INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 4.11]

NORMAL DISTRIBUTION

Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 7] ***[with GDC]***

The random variable $X \sim \mathrm{N}(100, 20^2)$, that is X is normally distributed with $\mu = 100$ and $\sigma = 20$.

(a) Find the probabilities

(i) $\mathrm{P}(X < 90)$

(ii) $\mathrm{P}(90 < X < 130)$

(iii) $\mathrm{P}(X > 130)$. [3]

(b) Sketch a diagram to represent the information in question (a). [2]

2. [Maximum mark: 8] ***[with GDC]***

The random variable X follows a normal distribution with $\mu = 100$ and $\sigma = 20$.

(a) Given that $\mathrm{P}(X < a) = 0.8$ find the value of a. [2]

(b) Given that $\mathrm{P}(X > b) = 0.3$ find the value of b. [2]

(c) Given that $\mathrm{P}(c < X < d) = 0.4$, where c and d are symmetric about the mean, find the value of c and of d. [2]

(d) Find Q_1 and Q_3. [2]

3. [Maximum mark: 4] ***[with / without GDC]***

The random variable X is normally distributed with $\mu = 100$ and $\sigma = 20$.

(a) Write down the standardised value of 100. [1]

(b) Find the standardised values of 90 and 130. [3]

4. [Maximum mark: 6] ***[without GDC]***

The r.v. X is normally distributed with $\mu = 100$. It is given that $\mathrm{P}(X > 130) = 0.2$.

(a) Sketch a diagram to represent the information. [2]

(b) Write down the values of the following probabilities

(i) $\mathrm{P}(X < 130)$ (ii) $\mathrm{P}(X < 70)$

(iii) $\mathrm{P}(100 < X < 130)$ (iv) $\mathrm{P}(70 < X < 130)$ [4]

5. [Maximum mark: 4] ***[without GDC]***

A random variable X is distributed normally with a mean of 100 and a variance of 100.

(a) Find the value of X that is 1.12 standard deviations **above** the mean. [2]

(b) Find the value of X that is 1.12 standard deviations **below** the mean. [2]

6. [Maximum mark: 4] ***[with GDC]***

The random variable X follows a normal distribution with mean μ and standard deviation $\sigma = 20$. Given that $\mathrm{P}(X < 130) = 0.8$ find the value of μ.

7. [Maximum mark: 4] ***[with GDC]***

The random variable X follows a normal distribution with mean $\mu = 100$ and standard deviation σ. Given that $\mathrm{P}(X < 130) = 0.8$ find the value of σ.

8. [Maximum mark: 6] ***[with GDC]***

For the normally distributed random variable X, it is given that $\mathrm{P}(X < 60) = 0.2$ and $\mathrm{P}(X > 130) = 0.1$. Find the values of μ and of σ.

9. [Maximum mark: 4] ***[with GDC]***

The random variable X follows a normal distribution with $\mu = 100$ and $\sigma = 5$.

(a) Write down the value of $\mathrm{P}(X = 102)$. [1]

(b) Find the probability that $X = 102$ if the value of X is rounded to the nearest whole number. [3]

10*. [Maximum mark: 7] ***[with GDC]***

The random variable X follows a normal distribution with $\mu = 100$ and $\sigma = 5$.

(a) Find $\mathrm{P}(X > 102)$. [1]

(b) Find the conditional probabilities:

(i) $\mathrm{P}(X > 102 \mid X > 100)$ (ii) $\mathrm{P}(X < 102 \mid X > 100)$. [4]

11*. [Maximum mark: 8] ***[with GDC]***

The weight X of a particular animal is normally distributed with $\mu = 200\,\mathrm{kg}$, $\sigma = 15\,\mathrm{kg}$. An animal of this population is **overweight** if it has weight greater than $230\,\mathrm{kg}$

(a) Find the probability than an animal is overweight. [1]

(b) We select 2 animals of this population. Find the probabilities that

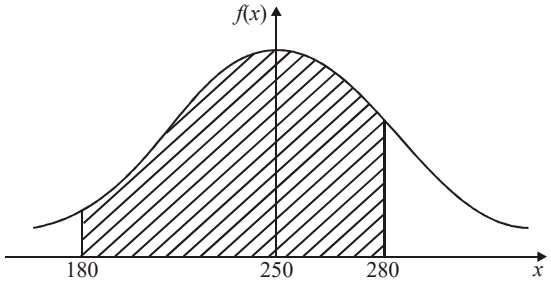
(i) both animals are overweight (ii) only one animal is overweight. [4]

(c) We select 7 animals of this population. Find the probability that **exactly** two of them are overweight. [3]

A. Exam style questions (SHORT)

12. [Maximum mark: 3] ***[with GDC]***

The following diagram shows the probability density function for the random variable *X*, which is normally distributed with mean 250 and standard deviation 50.



