INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math AA 2.8]

EXPONENTS

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O. Practice questions

1. [Maximum mark: 15] ***[without GDC]***

Find the following values in the form of an integer or a fraction $\frac{a}{b}$ of integers.

$3^3 =$	$3^{-2} =$	$3^{-3} =$
$\left(\frac{1}{3}\right)^2 =$	$\left(\frac{2}{3}\right)^2 =$	$\left(\frac{2}{3}\right)^3 =$
$\left(\frac{1}{3}\right)^{-2} =$	$\left(\frac{2}{3}\right)^{-2} =$	$\left(\frac{2}{3}\right)^{-3} =$
$4^{\frac{1}{2}} =$	$4^{\frac{3}{2}} =$	$4^{-\frac{1}{2}} =$
$25^{\frac{1}{2}} =$	$25^{-\frac{1}{2}} =$	$\left(\frac{25}{4}\right)^{\frac{1}{2}} =$

2. [Maximum mark: 6] ***[without GDC]***

Find the following values in the form of an integer or a fraction $\frac{a}{b}$ of integers

$3^2 \times 2^3 =$	$3^5 \times 3^{-3} =$	$3^{-5} \times 3^3 =$
$3^0 + 0^3 =$	$\frac{5^4 \times 2^3}{2^2 \times 5^3} =$	$\frac{7^4 \times 7^2}{7 \times 7^7} =$
$3^{-1} \times 2^3 =$	$3 \times 2^{-3} =$	$\frac{2^{-1}}{3^{-1}} =$

3. [Maximum mark: 9] ***[without GDC]***

Find the following values in the form of an integer or a fraction $\frac{a}{b}$ of integers

$\frac{30^2}{15^2} =$	$\frac{15^{-2}}{30^{-2}} =$	$\frac{12^{1/2}}{3^{1/2}} =$
$7^{0.3} \times 7^{0.7} =$	$7^{\frac{1}{2}} \times 7^{\frac{3}{2}} =$	$7^{-2} \times 7 =$
$7^2 \times 7^{-1} =$	$\frac{2^3 \times 3^3}{6^3} =$	$\frac{4^3 \times 3^3}{6^3} =$

4. [Maximum mark: 15] ***[without GDC]***

Express the following in the form of a single power (x^y)

$a^6 a^2 =$	$a^6 a =$	$(a^{\frac{1}{2}})^6 =$
$\left(a^6\right)^2 =$	$\left(a^2\right)^6 =$	$\frac{a^6}{a^2} =$
$a^2 a^3 a =$	$\frac{a^2 a^6}{a^5} =$	$\frac{a^2 b^6}{b^2 a^{-3} b^4} =$
$\left(a^{\frac{3}{5}}\right)^5 =$	$\left(a^{\frac{3}{5}}\right)^{10} =$	$\left(a^{=1}\right)^{=2} =$
$a^{x+3} a^{1-x} =$	$\frac{a^{n+5}}{a^{n+3}} =$	$\frac{a^{-8}}{a^{-10}} =$

5. [Maximum mark: 8] ***[without GDC]***

Given that $A > 0$, $B > 0$, $C > 0$, simplify the expressions

$\frac{A^6 B^3 C^{10}}{C^5 A^2 B} =$
$\frac{A^2 B + AB^3}{AB} =$
$\frac{2A + A(4B) + (2A)^2}{2A} =$
$\frac{A^4 B^3 + A^3 B^4}{A + B} =$

6. [Maximum mark: 5] ***[without GDC]***

Consider the following powers of e:

