# Quiz 1-C [15 marks]

**1.** [Maximum mark: 15] 24M.2.AHL.TZ2.10

A shop sells chocolates. The weight, in kilograms, of chocolates bought by a random customer can be modelled by a continuous random variable $X$ with probability density function $f$ defined by

$$f(x) = \begin{cases} \frac{6}{85}\left(4 + 3x - x^2\right), & 0.5 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}.$$

(a) Find the mode of $X$. [2]

Markscheme

recognizes that the mode is a value of $x$ at which $f$ has a maximum value ***(M1)***

a clearly labelled graph of $f$ OR states $f\prime(x) = 0$ OR considers the axis of symmetry

mode is $1.5$ (kg) ***A1***

**Note:** Award ***M1A0*** for $(1.5,\ 0.441)$ or $0.441$ stated as the final answer.

***[2 marks]***

Examiners report

Parts (a) and (b): These were well answered by most candidates. However, identifying the mode as the $x$-value with the highest probability was often incorrect. While most candidates used the graph, some used differentiation to obtain the mode.

Part (c): Most candidates wrote the correct equation. Solving it using their graphing calculators (GDC) was the most successful method to obtain the median, whereas solving by hand often resulted in errors.

Parts (d) and (e): These parts were very poorly answered or not attempted at all.

Part (d): Incorrect bounds were frequently seen, with zero often incorrectly used as the lower bound for the entire question or part of it.

Part (e): This part was very challenging, and most candidates only earned one mark for writing some expectation integral. Very few understood how to approach it, making part (e) of Question 10 the most difficult on the whole paper.

(b) Find $\mathrm{P}(1 \le X \le 2)$. [2]

Markscheme

attempts to find $\int_1^2 f(x)\ \mathrm{d}\ x$ *(M1)*

$= 0.435294\ldots$

$= 0.435\ \left(= \frac{37}{85}\right)$ ***A1***

***[2 marks]***

(c) Find the median of $X$. [3]

Markscheme

**METHOD 1**

recognizes that $\int_{0.5}^{m} f(x)\ \mathrm{d}\ x = 0.5$ *(M1)*

$m = 1.68701\ldots$

$m = 1.69$ (kg) ***A2***

**METHOD 2**

recognizes that $\int_{0.5}^{m} f(x)\,\mathrm{d}\,x = 0.5$ ***(M1)***

$\frac{6}{85}\left(4m + \frac{3}{2}m^2 - \frac{1}{3}m^3\right) - \frac{6}{85}\left(2 + \frac{3}{8} - \frac{1}{24}\right) = 0.5$ ***A1***

$m = 1.68701\ldots$

$m = 1.69$ (kg) ***A1***

***[3 marks]***

The shop sells chocolates to customers at $\$25$ per kilogram.

However, if the weight of chocolate bought by a customer is at least $0.75$ kilograms, the shop sells chocolate at a discounted rate of $\$24$ per kilogram.

(d) Find the probability that a randomly selected customer spends at most $\$48$. [3]

Markscheme

$0.5 \leq x \leq 2$ (can be seen in a definite integral) ***(A1)***

attempts to evaluate their definite integral ***(M1)***

$\int_{0.5}^{2} f(x)\,\mathrm{d}\,x = 0.635294\ldots$

$= 0.635$ ***A1***

***[3 marks]***

(e) Find the expected amount spent per customer. Give your answer correct to the nearest cent. [5]

Markscheme

an attempt at forming an expected value integral $\int_{x_1}^{x_2} xf(x)\ \mathrm{d}\ x$ ***(M1)***

$\int_{0.5}^{0.75} xf(x)\ \mathrm{d}\ x\ (= 0.060592\ldots)$ OR

$\int_{0.5}^{0.75} 25xf(x)\ \mathrm{d}\ x\ (= 1.51482\ldots)$ ***(A1)***

$\int_{0.5}^{3} xf(x)\ \mathrm{d}\ x\ (= 1.64345\ldots)$ OR

$\int_{0.5}^{3} 24xf(x)\ \mathrm{d}\ x\ (= 39.4428\ldots)$ ***(A1)***

sums their two definite integrals ***(M1)***

(expected amount spent per customer is)

$$= \int_{0.5}^{0.75} 25xf(x)\ \mathrm{d}\ x + \int_{0.5}^{3} 24xf(x)\ \mathrm{d}\ x$$

$$= 40.9576\ldots$$

(expected amount spent per customer is) $\$40.96$ ***A1***

***[5 marks]***