INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 4.9]

DISCRETE DISTRIBUTIONS IN GENERAL

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O. Practice questions

1. [Maximum mark: 6] ***[with / without GDC]***

The probability distribution of the discrete random variable X is given by the table

x	1	2	3	4	5
$\mathrm{P}(X=x)$	0.4	0.2	0.15	0.15	0.1

Find

(a) the expected value $\mathrm{E}(X)$ of X. [2]

(b) the mode of X. [1]

(c) the median of X. [1]

(d) the lower quartile Q_1 and the upper quartile Q_3. [2]

2. [Maximum mark: 6] ***[without GDC]***

The probability distribution of the discrete random variable X is given by the table

x	1	2	3
$\mathrm{P}(X=x)$	a	$2a$	b

Find the values of a and b given that $\mathrm{E}(X)=2.2$

3. [Maximum mark: 6] ***[without GDC]***

The probability distribution of the discrete random variable X is given by the table

x	1	2	3
$\mathrm{P}(X=x)$	0.2	0.4	0.4

Nikos selects a number at random.

If he selects 1 he earns 10 €. If he selects 2 he earns 5 €. If he selects 3 he loses 4 €

(a) Find the expected value of X. [3]

(b) Find the expected value of the profit for Nikos. Is the game fair? [3]

4. [Maximum mark: 5] ***[without GDC]***

A discrete random variable X has its probability distribution given by

$$\mathrm{P}(X=x)=\frac{x}{6}, \quad \text{where } x \text{ is } 1, 2, 3.$$

(a) Complete the following table showing the probability distribution for X. [2]

x	1	2	3
$\mathrm{P}(X=x)$			

(b) Find $\mathrm{E}(X)$. [3]

5. [Maximum mark: 5] ***[without GDC]***

Each of the following 10 words is placed on **a card** and put in a hat.

ONE, TWO, THREE, FOUR, FIVE, SIX, SEVEN, EIGHT, NINE, TEN

We pick a card at random. Let X be the size (number of letters) of the corresponding word.

(a) Give the probability distribution for X (i.e. the table of probabilities). [3]

(b) Find the expected number of X. [2]

6. [Maximum mark: 18] ***[with / without GDC] mainly for HL***

The table below shows the probability distribution of a discrete random variable X.

x	0	1	2
$\mathrm{P}(X=x)$	0.3	0.2	0.5

Complete the following table.

$\mathrm{E}(X)$	
$\mathrm{E}(X^2)$	
$\mathrm{E}(X^3)$	
$\mathrm{Var}(X)$ using $\mathrm{E}(X^2)-\mathrm{E}(X)^2$	
$\mathrm{Var}(X)$ using $\mathrm{E}(X-\mu)^2$	
MODE	
MEDIAN	
Q_1	
Q_3	
$\mathrm{E}(2X+1)$	
$\mathrm{Var}(2X+1)$	

7*. [Maximum mark: 8] ***[with / without GDC]*** ***only for HL***

A discrete random variable X has its probability distribution given by

$$\mathrm{P}(X = x) = \frac{x^2}{14}, \text{ where } x \text{ is } 1, 2, 3, \ldots, n.$$

(a) Find the value of n. [2]

(b) Find (i) $\mathrm{E}(X)$ (ii) $\mathrm{E}(X^2)$ (iii) $\mathrm{Var}(X)$. [6]

8. [Maximum mark: 5] ***[with GDC]*** ***only for HL***

The table below shows the probability distribution of a discrete random variable *X*.

x	0	1	2
$\mathrm{P}(X = x)$	a	b	c

Given that $\mathrm{E}(X) = 1$ and $\mathrm{Var}(X) = 0.8$ find the values of a, b and c.