Find the probability represented by the shaded region.

13. [Maximum mark: 3] ***[with GDC]***

The random variable X is distributed normally with mean 30 and standard deviation 2. Find $\mathrm{P}(27 \le X \le 34)$.

14. [Maximum mark: 3] ***[with GDC]***

The diameters of discs produced by a machine are normally distributed with a mean of 10 cm and standard deviation of 0.1 cm. Find the probability of the machine producing a disc with a diameter smaller than 9.8 cm.

15. [Maximum mark: 3] ***[with GDC]***

The weights of a certain species of bird are normally distributed with mean 0.8 kg and standard deviation 0.12 kg. Find the probability that the weight of a randomly chosen bird of the species lies between 0.74 kg and 0.95 kg.

16. [Maximum mark: 9] ***[with GDC]***

The r.v. X is normally distributed with mean 1000 and standard deviation 50.

(a) 30% is more than a. Find a. [2]

(b) 57% is less than b. Find b. [2]

(c) 60% is between c and d, where c and d are symmetric about the mean. Find c and d. [2]

(d) Find the interquartile range. [3]

17. [Maximum mark: 4] ***[with GDC]***

The heights, H, of the people in a certain town are normally distributed with mean 170 cm and standard deviation 20 cm.

(a) A person is selected at random. Find the probability that his height is less than 185 cm. [2]

(b) Given that $\mathrm{P}(H > d) = 0.6808$, find the value of d. [2]

18. [Maximum mark: 4] ***[with GDC]***

The heights of a group of students are normally distributed with a mean of 160 cm and a standard deviation of 20 cm.

(a) A student is chosen at random. Find the probability that the student's height is greater than 180 cm. [2]

(b) In this group of students,11.9% have heights less than d cm. Find the value of d. [2]

19. [Maximum mark: 5] ***[with GDC]***

It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

(a) Find the probability that a lion selected at random will have a mass of 350 kg or more. [2]

(b) The probability that the mass of a lion lies between a and b is 0.95, where a, b are symmetric about the mean. Find the value of a and of b. [3]

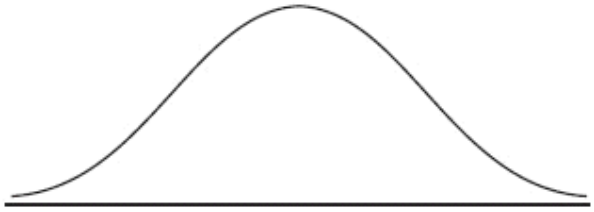
20. [Maximum mark: 5] ***[with GDC]***

The weights of a group of children are normally distributed with a mean of 22.5 kg and a standard deviation of 2.2 kg.

(a) Find the probability that a child selected at random weighs more than 25.8 kg. [2]

(b) Of the group 95% weigh less than k kilograms. Find the value of k. [2]

(c) The diagram below shows a normal curve. On the diagram, shade the region that represents the following information: 87% of the children weigh less than 25 kg. [1]



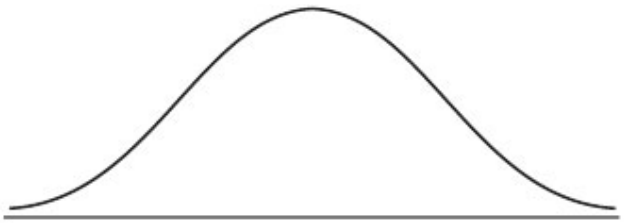
21. [Maximum mark: 4] ***[with GDC]***

The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92% are in category A. The tallest 10.38% are in category C.

All the other plants are in category B with heights between r cm and t cm.

(a) Complete the following diagram to represent this information.



[2]

(b) Given that the mean height is 6.84 cm and the standard deviation 0.25 cm, find the value of r and of t. [2]

22. [Maximum mark: 5] ***[with GDC]***

Intelligence Quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15.

(a) What percentage of the population has an IQ between 90 and 125? [2]

(b) If two persons are chosen at random from the population, what is the probability that both have an IQ greater than 125? [3]

23. [Maximum mark: 6] ***[with GDC]***

The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.

