INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

Math AA

EXERCISES [Math-AA 1.4]

GEOMETRIC SEQUENCES

Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 8] ***[with GDC]***

Consider the geometric sequence $10, 20, 40, 80, \ldots$

(a) Write down the common ratio r. [1]

(b) Find the 10th term of the sequence. [2]

(c) Find the sum of the first 10 terms. [2]

(d) Express the general term u_n in terms of n. [1]

(e) **Hence** find the value of n given that $u_n = 20480$. [2]

2. [Maximum mark: 11] ***[with GDC]***

Consider the geometric sequence $10, 5, 2.5, 1.25, \ldots$

(a) Write down the common ratio r. [1]

(b) Find the 10th term of the sequence. [2]

(c) Find the sum of the first 10 terms, correct **to 4sf**. [2]

(d) Express the general term u_n in terms of n. [1]

(e) **Hence** find the value of n given that $u_n = 0.3125$ [2]

(f) Explain why the sum of the infinite series exists (i.e. the series converges) and find its value. [3]

3. [Maximum mark: 10] ***[with GDC]***

The first term of a geometric sequence is 5 while the fourth term is 40.

(a) Find the common ratio r. [2]

(b) Find the fifth term of the sequence. [2]

(c) Find the sum of the first 10 terms. [2]

(d) Find the smallest value of n given that the n-th term exceeds 1000. [2]

(e) Find the first term that exceeds 1000. [1]

(f) Explain why the sum of the infinite series does not exist (i.e. the series diverges). [1]

4. [Maximum mark: 6] ***[without GDC]***

Find the sum of each infinite geometric series below.

(i) $1+\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\cdots$ (ii) $1-\frac{2}{5}+\frac{4}{25}-\frac{8}{125}+\cdots$

5. [Maximum mark: 10] ***[without GDC]***

Let k, $2k$, $k+60$ be consecutive terms of a sequence.

(a) (i) Find the value of k if the sequence is arithmetic.

(ii) Write down the three terms of the sequence and state the common difference. [5]

(b) (i) Find the value of k if the sequence is geometric.

(ii) Write down the three terms of the sequence and state the common ratio. [5]

6*. [Maximum mark: 8] ***[with GDC]***

Consider the geometric sequence

$$5, 15, 45, 135, \cdots$$

(a) Find the number of terms which are less than 100000. [3]

(b) Find the greatest term which is less than 100000. [2]

(c) Find the greatest value of n such that $S_n < 100000$. [3]

7*. [Maximum mark: 8] ***[with GDC]***

Consider the geometric sequence

$$1000,\ 500,\ 250,\ ...$$

(a) Find the number of terms which are **greater than** 1. [3]

(b) Find the first term which is **less than** 1. [2]

(c) Find the sum of the terms which are **greater than** 1, to the nearest integer. [2]

8*. [Maximum mark:14] ***[with GDC]***

A geometric sequence has first term u_1 and common ratio r. Find the values of u_1 and of r in each of the following cases:

(a) if $u_7 = 3645$ and $u_{10} = 98415$. [3]

(b) if $u_7 = 98415$ and $u_{10} = 3645$ [3]

(c) if $S_2 = 20$ and $S_4 = 200$. [5]

(d) if $S_3 = 35$ and $S_\infty = 40$. [3]

9. [Maximum mark: 6] ***[without GDC]***

Find the sums (i) $\sum_{k=1}^{10} 2^k$ (ii) $\sum_{k=0}^{10} 2^k$ [It is given that $2^{10} = 1024$]

10. [Maximum mark: 6] ***[without GDC]***

Find the sums (i) $\sum_{k=1}^{+\infty}(0.5)^k$ (ii) $\sum_{k=0}^{+\infty}(0.5)^k$

11. [Maximum mark: 6] ***[without GDC]***

Find the sums (i) $\sum_{k=1}^{+\infty}3\left(\frac{2}{5}\right)^k$ (ii) $\sum_{k=0}^{+\infty}3\left(\frac{2}{5}\right)^k$

