INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 4.10]

BINOMIAL DISTRIBUTION

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O. Practice questions

1. [Maximum mark: 10] ***[with GDC]***

A fair coin is tossed eight times.

(a) Calculate the probability of obtaining [7]

exactly 4 heads	
exactly 3 heads	
3, 4 or 5 heads	
no heads	
always heads	
at most 2 heads	
at least 3 heads	

(b) Find the expected number of heads and the variance of the number of heads [3]

$\mathrm{E}(X)$		$\mathrm{Var}(X)$	

2. [Maximum mark: 6] ***[with GDC]***

The random variable X follows the binomial distribution $\mathrm{B}(n,p)$. Given that $\mathrm{E}(X) = 10$ and $\mathrm{Var}(X) = 6$ find the values of n and p.

3. [Maximum mark: 7] ***[with GDC]***

A factory makes calculators. It has been noticed that 1.5% of the calculators are faulty**.**

A high school ordered 160 calculators.

(a) Find (i) the expected number of faulty calculators (ii) the corresponding variance. [2]

(b) Find the probability that none of the calculators is faulty. [1]

(c) Find the probability that at least one calculator is faulty. [1]

(d) Find the probability that the number of calculators is equal to the mode. [3]

4*. [Maximum mark: 9] ***[with GDC]***

The random variable X follows the binomial distribution $\mathrm{B}(n, 0.4)$. Find the value of n for each of the following cases:

(a) given that $\mathrm{E}(X) - \mathrm{Var}(X) = 0.8$. [2]

(b) given that $\mathrm{P}(X = 3) = 0.1774$ [using trial and error] [2]

(c) given that $\mathrm{P}(X \leq 2) = 0.0271$ [using trial and error] [2]

(d) given that $\mathrm{E}(X^2) = 18.4$ **[only for HL]** [3]

A. Exam style questions (SHORT)

5. [Maximum mark: 4] ***[with GDC]***

A fair coin is tossed five times. Calculate the probability of obtaining

(a) exactly three heads; [2]

(b) at least one head. [2]

6. [Maximum mark: 4] ***[with GDC]***

The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads. [2]

(b) Find the probability of obtaining **at least** two heads. [2]

7. [Maximum mark: 5] ***[with GDC]***

A factory makes switches. The probability that a switch is defective is 0.04.

The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample. [1]

(b) Find the probability that there are exactly six defective switches in the sample. [2]

(c) Find the probability that there is at least one defective switch in the sample. [2]

8. [Maximum mark: 5] ***[with GDC]***

A factory makes calculators. Over a long period, 2 % of them are found to be faulty. A random sample of 100 calculators is tested.

(a) Write down the expected number of faulty calculators in the sample. [1]

(b) Find the probability that three calculators are faulty. [2]

(c) Find the probability that more than one calculator is faulty. [2]

9. [Maximum mark: 7] ***[with GDC]***

A box contains 35 red discs and 5 black discs. A disc is selected at random and its colour noted. The disc is then replaced in the box.

(a) In eight such selections, what is the probability that a black disc is selected

(i) exactly once? (ii) at least once? [5]

(b) The process of selecting and replacing is carried out 400 times.

What is the expected number of black discs that would be drawn? [2]

10. [Maximum mark: 4] ***[with GDC]***

In a school, $\frac{1}{3}$ of the students travel to school by bus. Five students are chosen at random.

(a) Find the probability that exactly 3 of them travel to school by bus. [2]

(b) Find the probability that at most 3 of them travel to school by bus. [2]

11. [Maximum mark: 4] ***[with GDC]***

When John throws a stone at a target, the probability that he hits the target is 0.4. He throws a stone 6 times.

(a) Find the probability that he hits the target **exactly** 4 times. [2]

(b) Find the probability that he hits the target for the first time on his third throw. [2]

12. [Maximum mark: 4] ***[with GDC]***

When a boy plays a game at a fair, the probability that he wins a prize is 0.25. He plays the game 10 times. Let X denote the total number of prizes that he wins. Assuming that the games are independent, find

(a) $\mathrm{E}(X)$ [2]

(b) $\mathrm{P}(X \leq 2)$. [2]

13. [Maximum mark: 3] ***[with GDC]***

On a television channel the news is shown at the same time each day. The probability that Alice watches the news on a given day is 0.4. Calculate the probability that on five consecutive days, she watches the news on at most three days.

14. [Maximum mark: 9] ***[with GDC]***

The probability of obtaining heads on a biased coin is $\frac{1}{3}$.

(a) Sammy tosses the coin three times. Find the probability of getting

(i) three heads;

(ii) two heads and one tail. [4]

(b) Amir plays a game in which he tosses the coin 12 times.

(i) Find the expected number of heads.