(a) Find the probability that a packet chosen at random has mass less than 740 g;. [2]

(b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g? [2]

(c) The mass of 70% of the packets is more than x grams. Find the value of x. [2]

24. [Maximum mark: 7] ***[with GDC]***

Residents of a small town have savings which are normally distributed with a mean of $ 3000 and a standard deviation of $500.

(a) What percentage of townspeople have savings greater than $ 3200? [2]

(b) Two townspeople are chosen at random. What is the probability that **both** of them have savings between $ 2300 and $ 3300? [3]

(c) The percentage of townspeople with savings less than d dollars is 74.22%. Find d. [2]

25. [Maximum mark: 7] ***[with GDC]***

A company manufactures television sets. They claim that the lifetime of a set is normally distributed with a mean of 80 months and standard deviation of 8 months.

(a) What proportion of television sets break down in less than 72 months? [2]

(b) (i) Find the proportion of sets with a lifetime between 72 and 90 months.

(ii) Illustrate this proportion by a sketch of a normal distribution curve. [3]

(c) If a set breaks down in less than x months, the company replace it free of charge. They replace 4% of the sets. Find the value of x. [2]

26*. [Maximum mark: 6] ***[with GDC]***

The lengths of a particular species of lizard are normally distributed with a mean length of 50 cm and a standard deviation of 4 cm, A lizard is chosen at random.

(a) Find the probability that its length is greater than 45 cm. [2]

(b) Given that its length is greater than 45 cm, find the probability that its length is greater than 55 cm. [4]

27*. [Maximum mark: 5] ***[with GDC]***

In a country called *Tallopia,* the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.

(a) What percentage of adults in *Tallopia* have a height greater than 197 cm? [2]

(b) A standard doorway in *Tallopia* is designed so that 99 % of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in *Tallopia*. Give your answer to the nearest cm. [3]

28*. [Maximum mark: 10] ***[with GDC]***

The weight of the population of a particular animal is normally distributed with mean 300kg and standard deviation 40kg. Animals above 350kg are considered **overweight**.

(a) We select 1000 animals. Find the **expected number** of the overweight animals. [2]

(b) We select **two** animals. Find the probability that

(i) both animals are overweight.

(ii) only one of the animals is overweight.

(iii) at least one of the animals is overweight. [4]

(c) We select **eight** animals. Find the probability that

(i) exactly half of the animals are overweight.

(ii) at least one of the animals is overweight. [4]

29*. [Maximum mark: 9] ***[without GDC]***

The heights of trees in a forest are normally distributed with mean height 17 metres. One tree is selected at random. The probability that a selected tree has a height greater than 24 metres is 0.06.

(a) Find the probability that the tree selected has a height less than 24 metres. [2]

(b) The probability that the tree has a height less than D metres is 0.06. Find D. [3]

(c) A woodcutter randomly selects 200 trees. Find the expected number of trees whose height lies between 17 metres and 24 metres. [4]

30*. [Maximum mark: 9] ***[with GDC]***

A certain type of vegetable has a weight which follows a normal distribution with mean 450 grams and a standard deviation 50 grams.

(a) In a load of 2000 of these vegetables, calculate the expected number with a weight greater than 525 grams. [3]

(b) Find the upper quartile of this distribution. [2]

(c) In a load of 40 of these vegetables, calculate

(i) the probability that none of them has a weight greater than 525 grams.

(ii) the probability that at most 3 of them have a weight greater than 525 grams. [4]

31. [Maximum mark: 7] ***[with GDC]***

The times taken for buses travelling between two towns are normally distributed with a mean of 35 minutes and standard deviation of 7 minutes.

(a) Find the probability that a randomly chosen bus completes the journey in less than 40 minutes. [2]

(b) 90 % of buses complete the journey in less than t minutes. Find the value of t. [2]

(c) A random sample of 10 buses has their travel time between the two towns recorded. Find the probability that exactly 6 of these buses complete the journey in less than 40 minutes. [3]

32. [Maximum mark: 9] ***[with GDC]***

The r.v. X is normally distributed with mean 1000 and standard deviation 50.

(a) Find

(i) $\mathrm{P}(X < 925)$

(ii) $\mathrm{P}(925 < X < 1025)$

(iii) $\mathrm{P}(X < 1025)$. [3]

(b) Sketch a graph representing the information in (a). [2]

(c) Find the standardised value of 925 and of 1025. [2]

(d) Sketch the corresponding graph of standardised values. [2]

33. [Maximum mark: 6] ***[with GDC]***

The heights of boys at a particular school follow a normal distribution with a standard deviation of 5 cm. The probability of a boy being shorter than 153 cm is 0.705.

