INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 1.5]

PERCENTAGE CHANGE - FINANCIAL APPLICATIONS

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O. Practice questions

PERCENTAGE CHANGE

1. [Maximum mark: 8] ***[with GDC]***

(a) The population of village A **increases** by 8% every year. If the population today is 1000 people find

(i) the population after 5 years;

(ii) the population 5 years ago;

(iii) the number of full years after the population will exceed 2000. [4]

(b) The population of village B **decreases** by 8% every year. If the population today is 1000 people find

(i) the population after 5 years;

(ii) the population 5 years ago;

(iii) the number of full years after the population will fall under 500. [4]

FINANCIAL APPLICATIONS

2. [Maximum mark: 8] ***[with GDC]***

An amount of $ 10 000 is invested at an annual interest rate of 12%.

(a) Find the value of the investment after 5 years

(i) if the interest is compounded yearly;

(ii) if the interest is compounded half-yearly;

(iii) if the interest is compounded quarterly;

(iv) if the interest is compounded monthly. [4]

(b) The value of the investment will exceed $ 20 000 after n full years. Calculate the minimum value of n

(i) if the interest is compounded yearly;

(ii) if the interest is compounded monthly. [4]

METHOD A: By using the formula of FV

METHOD B: By using GDC - Financial mode

A. Exam style questions (SHORT)

3. [Maximum mark: 7] ***[with GDC]***

The population of a city at the end of 1972 was 250 000. The population increases by 1.3% per year.

(a) Write down the population at the end of 1973. [1]

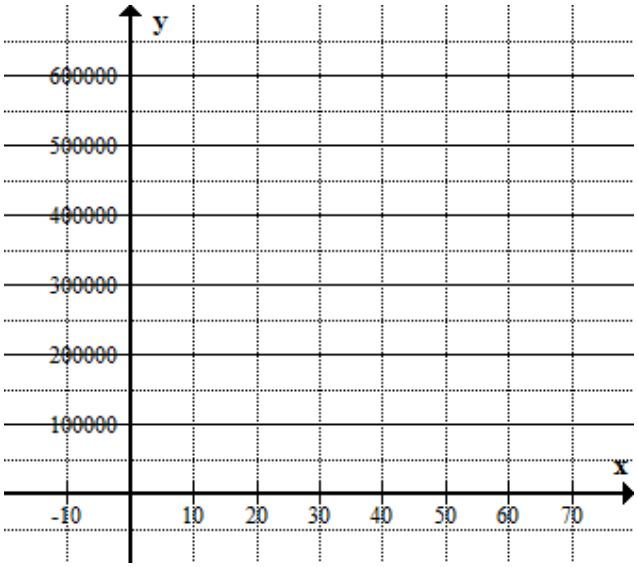
(b) Find the population at the end of 2002. [3]

(c) How long will it take for the size of the population to exceed 400 000? [3]

Extra question

Sketch a graph of Population (P) in terms of years (t) after 1972

Show the information found in questions (b) and (c)



4. [Maximum mark: 4] ***[with GDC]***

A population of bacteria is growing at the rate of 2.3% per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

5. [Maximum mark: 4] ***[with GDC]***

Each year for the past five years the population of a certain country has increased at a steady rate of 2.7% per annum. The present population is 15.2 million.

(a) What was the population one year ago? [2]

(b) What was the population five years ago? [2]

6. [Maximum mark: 7] ***[with GDC]***

An initial amount of \$ 10 000 and an extra \$ 10 000 **at the end of each year** are deposited into a savings account at an annual interest rate of 5%.

(a) Find the value of the investment

(i) at the end of the 1st year

(ii) at the end of the 2nd year

(iii) at the end of the 10th year [5]

(b) Find the least number of full years for the value of investment to exceed 200 000 [2]

METHOD A: By using the formula of FV

METHOD B: By using GDC - Financial mode

Extra question (mainly for HL)

Explain why the value of the investment at the end of the n^{th} year is given by

$$FV = 10000 \times \left(\frac{1.05^{n+1} - 1}{0.05} \right)$$

[if last payment is not included we subtract 10000]

7. [Maximum mark: 7] ***[with GDC]***

An amount of \$10 000 is deposited into a savings account at an annual interest rate of 5%. An extra amount of \$ 1 000 is deposited **at the end of each year**.

(a) Find the value of the investment

(i) at the end of the 1st year

(ii) at the end of the 2nd year

(iii) at the end of the 10th year [5]

(b) Find the least number of full years for the value of investment to exceed 50 000 [2]

METHOD A: By using the formula of FV

METHOD B: By using GDC - Financial mode

Extra question (mainly for HL)

Explain why the value of the investment at the end of the n^{th} year is given by

$$FV = 10000 \times 1.05^{n} + 1000 \times \left(\frac{1.05^{n} - 1}{0.05} \right)$$

[if last payment is not included we subtract 1000]

8. [Maximum mark: 6] ***[with GDC]***

An amount of $ 10 000 is deposited into a savings account at an annual interest rate of 5% compounded **monthly**. An extra amount of $1000 is deposited **at the end of each year.**

(a) Find the value of the investment

(i) at the end of the 1st year

(ii) at the end of the 10th year [4]

(b) Find the least number of full years for the value of investment to exceed 50 000 [2]

9. [Maximum mark: 6] ***[with GDC]***

An amount of $ 10 000 is deposited into a savings account at an annual interest rate of 5% compounded **monthly**. An extra amount of $ 100 is deposited **at the end of each month**.