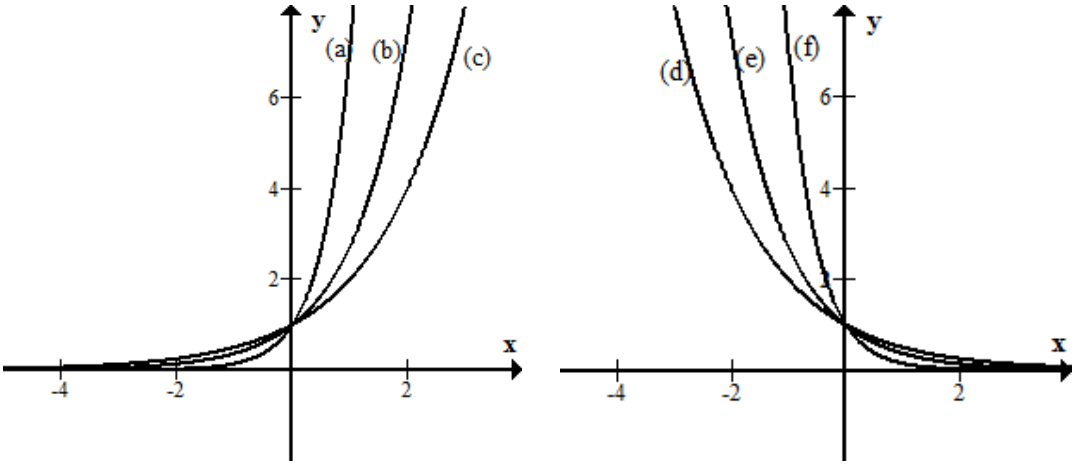
A. e^{-2}	**B.** $e^{\frac{1}{2}}$	**C.** e^{x}	**D.** e^{-x}	**E.** $e^{x/2}$	**F.** e^{2x}	**G.** e^{x-2}

Find the equivalent expressions in the table below, by stating the corresponding letter.

$(e^{x})^{2}$	$\frac{1}{e^{x}}$	$\frac{e^{2x}}{e^{x}}$	$\frac{1}{e^{2}}$	$\sqrt{e}$	$\frac{e^{x}}{e^{2}}$	$\sqrt{e^{x}}$
F						

7. [Maximum mark: 8] ***[without GDC]***

(a) The graphs of 6 functions are shown below



Match the graphs (a) (b) (c) (d) (e) and (f) to the following functions

$y = 2^{x}$	$y = 5^{x}$	$y = e^{x}$

$y = 2^{-x}$	$y = 5^{-x}$	$y = e^{-x}$

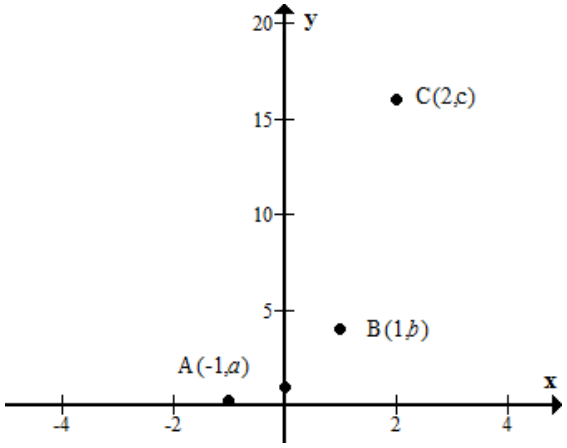
(b) Complete the following table (it contains the common details for all six graphs)

y – intercept	
Horizontal asymptote	
Domain	
Range	

8. [Maximum mark: 10] ***[without GDC]***

Let $f(x) = 4^x$.

The points A(-1,a), B(1,b) and C(2,c) of the graph are shown in the diagram below.



(a) Write down the coordinates of the y-intercept of the graph. [1]

(b) Write down the value of a in decimal form. [2]

(c) Write down the values of b and of c. [2]

(d) Write down the equation of the horizontal asymptote. [1]

(e) On the diagram above, sketch the graph of f. [2]

(f) Write down the domain and the range of f. [2]

9. [Maximum mark: 6] ***[without GDC]***

The graph of $f(x) = 2^x + 2$ passes through the points A(1, a), B(2, b) and C(3, c).