(a) Calculate the mean height of the boys. [4]

(b) Find the probability of a boy being taller than 156 cm. [2]

34. [Maximum mark: 6] ***[with GDC]***

The weights in grams of bread loaves sold at a supermarket are normally distributed with mean 200 g. The weights of 88% of loaves are less than 220 g. Find the standard deviation.

35. [Maximum mark: 6] ***[with GDC]***

The scores of a test given to students are normally distributed with a mean of 21.
80 % of the students have scores less than 23.7.

(a) Find the standard deviation of the scores. [3]

A student is chosen at random. This student has the same probability of having a score less than 25.4 as having a score greater than b.

(b) (i) Find the probability the student has a score less than 25.4.

(ii) Find the value of b. [3]

36. [Maximum mark: 4] ***[with GDC]***

A factory has a machine designed to produce 1 kg bags of sugar. It is found that the average weight of sugar in the bags is 1.02 kg. Assuming that the weights of the bags are normally distributed, find the standard deviation if 1.7% of the bags weigh below 1 kg. Give your answer correct to the nearest 0.1 gram.

37. [Maximum mark: 6] ***[with GDC]***

The random variable X is normally distributed. It is given that $\mathrm{P}(X \leq 10) = 0.670$ and $\mathrm{P}(X \leq 12) = 0.937$. Find $\mathrm{E}(X)$.

38. [Maximum mark: 6] ***[with GDC]***

The random variable X is normally distributed. 17% is less than 100 while 57% is less than 200. Find the mean and the standard deviation.

39. [Maximum mark: 6] ***[with GDC]***

The heights of certain flowers follow a normal distribution. It is known that 20% of these flowers have a height less than 3 cm and 10% have a height greater than 8 cm.
Find the value of the mean μ and the standard deviation σ.

40. [Maximum mark: 6] ***[with GDC]***

The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15 % of the cars travelled at speeds greater than 90 km h^{-1} and 12 % of them at speeds less than 40 km h^{-1}. Find μ and σ.

41. [Maximum mark: 6] ***[with GDC]***

A furniture manufacturer makes tables. A table leg is considered to be oversize if its width is greater than 10.5 cm and undersize if its width is less than 9.5 cm. From past experience it is found than 2% of the table legs that are made are oversize and 4% of the table legs are undersize. The widths of the table legs are normally distributed with mean μ cm and standard deviation μ cm. Find the value of μ and of σ.

42. [Maximum mark: 3] ***[with GDC]***

Z is the standardised normal random variable with mean 0 and variance 1. Find the value of a such that $\mathrm{P}(|Z| \leq a) = 0.75$.

43*. [Maximum mark: 5] ***[with GDC]***

Let X be a normal random variable with mean 25 and variance 4. Find $\mathrm{P}(|X - 25| < 3)$.

44*. [Maximum mark: 8] ***[with GDC]***

A random variable X is normally distributed with mean μ and standard deviation σ, such that $\mathrm{P}(X > 50.32) = 0.119$, and $\mathrm{P}(X < 43.56) = 0.305$.

(a) Find μ and σ. [5]

(b) Hence find $\mathrm{P}(|X - \mu| < 5)$. [3]

B. Exam style questions (LONG)

45. [Maximum mark: 13] ***[with GDC]***

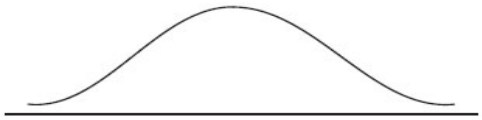
A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

(i) weighs less than 8 g; (ii) weighs between 6 g and 8 g. [4]

(b) Five percent of the biscuits in the box weigh less than *d* grams.

(i) Complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.



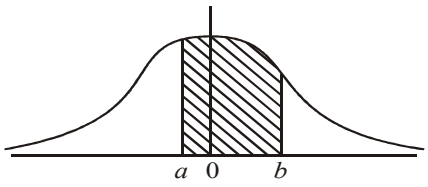
(ii) Find the value of d. [5]

(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20% of the biscuits in this second box weigh less than 5 g. Find the value of μ. [4]

46. [Maximum mark: 10] ***[with GDC]***

The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.