12.** [Maximum mark: 6] ***[without GDC]***

Find the sum $\sum_{k=1}^{+\infty}\left(\frac{2^k+3^k}{5^k}\right)$

13.** [Maximum mark: 6] ***[without GDC]***

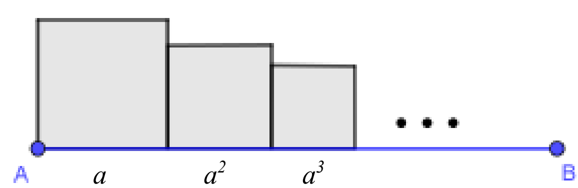
Find the sum $\sum_{k=1}^{+\infty}\left(\frac{3^k+1}{5^k}\right)$

14.** [Maximum mark: 7] ***[without GDC]***

Find the sum $\sum_{k=1}^{8}\left(3^k+3k+5\right)$ [It is given that $3^8=6561$]

15. [Maximum mark: 8] ***[without GDC]***

An **infinite** number of squares are placed next to each other on a line segment [AB], as in the following diagram. The sides of the squares are $a, a^2, a^3, \ldots$ where $a<1$.



(a) Find in terms of a

(i) the **total length** AB (ii) the **total area** of the squares. [4]

It is given that the total length AB is 4

(b) Find

(i) the value of a (ii) the **total area** of the squares. [4]

16.** [Maximum mark: 8] ***[without GDC]***

The **1st, 7th** and **25th** terms of an arithmetic sequence are also consecutive terms of a geometric sequence (in that order). The terms of the geometric sequence are all different.

(a) Show that $u_1=3d$. [4]

(b) Find the common ratio of the geometric sequence. [2]

A. Exam style questions (SHORT)

17. [Maximum mark: 5] ***[without GDC]***

Consider the infinite geometric sequence $3,\ 3(0.9),\ 3(0.9)^2,\ 3(0.9)^3,\ \ldots$

(a) Write down the 10th term of the sequence. Do not simplify your answer. [1]

(b) Find the sum of the infinite series. [4]

18. [Maximum mark: 6] ***[with / without GDC]***

Consider the infinite geometric sequence 25, 5, 1, 0.2, …

(a) Find the common ratio. [1]

(b) Find (i) the 10th term; (ii) an expression for the n^{th} term. [3]

(c) Find the sum of the infinite series. [2]

19. [Maximum mark: 6] ***[with GDC]***

Consider the infinite geometric series $405 + 270 + 180 + \ldots$

(a) For this series, find the common ratio, giving your answer as a fraction in its simplest form. [2]

(b) Find the fifteenth term of this series. [2]

(c) Find the **exact** value of the sum of the infinite series. [2]

20. [Maximum mark: 6] ***[with GDC]***

The first four terms of a sequence are 18, 54, 162, 486.

(a) Use all four terms to show that this is a geometric sequence. [2]

(b) (i) Find an expression for the n^{th} term of this geometric sequence.

(ii) If the n^{th} term of the sequence is 1062 882, find the value of n. [4]

21. [Maximum mark: 6] ***[with GDC]***

Consider the geometric sequence $8,\ a,\ 2, \ldots$ for which the common ratio is $\frac{1}{2}$.

(a) Find the value of a. [1]

(b) Find the value of the eighth term. [2]

(c) Find the sum of the first twelve terms. [3]

22. [Maximum mark: 6] ***[with GDC]***

Consider the geometric sequence $16,\ 8,\ a,\ 2,\ b,\ \ldots$

(a) Write down the common ratio. [1]

(b) Write down the value of (i) a; (ii) b. [2]

(c) The sum of the first n terms is 31.9375. Find the value of n. [3]

23. [Maximum mark: 6] ***[with GDC]***

Consider the infinite geometric sequence

3000, – 1800, 1080, – 648, …

(a) Find the common ratio. [2]

(b) Find the 10th term. [2]

(c) Find the **exact** sum of the infinite series. [2]

24. [Maximum mark: 5] ***[without GDC]***

The first three terms of an infinite geometric sequence are 32, 16 and 8.