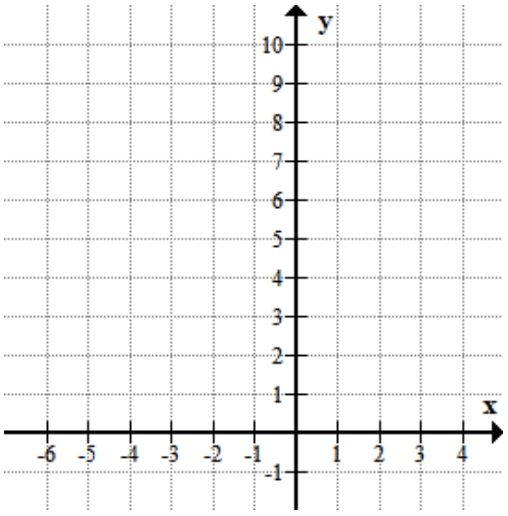
(a) Find the values of a, b and c. [2]

(b) Sketch the graph of the function. Indicate the y-intercept, the horizontal asymptote and the points A, B, C of the graph. [4]

$a =$ .....

$b =$ .....

$c =$ .....



10. [Maximum mark: 6] ***[without GDC]***

The graph of $f(x) = 2^x - 2$ passes through the points A(1, a), B(2, b) and C(3, c).

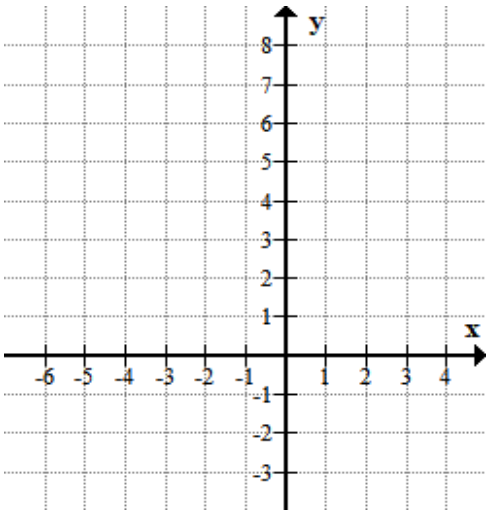
(a) Find the values of a, b and c. [2]

(b) Sketch the graph of the function. Indicate the y-intercept, the horizontal asymptote and the points A, B, C of the graph. [4]

$a =$ .....

$b =$ .....

$c =$ .....



11. [Maximum mark: 18] ***[without GDC]***

Solve the equations

(a) $2^{2x} = 2^{1-x}$ [3]

(b) $2^{2x} = 8^{1-x}$ [3]

(c) $4^{2x} = 8^{1-x}$ [3]

(d) $8^{x+3} = 16^{x}$ [3]

(e) $\frac{1}{2^x} = 4^{x-3}$ [3]

(f) $\sqrt{2^x} = 4^{1-x}$ [3]

12. [Maximum mark: 15] ***[without GDC]***

Solve the equations

(a) $25^{x+1} = 5^3$ [3]

(b) $25^{x+1} = \frac{1}{5^x}$ [3]

(c) $25^{x+1} = \sqrt{5^x}$ [3]

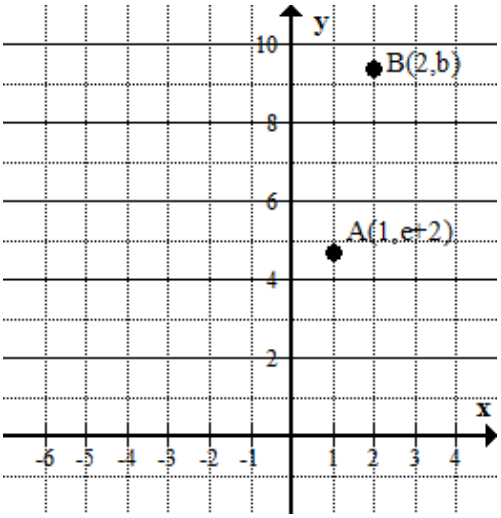
(d) $25^{x^2} = 125^x$ [3]

(e) $7^{x^2-5x} = 1$ [3]

A. Exam style questions (SHORT)

13. [Maximum mark: 9] ***[with GDC]***

Let $f(x) = e^x + 2$. The points A(1, e+2) and B(2,b) of the graph are shown in the diagram below.