(a) The probability that the lifespan of an insect of this species lies between 55 and 60 hours is represented by the shaded area in the following diagram. This diagram represents the standard normal curve.



(i) Write down the values of a and b. [2]

(ii) Find the probability that the lifespan of an insect of this species is

(a) more than 55 hours; (b) between 55 and 60 hours. [3]

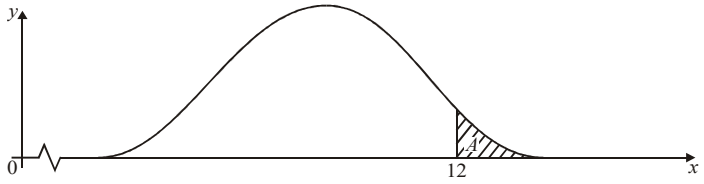
(b) 90% of the insects die after t hours.

(i) Represent this information on a standard normal curve diagram, similar to the one given in part (a), indicating clearly the area representing 90%. [2]

(ii) Find the value of t. [3]

47. [Maximum mark: 10] ***[with GDC]***

The graph shows a normal curve for the random variable X, with mean μ and standard deviation σ.



It is known that $\mathrm{P}(X > 12) = 0.1$.

(a) The shaded region A is the region under the curve where $x > 12$. Write down the area of the shaded region A. [1]

It is also known that $\mathrm{P}(X < 8) = 0.1$.

(b) Find the value of μ, explaining your method in full. [3]

(c) Show that $\sigma = 1.56$ to an accuracy of three significant figures. [4]

(d) Find $\mathrm{P}(X < 11)$. [2]

48. [Maximum mark: 12] ***[with GDC]***

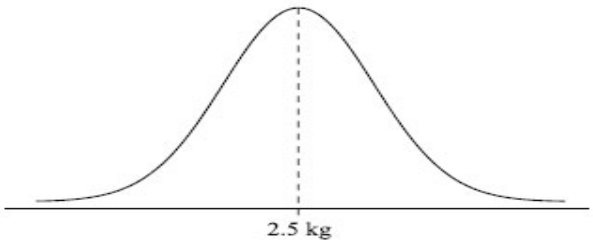
The weights of chickens for sale in a shop are normally distributed with mean 2.5 kg and standard deviation 0.3 kg.

(a) A chicken is chosen at random.

(i) Find the probability that it weighs less than 2 kg.

(ii) Find the probability that it weighs more than 2.8 kg.

(iii) In the diagram below, shade the areas that represent the probabilities from parts (i) and (ii).



(iv) Find the probability that it weighs between 2 kg and 2.8 kg (4sf). [7]

(b) A customer buys 10 chickens.

(i) Find the probability that all 10 chickens weigh between 2 kg and 2.8 kg.

(ii) Find the probability that at least 7 of the chickens weigh between 2 kg and 2.8 kg. [5]

49. [Maximum mark: 10] ***[with GDC]***

Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25.7 kg and standard deviation 0.50 kg.

(a) What is the probability a bag selected at random will weigh less than 25.0 kg? [2]

In order to reduce the number of underweight bags (bags weighing less than 25 kg) to 2.5 % of the total, the mean is increased without changing the standard deviation.

(b) Show that the increased mean is 26.0 kg. [3]

It is decided to purchase a more accurate machine for filling the bags. The requirements for this machine are that only 2.5 % of bags be under 25 kg and that only 2.5 % of bags be over 26 kg.

(c) Calculate the mean and standard deviation that satisfy these requirements. [5]

50. [Maximum mark: 10] ***[with GDC]***

A company produces computer microchips, which have a life expectancy that follows a normal distribution with a mean of 90 months and a standard deviation of 3.7 months.

(a) If a microchip is guaranteed for 84 months find the probability that it will fail before the guarantee ends. [3]

(b) The probability that a microchip does not fail before the end of the guarantee is required to be 99 %. For how many months should it be guaranteed? [3]

(c) A rival company produces microchips where the probability that they will fail after 84 months is 0.88. Given that the life expectancy also follows a normal distribution with standard deviation 3.7 months, find the mean. [4]

51. [Maximum mark: 13] ***[with GDC]***

In a large school, the heights of all fourteen-year-old students are measured.
The heights of the girls are normally distributed with mean 155cm and standard deviation 10cm.
The heights of the boys are normally distributed with mean 160cm and standard deviation 12cm.