(a) Write down the value of r. [1]

(b) Find u_6. [2]

(c) Find the sum to infinity of this sequence. [2]

25. [Maximum mark: 3] ***[without GDC]***

Find the sum of the infinite geometric sequence 27, −9, 3, −1, ...

26. [Maximum mark: 4] ***[without GDC]***

Find the sum of the infinite geometric series $\frac{2}{3}-\frac{4}{9}+\frac{8}{27}-\frac{16}{81}+...$

27. [Maximum mark: 4] ***[without GDC]***

Find the sum to infinity of the geometric series $-12+8-\frac{16}{3}+....$

28. [Maximum mark: 4] ***[without GDC]***

The n^{th} term, u_n, of a geometric sequence is given by $u_n = 3(4)^{n+1}$, $n \in \mathbb{Z}^+$.

(a) Find the first two terms and the common ratio r. [2]

(b) Hence, or otherwise, find S_n, the sum of the first n terms of this sequence. [2]

29*. [Maximum mark: 6] ***[with GDC]***

A geometric sequence has a first term of 2 and a common ratio of 1.05.

Find the value of the smallest term that is greater than 500.

30. [Maximum mark: 5] ***[with GDC]***

Let $u_n = 3\times 2^n$.

(a) Express $\sum_{n=1}^{3} u_n$ as a sum of three terms and find the result. [3]

(b) Find $\sum_{n=1}^{12} u_n$. [2]

31. [Maximum mark: 4] ***[with GDC]***

The tuition fees for the first three years of high school are given in the table below.

Year	Tuition fees (in dollars)
1	2000
2	2500
3	3125

These tuition fees form a geometric sequence.

(a) Find the common ratio, r, for this sequence. [2]

(b) If fees continue to rise at the same rate, calculate (to the nearest dollar) the total cost of tuition fees for the first six years of high school. [2]

32. [Maximum mark: 6] ***[with GDC]***

The annual fees paid to a school for the school years 2000, 2001 and 2002 increase as a geometric progression. The table below shows the fee structure.

Year	**Fees (USD)**
2000	8000.00
2001	8320.00
2002	8652.80

(a) Calculate the common ratio for the increasing sequence of fees. [2]

In parts (b) and (c) give your answer correct to 2 decimal places.

The fees continue to increase in the same ratio.

(b) Find the fees paid for 2006. [2]

A student attends the school for eight years, starting in 2000.

(c) Find the **total** fees paid for these eight years. [2]

33. [Maximum mark: 10] ***[with GDC]***

Ann and John go to a swimming pool.

They both swim the first length of the pool in 2 minutes.

The time John takes to swim a length is 6 seconds more than he took to swim the previous length.

The time Ann takes to swim a length is 1.05 times that she took to swim the previous length.

(a) (i) Find the time John takes to swim the third length.

(ii) Show that Ann takes 2.205 minutes to swim the third length. [3]

(b) Find the time taken for Ann to swim a total of 10 lengths of the pool. [3]

(c) Find the time taken for John to swim a total of 10 lengths of the pool. [4]

34. [Maximum mark: 5] ***[with GDC]***

The population of Bangor is growing each year. At the end of 1996, the population was 40 000. At the end of 1998, the population was 44 100. Assuming that these annual figures follow a geometric progression, calculate

(a) the population of Bangor at the end of 1997; [2]

(b) the population of Bangor at the end of 1992. [3]

35. [Maximum mark: 6] ***[with GDC]***

A geometric sequence has all its terms positive. The 1st term is 7 and the 3rd term is 28.