A. Exam style questions (SHORT)

9. [Maximum mark: 6] ***[with GDC]***

The probability distribution of the discrete random variable X is given by the following table.

x	1	2	3	4	5
$\mathrm{P}(X = x)$	0.4	p	0.2	0.07	0.02

(a) Find the value of p. [3]

(b) Calculate the expected value of X. [3]

10. [Maximum mark: 6] ***[without GDC]***

A discrete random variable X has a probability distribution as shown in the table below.

x	0	1	2	3
$\mathrm{P}(X = x)$	0.1	a	0.3	b

(a) Find the value of $a + b$. [2]

(b) Given that $\mathrm{E}(X) = 1.5$, find the value of a and of b. [4]

11. [Maximum mark: 5] ***[with GDC]***

The table below shows the probability distribution of a discrete random variable X.

x	0	1	2	3
$\mathrm{P}(X = x)$	0.2	a	b	0.25

Given that $\mathrm{E}(X) = 1.55$, find the value of a and of b.

12*. [Maximum mark: 6] ***[with GDC]***

In a game a player rolls a biased tetrahedral (four-faced) die. The probability of each possible score is shown below.

Score	1	2	3	4
Probability	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	x

Find the probability of a total score of six after two rolls.

13. [Maximum mark: 6]] ***[with GDC]***

The following table shows the probability distribution of a discrete random variable X.

x	-1	0	2	3
$\mathrm{P}(X=x)$	0.2	$10k^2$	0.4	$3k$

(a) Find the value of k. [3]

(b) Find the expected value of X. [3]

14. [Maximum mark: 6] ***[without GDC]***

A discrete random variable X has its probability distribution given by

$\mathrm{P}(X=x)=k(x+1)$, where x is $0, 1, 2, 3, 4$.

(a) Complete the following table showing the probability distribution for X (in terms of k) [2]

x	0	1	2	3	4
$\mathrm{P}(X=x)$					

(b) Show that $k=\frac{1}{15}$. [1]

(c) Find $\mathrm{E}(X)$. [3]

15. [Maximum mark: 6] ***[without GDC]***

The probability distribution of a discrete random variable X is defined by

$$\mathrm{P}(X=x)=cx(5-x),\ x=1,2,3,4.$$

(a) Find the value of c [3]

(b) Find $\mathrm{E}(X)$. [3]

16. [Maximum mark: 5]

The probability distribution of a discrete random variable X is given by

$$\mathrm{P}(X=x)=k\left(\frac{2}{3}\right)^x \text{ for } x=0,1,2,\ldots$$

Find the value of k.

17. [Maximum mark: 8] ***[without GDC]***

Three students, Kim, Ching Li and Jonathan each have a pack of cards, from which they select a card at random. Each card has a 0, 3, 4, or 9 printed on it.

(a) Kim states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$P(X=x)$	0.3	0.45	0.2	0.35

Explain why Kim is incorrect. [2]

(b) Ching Li correctly states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$P(X=x)$	0.4	k	$2k$	0.3

Find the value of k. [2]

(c) Jonathan correctly states that the probability distribution for his pack of cards is given by $P(X=x)=\frac{x+1}{20}$. One card is drawn at random from his pack. Calculate the probability that the number on the card drawn

(i) is 0. (ii) is greater than 0. [4]

18. [Maximum mark: 8]

A biased die with four faces is used in a game. A player pays 10 counters to roll the die. The table below shows the possible scores on the die, the probability of each score and the number of counters the player receives in return for each score.

Score	1	2	3	4
Probability	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$
Number of counters player receives	4	5	15	n

(a) The player throws the die twice. Find the probability that

(i) he has a total score of 3. (ii) he has a total score of 4. [4]

(b) Find the value of n in order for the player to get an expected return of 9 counters per roll. [4]

B. Exam style questions (LONG)

19. [Maximum mark: 15] ***[without GDC]***

Two fair **four**-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

The sample space is shown below:

	1	2	3	4
1	●	●	●	●
2	●	●	●	●
3	●	●	●	●
4	●	●	●	●

(a) Write down the probability that two scores of 4 are obtained. [1]

Let X be the number of 4s that land face down.

(b) Complete the following probability distribution for X. [3]

x	0	1	2
$\mathrm{P}(X = x)$			

(c) Find $\mathrm{E}(X)$. [3]

Chris plays a game where he rolls the dice above.

If two 4s are obtained he wins 20€.

If only one 4 is obtained he wins 5€.