(a) Find the probability that a girl is taller than 170 cm. [2]

(b) Given that 10 % of the girls are shorter than x cm, find x. [2]

(c) Given that 90 % of the boys have heights between q cm and r cm where q and r are symmetrical about 160 cm, and $q < r$, find the value of q and of r. [3]

In the group of fourteen-year-old students, 60 % are girls and 40 % are boys.
The probability that a girl is taller than 170 cm was found in part (a).
The probability that a boy is taller than 170 cm is 0.202.

A fourteen-year-old student is selected at random.

(d) Calculate the probability that the student is taller than 170 cm. [3]

(e) Given that the student is taller than 170cm, find the probability that the student is a girl. [3]

52. [Maximum mark: 11] ***[with GDC]***

(a) A box of biscuits is considered to be underweight if it weighs less than 228 grams. It is known that the weights of these boxes of biscuits are normally distributed with a mean of 231 grams and a standard deviation of 1.5 grams. What is the probability that a box is underweight? [2]

(b) The manufacturer decides that the probability of a box being underweight should be reduced to 0.002.

(i) Bill's suggestion is to increase the mean and leave the standard deviation unchanged. Find the value of the new mean.

(ii) Sarah's suggestion is to reduce the standard deviation and leave the mean unchanged. Find the value of the new standard deviation. [6]

(c) After the probability of a box being underweight has been reduced to 0.002, a group of customers buys 100 boxes of biscuits. Find the probability that at least two of the boxes are underweight. [3]

53. [Maximum mark: 12] ***[with GDC]***

A machine is set to produce bags of salt, whose weights are distributed normally, with a mean of 110 g and standard deviation of 1.142 g. If the weight of a bag of salt is less than 108 g, the bag is rejected. With these settings, 4% of the bags are rejected.
The settings of the machine are altered and it is found that 7% of the bags are rejected.

(a) (i) If the mean has not changed, find the new standard deviation, **correct to three decimal places.**

The machine is adjusted to operate with this new value of the standard deviation.

(ii) Find the value, **correct to two decimal places,** at which the mean should be set so that only 4% of the bags are rejected. [8]

(b) With the new settings from part (a), it is found that 80% of the bags of salt have a weight which lies between A g and B g, where A and B are symmetric about the mean. Find the values of A and B, giving your answers **correct to two decimal places.** [4]

54. [Maximum mark: 16] ***[with GDC]***

A company buys 44 % of its stock of bolts from manufacturer A and the rest from manufacturer B. The diameters of the bolts produced by each manufacturer follow a normal distribution with a standard deviation of 0.16 mm.

The mean diameter of the bolts produced by manufacturer A is 1.56 mm.

The mean diameter of the bolts produced by manufacturer B is 1.63 mm.

(a) Find the percentage of the bolts produced by manufacturer B which have a diameter less than 1.52. [3]

A bolt is chosen at random from the company's stock.

(b) Show that the probability that the diameter is less than 1.52 mm is 0.312, to three significant figures. [4]

(c) The diameter of the bolt is found to be less than 1.52 mm. Find the probability that the bolt was produced by manufacturer B. [3]

(d) Manufacturer B makes 8000 bolts in one day. It makes a profit of $ 1.50 on each bolt sold, on condition that its diameter measures between 1.52 mm and 1.83 mm Bolts whose diameters measure less than 1.52 mm must be discarded at a loss of $0.85 per bolt. Bolts whose diameters measure over 1.83 mm are sold at a reduced profit of $0.50 per bolt.

Find the expected profit for manufacturer B. [6]

55. [Maximum mark: 14] ***[with GDC]***

Ian and Karl have been chosen to represent their countries in the Olympic discus throw. Assume that the distance thrown by each athlete is normally distributed. According to the records in the past year

Karl's throws have a mean of $59.50\ \text{m}$ and a standard deviation of $3.00\ \text{m}$

Ian's throws have a mean of $60.33\ \text{m}$ and a standard deviation of $1.95\ \text{m}$

(a) Find the percentage of Karl's throws which have been longer than $56\ \text{m}$. [2]

(a) 80 % of Ian's throws have been longer than x metres. Find x, correct to **two** decimal places. [2]

(c) This year, Karl's and Ian's throws still have the same means and standard deviations. In a competition an athlete must have at least one throw of $65\ \text{m}$ or more in the first round to qualify for the final round. Each athlete is allowed three throws in the first round.

(i) Determine which of these two athletes is more likely to qualify for the final on their first throw.

(ii) Find the probability that **both** athletes qualify for the final. [10]