(a) Find the common ratio. [3]

(b) Find the sum of the first 14 terms. [3]

36. [Maximum mark: 8] ***[without GDC]***

The first term of an infinite geometric sequence is 18, while the third term is 8. There are two possible sequences.

(a) Find the common ratio of each sequence. [3]

(b) Find the sum of each sequence. [3]

(c) Write down the first three terms of each sequence. [2]

37*. [Maximum mark: 5] ***[without GDC]***

The first and fourth terms of a geometric series are 18 and $-\frac{2}{3}$ respectively.

(a) Find the common ratio of the series; [3]

(b) Find the sum to infinity of the series. [2]

38. [Maximum mark: 6] ***[with GDC]***

The seventh term, u_7, of a geometric sequence is 108. The eighth term, u_8, of the sequence is 36.

(a) Write down the common ratio of the sequence. [1]

(b) Find u_1. [2]

(c) The sum of the first k terms in the sequence is $118\ 096$. Find the value of k. [3]

39. [Maximum mark: 9] ***[with GDC]***

A geometric sequence has second term 12 and fifth term 324.

(a) Calculate the value of the common ratio. [4]

(b) Calculate the 10th term of this sequence. [2]

(c) The k^{th} term is the first term that exceeds 2000. Find the value of k. [3]

40*. [Maximum mark: 8] ***[with GDC]***

In a geometric series, $u_1 = \dfrac{1}{81}$ and $u_4 = \dfrac{1}{3}$.

(a) Find the value of r. [3]

(b) Find the smallest value of n for which $S_n > 40$. [3]

(c) Confirm the result in (b) by finding two consecutive values of S_n. [2]

41*. [Maximum mark: 10] ***[with GDC]***

The third term of a geometric sequence is –108 and the sixth term is 32.

(a) Find (i) The common ratio. (ii) The first term. [5]

(b) Find the sum of the first six terms. [3]

(c) The sum to infinity. [2]

42. [Maximum mark: 6] ***[with GDC]***

Let $5,\ x, 45,\ y$ be consecutive terms of a geometric sequence.

(a) Find the possible values of x. [3]

(b) **Hence** find the possible values of y. [3]

43*. [Maximum mark: 6] ***[with / without GDC]***

The three terms a, 1, b are in arithmetic progression. The three terms 1, a, b are in geometric progression. Find the value of a and of b given that $a \neq b$.

44. [Maximum mark: 4] ***[without GDC]***

Let $\sum_{x=0}^{\infty} k\left(\frac{2}{3}\right)^x = 1$. Find the value of k.

45*. [Maximum mark: 6] ***[without GDC]***

Consider the infinite geometric series $1+\left(\frac{2x}{3}\right)+\left(\frac{2x}{3}\right)^2+\left(\frac{2x}{3}\right)^3+.....$

(a) For what values of x does the series converge? [4]

(b) Find the sum of the series if $x = 1.2$. [2]

46*. [Maximum mark: 8] ***[with GDC]***

An infinite geometric series is given by $\sum_{k=1}^{\infty} 2(4-3x)^k$

(a) Write down the common ratio of the series in terms of x. [1]

(b) Find the values of x for which the series has a finite sum. [3]

(c) When $x = 1.2$, find

(i) the sum of the series.

(ii) the minimum number of terms needed to give a sum which exceeds 1.328 [4]

47*. [Maximum mark: 6] ***[without GDC]***

A geometric sequence has all positive terms. The sum of the first two terms is 15 and the sum to infinity is 27. Find the value of

(a) the common ratio; [4]

(b) the first term. [2]

48*. [Maximum mark: 6] ***[without GDC]***

The sum of an infinite geometric sequence is $13\frac{1}{2}$, and the sum of the first three terms is 13. Find the first term.

49*. [Maximum mark: 6] ***[with GDC]***

The sum to infinity of a geometric series is 32. The sum of the first four terms is 30 and all the terms are positive. Find the difference between the sum to infinity and the sum of the first eight terms.