(a) Write down value of b

(i) in **exact** form (ii) correct to 3s.f. [2]

(b) Write down, correct to 3.s.f.,

(i) the value of $f(1)$ (ii) the value of $f^{-1}(6)$ [2]

(c) On the diagram above, sketch the graph of f. Indicate the y-intercept and the horizontal asymptote of the graph. [3]

(d) Write down the domain and the range of f. [2]

14. [Maximum mark: 5] ***[without GDC]***

Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$.

15. [Maximum mark: 5] ***[without GDC]***

Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

16. [Maximum mark: 5] ***[without GDC]***

Solve the equation $25^{x^2} = \sqrt{5}$

17. [Maximum mark: 5] ***[without GDC]***

Solve the equation $4^{x^2} = 8^x$.

18*. [Maximum mark: 5] ***[without GDC]***

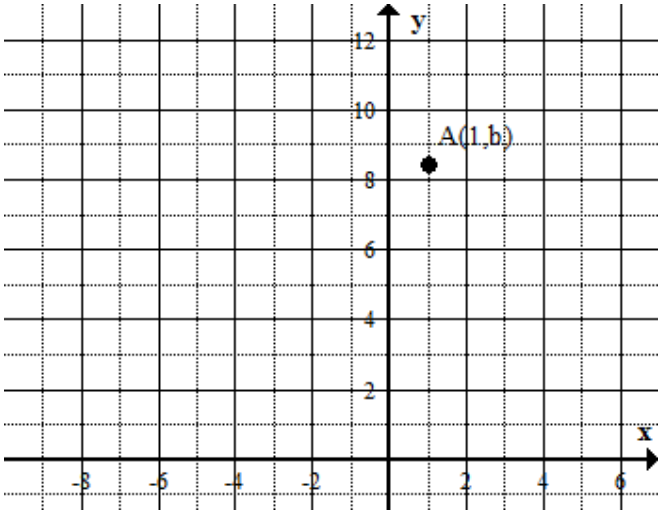
Solve the equation $5^{x^2-3} = \left(\frac{1}{25}\right)^{2x-1}$.

B. Exam style questions (LONG)

19. [Maximum mark: 12] ***[without GDC]***

For each of the following functions sketch the graph and complete the table

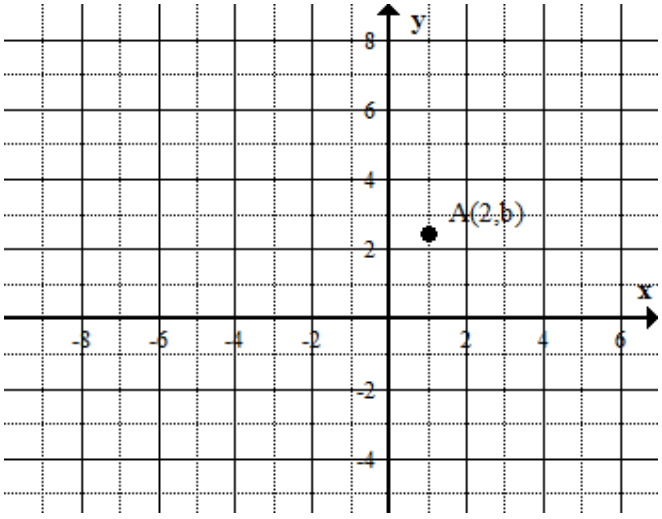
(a) $f(x) = 2e^x + 3$ (the point A lies on the curve).



y - intercept:	
horizontal asymptote:	**Exact** value of b:
Domain:	Range:

[6]

(b) $f(x) = 2e^x - 3$ (the point A lies on the curve).



y - intercept:	
horizontal asymptote:	**Exact** value of b:
Domain:	Range:

[6]

20*. [Maximum mark: 12] ***[with GDC]***

Consider the function $y = f(x)$ with $f(x) = 10e^{0.3x} + 5$

(a) Write down the domain of f. [1]

(b) Find the y-intercept of the graph. [2]

(c) Find (i) $f(5)$ correct to 3sf. (ii) $f^{-1}(100)$ correct to 3sf. [3]

(d) Find the first integer value of x for which the value of y will exceed 120. [2]

(e) Find the value of $f(-20)$ and deduce the equation of the horizontal asymptote of the graph. [2]

(f) Write down the range of f. [2]