(a) Find the value of the investment

(i) at the end of the 1^{st} year

(ii) at the end of the 10^{th} year [4]

(b) Find the least number of full **months** for the value of investment to exceed 50 000 [2]

10*. [Maximum mark: 9] ***[with GDC]***

An amount of $ 10 000 is deposited into a savings account at an annual interest rate of 5% (compounded yearly).

(a) Find the value of the **interest** after 1^{st} year. [1]

If an amount of $ 500 is **withdrawn** at the end of each year;

(b) Find the amount remained in the account after 10 years. [2]

If an amount of $ 400 is **withdrawn** at the end of each year;

(c) Find the amount remained in the account after 10 years. [2]

If an amount of $ 1 000 is **withdrawn** at the end of each year;

(d) Find the amount remained in the account after 10 years. [2]

(e) The last withdrawal L will be made after n years. Find the values of n and of L. [2]

11. [Maximum mark: 7] ***[with GDC]***

Ann gets a loan of € 20000 for 10 years at a nominal annual interest rate of 15 % compounded **yearly**.

(a) Find the repayment made each **year** (correct to 2 decimal places) [3]

(b) Find the total amount paid. [2]

(c) Find the interest paid on the loan. [2]

12. [Maximum mark: 7] ***[with GDC]***

Bill gets a loan of € 20000 for 10 years at a nominal annual interest rate of 15 % compounded **monthly**.

(a) Find the repayment made each **month**. (correct to 2 decimal places) [3]

(b) Find the total amount paid. [2]

(c) Find the interest paid on the loan. [2]

13. [Maximum mark: 7] ***[with GDC]***

Chris gets a loan of € 20000 for 10 years at a nominal annual interest rate of 15 % compounded **quarterly**.

(a) Find the repayment made each **quarter** (correct to 2 decimal places) [3]

(b) Find the total amount paid. [2]

(c) Find the interest paid on the loan. [2]

14. [Maximum mark: 7] ***[with GDC]***

Dimitris gets a loan of € 20000 for 10 years at a nominal annual interest rate of r % compounded **monthly**. Terms of the loan require monthly repayments of € 300.

(a) Find the value of r. [3]

(b) Find the total amount paid. [2]

(c) Find the interest paid on the loan. [2]

15. [Maximum mark: 5] ***[with GDC]***

A sum of $ 5 000 is invested at a compound interest rate of 6.3 % per annum. The value of the investment will exceed $ 10 000 after n full years. Calculate the minimum value of n.

16*. [Maximum mark: 6] ***[with GDC]***

$1000 is invested at the beginning of each year for 10 years.

The rate of interest is fixed at 7.5% per annum. Interest is compounded annually.

Calculate, giving your answers to the nearest dollar

(a) how much the first $1000 is worth at the end of the ten years; [2]

(b) the total value of the investments at the end of the ten years. [4]

17*. [Maximum mark: 5] ***[with GDC]***

$1000 is invested at 15% per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed $3000.

18*. [Maximum mark: 8] ***[with GDC]***

Michele invested 1500 francs at an annual rate of interest of 5.25 percent, compounded annually.

(a) Find the value of Michele's investment after 3 years. Give your answer to the nearest franc. [2]

(b) How many complete years will it take for Michele's initial investment to double in value? [3]

(c) What should the interest rate be if Michele's initial investment were to double in value in 10 years? [3]

B. Exam style questions (LONG)

19. [Maximum mark: 18] ***[with GDC]***

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % compounded quarterly.

No deposit required and repayments are made each quarter.

(a) (i) Find the repayment made each quarter.

(ii) Find the total amount paid for the car.

(iii) Find the interest paid on the loan. [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % compounded monthly. Terms of the loan require a 10 % deposit and monthly repayments of €250.

(b) (i) Find the amount to be borrowed for this option.

(ii) Find the annual interest rate, r . [5]

(c) State which option Bryan should choose. Justify your answer. [2]

Bryan chooses option B. The car dealership invests the money Bryan pays as soon as they receive it.

(d) If they invest it in an account paying 0.4% interest per month and inflation is 0.1% per month, calculate the real amount of money the car dealership has received by the end of the 6 year period. [4]

20*. [Maximum mark: 14] ***[with GDC]***

Themis sold his car at the beginning of 2020 and received an amount of € 7000.

(a) Find the **real** value of Themis' amount at the beginning of 2021,

(i) given that the inflation rate is 1.1 % per year.

(ii) given that the inflation rate is 0.1 % per month. [4]

(b) Find the **real** value of Themis' amount at the beginning of 2025,

(i) given that the inflation rate is 1.1 % per year.

(ii) given that the inflation rate is 0.1 % per month. [4]

Themis decides to invest his money at 12% per year, **compounded monthly.**

(c) Find the amount he will receive back at the beginning of 2025. [2]

(d) Find the **real** value of the money he will receive at the beginning of 2025 if the inflation rate is 1.1 % per year. [2]

(e) Find the **real** value of the money he will receive at the beginning of 2025 if the inflation rate is 0.1 % per month. [2]