^{10} in the expansion of $(5 + 2x^2)^7$.

34. [Maximum mark: 6] ***[with GDC]***

Consider the expansion of $(x^2 - 2)^5$.

(a) Write down the number of terms in this expansion. [1]

(b) The first four terms of the expansion in descending powers of *x* are

$$x^{10} - 10x^8 + 40x^6 + Ax^4 + ...$$

Find the value of A. [5]

35. [Maximum mark: 6] ***[with GDC]***

Consider the expansion of the expression $(x^3 - 3x)^6$.

(a) Write down the number of terms in this expansion. [1]

(b) Find the term in x^{12}. [5]

36. [Maximum mark: 6] ***[with / without GDC]***

Find the term in x^4 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^5$.

37. [Maximum mark: 6] ***[with GDC]***

Consider the expansion of $\left(3x^2 - \frac{1}{x}\right)^9$.

(a) How many terms are there in this expansion? [1]

(b) Find the constant term in this expansion. [5]

38. [Maximum mark: 4] ***[with GDC]***

Determine the constant term in the expansion of $\left(x - \frac{2}{x^2}\right)^9$.

39. [Maximum mark: 6] ***[without GDC]***

Find the constant term in the expansion of $\left(x^2 - \frac{2}{x^2}\right)^6$.

40. [Maximum mark: 6] ***[with / without GDC]***

Find the coefficient of x in the expansion of $\left(3x - \frac{2}{x}\right)^5$.

41. [Maximum mark: 6] ***[with GDC]***

When the expression $(2 + ax)^{10}$ is expanded, the coefficient of the term in x^3 is $414\,720$.

Find the value of a.

42. [Maximum mark: 6] ***[with GDC]***

The coefficient of x in the expansion of $\left(x+\frac{1}{ax^2}\right)^7$ is $\frac{7}{3}$. Find the possible values of a.

43. [Maximum mark: 6] ***[without GDC]***

(a) Expand $(x-2)^4$ and simplify your result. [3]

(b) Find the term in x^3 in $(3x+4)(x-2)^4$. [3]

44*. [Maximum mark: 6] ***[without GDC]***

(a) Expand $(2+x)^4$ and simplify your result. [3]

(b) Hence, find the term in x^2 in $(2+x)^4\left(1+\frac{1}{x^2}\right)$. [3]

45*. [Maximum mark: 8] ***[without GDC]***

(a) Find the expansion of $(2+x)^5$, giving your answer in ascending powers of x. [3]

(b) By letting $x=0.01$ or otherwise, find the **exact** value of 2.01^5. [2]

(c) Find the **exact** value of 1.99^5. [3]

46.** [Maximum mark: 6] ***[with GDC]***

Determine the first three terms in the expansion of $(1-2x)^5(1+x)^7$ in ascending powers of x.

47.** [Maximum mark: 6] ***[without GDC]***

Given that

$$(1+x)^5(1+ax)^6 \equiv 1+bx+10x^2+\cdots+a^6x^{11},$$

find the values of $a, b \in \mathbb{Z}^*$.

48*. [Maximum mark: 6] ***[without GDC]***

Find the term in x^2 in $(1+x)^4\left(3+\frac{2}{x^2}\right)$

49.** [Maximum mark: 6] ***[with / without GDC]***

Find the coefficient of x^3 in the expansion of $(2+x)^4(1+x^2)^5$

50.** [Maximum mark: 6] ***[with GDC]***

Find the constant term in the expansion of $(2+x)^4\left(2x+\frac{1}{x}\right)^5$

51.** [Maximum mark: 6] ***[with GDC]***

(a) Write down the quadratic expression $2x^2 + x - 3$ as the product of two linear factors.

(b) Hence, or otherwise, find the coefficient of x in the expansion of $(2x^2 + x - 3)^8$.

52.** [Maximum mark: 9] ***[with GDC]***

In the binomial expansion of $(a + x)^n$, where $n \geq 4$, the coefficient of x^3 is twice the coefficient of x^4.

(a) Show that $n = 2a + 3$ [5]

(b) Given that $a = 3$, find the coefficients of x^3 and x^4. [4]

53.** [Maximum mark: 6] ***[with GDC]***

In the expansion of $(1 + ax)^n$ the first term is 1, the second term is $24x$ and the third term is $252x^2$. Find the values of a and n.

54*. [Maximum mark: 8] ***[with GDC]***

When $\left(1 + \frac{x}{2}\right)^n, n \in \mathbb{N}$, is expanded in ascending powers of x, the coefficient of x^3 is 70.

(a) Find the value of n. [4]

(b) Hence, find the coefficient of x^2. [4]

B. Exam style questions (LONG)

55. [Maximum mark: 12] ***[without GDC]***

Consider the binomial expansion $(1 + x)^4 = 1 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + x^4$

(a) By substituting $x = 1$ into both sides, or otherwise, evaluate ${}^4C_1 + {}^4C_2 + {}^4C_3$ [2]

(b) Evaluate ${}^4C_1 - {}^4C_2 + {}^4C_3$ in a similar way. [3]

(c) Evaluate ${}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4 + {}^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8$. [4]

(d) Show that ${}^9C_1 + {}^9C_3 + {}^9C_5 + {}^9C_7 = {}^9C_2 + {}^9C_4 + {}^9C_6 + {}^9C_8$. [3]

56.** [Maximum mark: 9] ***[without GDC]***

Let $S = a + b$ and $P = ab$. By using the appropriate expansions of $(a + b)^n$, express the following in terms of S and P.

(a) $a^2 + b^2$ [2]

(b) $a^3 + b^3$ [3]

(c) $a^4 + b^4$ [4]