If no 4 is obtained he loses 2€

(d) Find the expected amount earned in one game. [3]

(e) If Chris plays this game 100 times find the amount he is expected to win. [2]

(f) If Chris plays this game twice find the probability that he earns 18€. [3]

20. [Maximum mark: 19] ***[with GDC]***

Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X.

x	0	1	2
$\mathrm{P}(X = x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

(a) Calculate $\mathrm{E}(X)$, the mean number of red balls chosen. [3]

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.

(ii) Hence find the probability distribution for Y, where Y is the number of red balls chosen. [8]

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen. [5]

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die. [3]

21. [Maximum mark: 16] ***[with GDC]***

A **four-sided** die has three blue faces and one red face. The die is rolled.

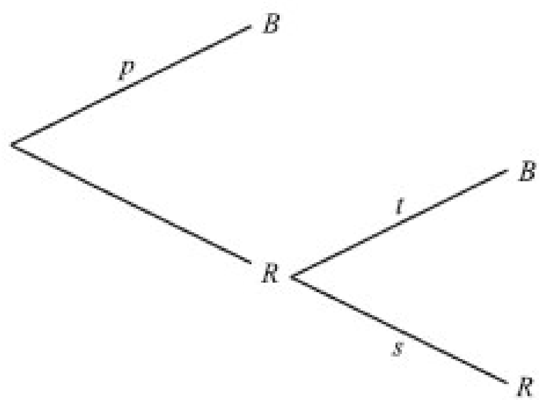
Let B be the event a blue face lands down, and R be the event a red face lands down.

(a) Write down the values of

(i) $\mathrm{P}(B)$

(ii) $\mathrm{P}(R)$ [2]

(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where p, s, t are probabilities.



Find the value of p, of s and of t. [2]

Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let X be the total score obtained.

(c) (i) Show that $\mathrm{P}(X=3)=\frac{3}{16}$.

(ii) Find $\mathrm{P}(X=2)$ [3]

(d) (i) Construct a probability distribution table for X.

(ii) Calculate the expected value of X. [5]

(e) If the total score is 3, Guiseppi wins $\$10$. If the total score is 2, Guiseppi gets nothing. He plays the game twice. Find the probability that he wins exactly $\$10$. [4]

22*. [Maximum mark: 18] ***[with GDC]***

A box contains 2 red and 8 blue balls. Find

(a) the expected number of **red balls** if we select two balls with replacement. [4]

(b) the expected number of **red balls** if we select two balls without replacement. [4]

(c) the expected number of **red balls** if we select three balls without replacement. [5]

(d) the expected number of **trials** if we select a ball without replacement until we find a blue one. [5]

23*. [Maximum mark: 10] ***[with GDC]***

John removes the labels from three cans of tomato soup and two cans of chicken soup in order to enter a competition, and puts the cans away. He then discovers that the cans are identical, so that he cannot distinguish between cans of tomato soup and chicken soup. Some weeks later he decides to have a can of chicken soup for lunch. He opens the cans at random until he opens a can of chicken soup. Let Y denote the number of cans he opens. Find

(a) the possible values of Y, [1]

(b) the probability of each of these values of Y, [4]

(c) the expected value of Y. [2]

(d) Let X denote the number of cans he opens until he opens a can of tomato soup. Find, by completing a table, the probability distribution of X [3]

24.** [Maximum mark: 15] ***[with GDC]***

Two children, Alan and Belle, each throw two fair cubical dice simultaneously. The score for each child is the sum of the two numbers shown on their respective dice.

(a) (i) Calculate the probability that Alan obtains a score of 9.

(ii) Calculate the probability that Alan and Belle both obtain a score of 9. [3]

(b) (i) Calculate the probability that Alan and Belle obtain the same score,

(ii) Deduce the probability that Alan's score exceeds Belle's score. [5]

(c) Let X denote the largest number shown on the four dice.

(i) Show that for $\mathrm{P}(X \le x) = \left(\frac{x}{6}\right)^4$, for $x = 1,2,3,4,5,6$

(ii) Complete the following probability distribution table.

x	1	2	3	4	5	6
$\mathrm{P}(X = x)$	$\frac{1}{1296}$	$\frac{15}{1296}$				$\frac{671}{1296}$

(iii) Calculate $\mathrm{E}(X)$. [7]