50*. [Maximum mark: 8] ***[without GDC]***

(a) The first term of an arithmetic sequence is -16 and the eleventh term is 39. Calculate the value of the common difference. [3]

(b) The third term of a geometric sequence is 12 and the fifth term is $\frac{16}{3}$. All the terms in the sequence are positive. Calculate the value of the common ratio. [5]

51.** [Maximum mark: 8] ***[with GDC]***

The first term of an arithmetic sequence is 0 and the common difference is 12.

(a) Find the value of the 96^{th} term of the sequence. [2]

The first term of a geometric sequence is 6. The 6^{th} term of the geometric sequence is equal to the 17^{th} term of the arithmetic sequence given above.

(b) Write down an equation using this information and **hence** calculate the common ratio of the geometric sequence. [3]

The n-th term of the arithmetic sequence is equal to the n-th term of the geometric sequence above.

(c) Find the possible value of n. [3]

52.** [Maximum mark: 6] ***[with GDC]***

The sum of the first n terms of an arithmetic sequence $\{u_n\}$ is given by the formula $S_n = 4n^2 - 2n$. Three terms of this sequence, u_2, u_m and u_{32}, are consecutive terms in a geometric sequence. Find m.

53.** [Maximum mark: 6] ***[without GDC]***

An arithmetic sequence has first term a and common difference d, $d \neq 0$.

The 3rd, 4th and 7th terms of the arithmetic sequence are the first three terms of a geometric sequence.

(a) Show that $a = -\frac{3}{2}d$. [4]

(b) Find the common ratio of the geometric sequence. [2]

54.** [Maximum mark: 8] ***[without GDC]***

The first three terms of a geometric sequence are also the first, eleventh and sixteenth term of an arithmetic sequence. The terms of the geometric sequence are all different. The sum to infinity of the geometric sequence is 18.

(a) Find the common ratio of the geometric sequence, clearly showing all working. [5]

(b) Find the common difference of the arithmetic sequence. [3]

B. Exam style questions (LONG)

55. [Maximum mark: 8] ***[with GDC]***

The first term of a geometric sequence is 500 while the fourth term is 62.5. Find

(a) Find the common ratio r. [2]

(b) Find the fifth term. [1]

(c) Find the sum of the first **ten** terms correct to 3 decimal places.

(d) Find the smallest value of n given that the n-th term is less than 10. [2]

(e) Find the first term which is less than 10. [1]

56. [Maximum mark: 11] ***[with GDC]***

Portable telephones are first sold in the country *Cellmania* in 1990. During 1990, the number of units sold is 160. In 1991, the number of units sold is 240 and in 1992, the number of units sold is 360.

In 1993 it was noticed that the annual sales formed a geometric sequence with first term 160, the 2nd and 3rd terms being 240 and 360 respectively.

(a) What is the common ratio of this sequence? [1]

Assume that this trend in sales continues.

(b) How many units will be sold during 2002? [3]

(c) In what year does the number of units sold first exceed 5000? [4]

Between 1990 and 1992, the total number of units sold is 760.

(d) What is the total number of units sold between 1990 and 2002? [2]

During this period, the total population of *Cellmania* remains approximately 80 000.

(e) Use this information to suggest a reason why the geometric growth in sales would not continue. [1]

57*. [Maximum mark: 12] ***[with GDC]***

(a) Consider the geometric sequence $-3, 6, -12, 24, \ldots$

(i) Write down the common ratio.

(ii) Find the 15th term. [3]

Consider the sequence $x-3$, $x+1$, $2x+8$, ….

(b) When $x=5$, the sequence is geometric.

(i) Write down the first three terms.