(ii) Amir wins $\$ 10$ for each head obtained and loses $\$ 6$ for each tail. Find his expected winnings. [5]

15. [Maximum mark: 6] ***[with GDC]***

In an examination of 20 multiple-choice questions each question has four possible answers, only one of which is correct. Robert randomly guesses the answer to each question.

(a) Find his expected number of correct answers. [2]

(b) Find the probability that Robert obtains this expected number of correct answers. [2]

(c) Find the probability that Robert obtains **less than** five correct answers. [2]

16*. [Maximum mark: 7] ***[with GDC]***

A biology test consists of seven multiple choice questions. Each question has five possible answers, only one of which is correct. At least four correct answers are required to pass the test. Juan does not know the answer to any of the questions so, for each question, he selects the answer at random.

(a) Find the probability that Juan answers exactly four questions correctly. [2]

(b) Find the probability that Juan passes the biology test. [2]

Ten students take the same test and select the answers at random

(c) Find the probability that nobody passes the biology test. [3]

17. [Maximum mark: 7] ***[with GDC]***

A coin is biased so that when it is tossed the probability of obtaining heads is $\frac{2}{3}$.

The coin is tossed 1800 times. Let X be the number of heads obtained.

(a) Find the mean of X; [2]

(b) Find the standard deviation of X. [2]

. The coin is tossed 18 times.

(c) Find the probability that more heads than tails are obtained. [3]

18. [Maximum mark: 4] ***[with GDC]***

A satellite relies on solar cells for its power and will operate provided that at least one of the cells is working. Cells fail independently of each other, and the probability that an individual cell fails within one year is 0.8.

(a) For a satellite with ten solar cells, find the probability that all ten cells fail within one year. [2]

(b) For a satellite with ten solar cells, find the probability that the satellite is still operating at the end of one year. [2]

19. [Maximum mark: 5] ***[with GDC]***

Let $X \sim \mathrm{B}(10,0.4)$. Find for X

(a) the mean [1]

(b) the mode [2]

(c) the variance [1]

(d) the standard deviation. [1]

20. [Maximum mark: 4] ***[with GDC]***

Let $X \sim \mathrm{B}(10,\frac{1}{4})$. Find for X

(a) the mean [1]

(b) the mode [2]

(c) the variance [1]

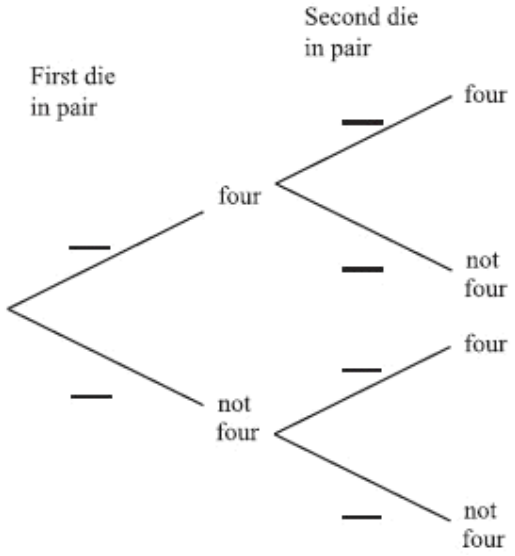
(d) the standard deviation. [1]

B. Exam style questions (LONG)

21. [Maximum mark: 12] ***[with GDC]***

A pair of fair dice is thrown.

(a) Complete the tree diagram below, which shows the possible outcomes.



[3]

Let E be the event that **exactly** one four occurs when the pair of dice is thrown.

(b) Calculate $\mathrm{P}(E)$. [3]

The pair of dice is now thrown five times.

(c) Calculate the probability that event E occurs exactly three times in the five throws. [3]

(d) Calculate the probability that event E occurs **at least** three times in the five throws. [3]

22. [Maximum mark: 12] ***[with GDC]***

A bag contains a very large number of ribbons. One quarter of the ribbons are yellow and the rest are blue. Ten ribbons are selected at random from the bag.

(a) Find the expected number of yellow ribbons selected. [2]

(b) Find the probability that exactly six of the ribbons are yellow. [2]

(c) Find the probability that at least two of the ribbons are yellow. [3]

(d) Find the most likely number of yellow ribbons selected. [4]

(e) What assumption have you made about the probability of selecting a yellow ribbon? [1]

23. [Maximum mark: 12] ***[with GDC]***

Andrew shoots 20 arrows at a target. He has a probability of 0.3 of hitting the target. All shots are independent of each other. Let X denote the number of arrows hitting the target.

(a) Find the mean and the variance of X. [4]

(b) Find (i) $\mathrm{P}(X = 5)$ (ii) $\mathrm{P}(4 \leq X \leq 8)$ [4]

Bill also shoots 20 arrows at a target with probability of 0.3 of hitting the target. All shots are independent of each other.

(c) Calculate the probability that Bill hits the target for the first time on his first shot. [1]

(d) Calculate the probability that Bill hits the target for the first time on his third shot. [3]