(ii) Find the common ratio. [2]

(c) Find the other value of x for which the sequence is geometric. [4]

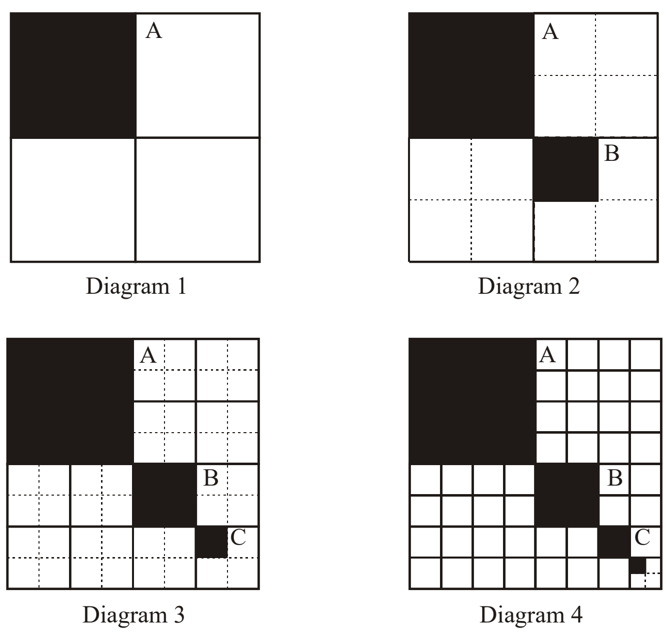
(d) For this value of x, find

(i) the common ratio;

(ii) the sum of the infinite sequence. [3]

58. [Maximum mark: 11] ***[with GDC]***

The diagrams below show the first four squares in a sequence of squares which are subdivided in half. The area of the shaded square A is $\frac{1}{4}$.



(a) (i) Find the area of square B and of square C.

(ii) Show that the areas of squares A, B and C are in geometric progression.

(iii) Write down the common ratio of the progression. [5]

(b) (i) Find the **total** area shaded in diagram 2.

(ii) Find the **total** area shaded in the 8th diagram (correct to 6 sf.) [4]

(c) The dividing and shading process illustrated is continued indefinitely.

Find the total area shaded. [2]

59*. [Maximum mark: 10] ***[with GDC]***

Diagram 1 shows a square ABCD of side 4 cm. The midpoints P, Q, R, S of the sides are joined to form a **second** square.

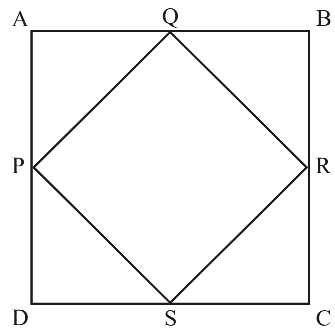


Diagram 1

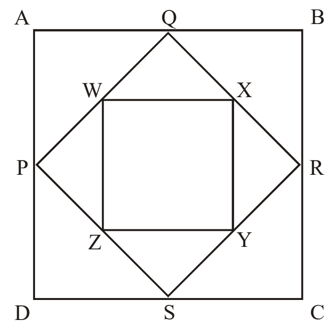


Diagram 2

(a) (i) Show that PQ = $2\sqrt{2}$ cm.

(ii) Find the area of PQRS. [3]

The midpoints W, X, Y, Z of the sides of PQRS are now joined to form a **third** square as shown in **Diagram 2.**

(b) (i) Write down the area of the **third** square, WXYZ.

(ii) Show that the areas of ABCD, PQRS, and WXYZ form a geometric sequence. Find the common ratio of this sequence. [3]

The process of forming smaller and smaller squares (by joining the midpoints) is **continued indefinitely.**

(c) (i) Find the area of the 11^{th} square.

(ii) Calculate the sum of the areas of **all** the squares. [4]

60*. [Maximum mark: 12] ***[with GDC]***

A geometric sequence has 1024 as its first term and 128 as its fourth term.

(a) Show that the common ratio is $\frac{1}{2}$. [2]

(b) Find the value of the eleventh term. [2]

(c) Find the sum of the first eight terms. [2]

(d) Find the number of terms in the sequence for which the **sum** first exceeds 2047.968. [4]

(e) Confirm the result in (d) by finding the appropriate sums. [2]

61. [Maximum mark: 12] ***[with GDC]***

A National Lottery is offering prizes in a new competition. The winner may choose one of the following.

Option one: \$1000 each week for 10 weeks.

Option two: \$250 in the first week, \$450 in the second week, \$650 in the third week, increasing by \$200 each week for a total of 10 weeks.

Option three: \$10 in the first week, \$20 in the second week, \$40 in the third week continuing to double for a total of 10 weeks.

(a) Calculate the amount you receive in the tenth week, if you select

(i) **option two**;

(ii) **option three**. [6]

(b) What is the total amount you receive if you select **option two**? [2]

(c) Which option has the greatest total value? Justify your answer by showing all appropriate calculations. [4]

62.** [Maximum mark: 14] ***[with GDC]***

A geometric progression G_1 has 1 as its first term and 3 as its common ratio.

(a) The sum of the first n terms of G_1 is $29\ 524$. Find n. [3]

A second geometric progression G_2 has the form $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \dots$

(b) State the common ratio for G_2. [1]

(c) Calculate the sum of the first 10 terms of G_2. [2]

(d) Explain why the sum of the first 1000 terms of G_2 will give the same answer as the sum of the first 10 terms, when corrected to three significant figures. [1]

(e) Using your results from parts (a) to (c), or otherwise, calculate the sum of the first 10 terms of the sequence $2, 3\frac{1}{3}, 9\frac{1}{9}, 27\frac{1}{27} \dots$

Give your answer **correct to one decimal place**.'

[remember that $3\frac{1}{3}$ is in fact $3+\frac{1}{3}$ [3]

(f) Find the sum of the **infinite** series $2, \left(\frac{1}{2}+\frac{1}{3}\right), \left(\frac{1}{4}+\frac{1}{9}\right), \left(\frac{1}{8}+\frac{1}{27}\right), \dots$ [4]

63*. [Maximum mark: 13] ***[with GDC]***

An arithmetic sequence is defined as $u_n = 135 + 7n, \quad n = 1, 2, 3, \ldots$

(a) Calculate u_1, the first term in the sequence. [2]

(b) Show that the common difference is 7. [2]

S_n is the sum of the first n terms of the sequence.

(c) Find an expression for S_n. Give your answer in the form $S_n = An^2 + Bn$, where A and B are constants. [3]

The first term, v_1, of a geometric sequence is 20 and its fourth term v_4 is 67.5.

(d) Show that the common ratio, r, of the geometric sequence is 1.5. [2]

T_n is the sum of the first n terms of the geometric sequence.

(e) Calculate T_7, the sum of the first seven terms of the geometric sequence. [2]

(f) Use your GSC to find the smallest value of n for which $T_n > S_n$. [2]

64*. [Maximum mark: 14] ***[without GDC]***

(a) The **sum** of 3 consecutive terms of an **arithmetic sequence** is 12.

(i) Show that one of them is 4.

(ii) Given that their product is 28, find the remaining two terms. [6]

(b) The **product** of 3 consecutive terms of a **geometric sequence** is 64.

(i) Show that one of them is 4.

(ii) Given that their sum is 14, find the remaining two terms. [8]

65.** [Maximum mark: 12] ***[without GDC]***

A set of n numbers form a geometric sequence with first term $\frac{1}{4}$ and common ratio 2.

The sum of the numbers is $\frac{63}{4}$.

(a) Find n.

(b) Show that the product of the numbers is equal to the largest term. [6]

A set of m numbers form a geometric sequence with first term 9 and common ratio $\frac{1}{3}$.

The product of the terms is equal to the largest term.

(c) Find the sum of the numbers. [4]

A set of k numbers form a geometric sequence with first term a^{10} and common ratio $\frac{1}{a}$

(d) Given that the product of the terms is 1, deduce the value of k. [2]