Name: **Math Academy 1 Fall Study Guide**

CHAPTER ONE: ARITHMETIC AND ALGEBRA — Due Friday, December 2

Thorough understanding and fluency of the concepts and methods of arithmetic and algebra, covered in this chapter and the next, is a cornerstone to success in this course and most future math courses as well. This chapter focuses on the fundamental concepts of numbers (including fractions, decimals, and negatives), exponents, and functions.

1-A Fractions

scientific notation

❶ Reduce a fraction.
❶ $\frac{140}{350}$.

❷ Multiply or divide by a fraction.
❷ a) $\frac{2}{5} \times \frac{3}{4}$ b) $\frac{2}{5} \times 3$

❸ a) $\frac{2}{5} \div \frac{4}{3}$ b) $\frac{2}{5} \div 3$

❸ Add or subtract fractions with the same denominator.
❸ $\frac{12}{20} - \frac{5}{20}$

❹ Add or subtract fractions with different denominators.
❹ $\frac{3}{5} - \frac{1}{4}$

❺ Divide zero or by zero.
❺ a) $\frac{2}{0}$ b) $\frac{0}{2}$

❻ Convert a percentage to a decimal.
❻ a) 9% b) .9%

❼ Convert calculator notation to scientific notation and to standard notation.
❼ a) `2.57E3` b) `2.57E-3`

1-B Order of Operations and Negatives

sign

❶ Subtract, multiply, or divide by a negative.
❶ a) $20 \times (-10)$ b) $20 \div (-10)$

c) $-20 \div (-10)$ d) $20 - (-10)$

❷ Apply order of operations for negatives and parentheses.
❷ a) -2^3 b) $(-2)^3$

c) (-2^3) d) $(-2)^3$

e) -2^4 f) $(-2)^4$

g) (-2^4) h) $(-2)^4$

❸ Apply order of operations for basic arithmetic and powers.
❸ a) $4 + 5 \times 2$

b) $4 + 5(2)^3$

c) $-5(-2)^3 + 3$

❹ Use a calculator to evaluate a fraction.
❹ $\frac{-8 + 2\sqrt{100}}{\sqrt{49} - 5}$

1-C Properties of Exponents

❶ Simplify an expression using properties of exponents.
❶ a) $ab^3(a^2b)^5$ b) $\left(\frac{5a}{2}\right)^3$

c) $\left(\frac{5a}{2}\right)^{-3}$ d) $10\left(\frac{5a}{2}\right)^{-3}$

❶ Simplify $\frac{(2a^4b)^3b^6}{a^{12}bc^2}$, and write it without a fraction. State each property of exponents used.

❷ Rewrite an expression without negative exponents.

❷ Rewrite with no parentheses or negative exponents.

a) $\frac{a^{-2}b}{c^3d}$ b) $\frac{a^2b}{c^{-3}d}$

c) $\frac{ab^{-2}}{c^3d}$ d) $\frac{(ab)^{-2}}{c^3d}\frac{a^2b}{c^{-3}d}$

e) $\left(\frac{a^2b}{c^3d}\right)^{-2}$ f) $\left(\frac{a^{-2}b}{c^3d}\right)^{-2}$

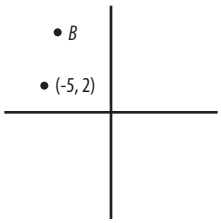
g) $(2a)^{-1}$ h) $2a^{-1}$

1-D Functions and their Graphs

x-axis • *y*-axis • origin • coordinates • domain • range • argument • function

❶ Estimate coordinates on a graph.

❶ Estimate the coordinates of point *B* below.



❷ Use function notation.

❷ Given $f(x) = 5x$ and $g(x) = x^2 + 2x - 10$, find the following.

a) $f(4)$

b) $g(4)$

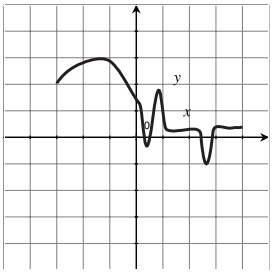
c) $f(x + 2)$

❸ Sketch a function by plotting points.

❸ $f(x) = x^2 - 2x - 3$

❹ Identify the domain and range of a graphed function.

❹ Identify the domain and range of the function graphed below.



❺ Read expressions in function notation, and identify the arguments.

❺ Read the notation $f(x) = \sqrt{x+1}$, and identify the arguments.

❻ Identify the domain of a function in function notation.

❻ a) $a(x) = 6x^2 - x + 4$ b) $b(x) = \frac{x+8}{2x-6}$

c) $c(x) = 4\sqrt{2x-6}$ d) $d(x) =$ *x*'s student ID number

❼ Identify whether or not a relation is a function.

❼ State whether or not the following relations are functions. For ones that are not functions, write an *x* value and two different *y* values for that *x* value.

a) $y = 5x^2 + 9x - 10$ b) $y = \pm\sqrt{x}$

c) $y =$ age of person *x* d) $y =$ brother of person *x*

❽ Identify whether or not a graph represents a function.

❽ a)

b)

1-E Terms versus Factors

❶ Identify terms and factors of an expression.

❶ List the factors of the expression $5x^2(3x+4)(x^2-9x+2)$, and state how many terms each factor has.

❶ State the terms and the factors of the expression $5a-5b$.

❷ Identify a coefficient.

❷ Identify the coefficients of the expression $5x^6+x^3+\frac{8x^2}{7}-9x+2$.

❸ Add or subtract expressions.

❸ $(4x^2+10x-5)+(x^2+2)$

❹ Classify a polynomial in one variable.

❹ Classify the following polynomials, write them in standard form, and identify the leading coefficient.

a) $x+4x^3$

b) $-15x$

c) $8x^2-2x^9+3$

d) $2-\frac{7x^3}{5}+6x^2+x$

❺ Multiply a polynomial by a monomial.

❺ $2(5x^3-4x+10)$

❻ Distribute a monomial.

❻ Multiply the expression $(x+3)(2x-1)+4x(9x^2+3)+\sqrt{5}$ by 10.

❻ Identify the error, if any, in each of the following attempts to multiply the expression $\frac{11(4x)-\sqrt{10x}}{5}$ by 2.

a) $\frac{22(4x)-\sqrt{20x}}{5}$

b) $\frac{22(4x)-2\sqrt{20x}}{10}$

c) $\frac{22(8x)-2\sqrt{10x}}{5}$

d) $\frac{22(4x)-2\sqrt{10x}}{5}$

❼ Simplify a fraction with multiple terms in the numerator or denominator.

❼ $\frac{6x^2-9x\sqrt{30x}}{6x^2-9x}$

❽ Take a power or root of an expression.

❽ a) $10xy$ to the third power b) the square root of $49x^8y^2$

1-F Polynomial Multiplication

❶ Multiply two polynomials.

❶ $(4x-3)(x+5)$

❷ Multiply more than two polynomials.
❷ $(x+2)(x+5)(x-10)$

❸ Multiply a binomial by its conjugate.
❸ $(3x-10)(3x+10)$

❹ Square a binomial.
❹ $(3x-10)^2$

1-G Solving Equations

equation • solution • solve • inverse

❶ Identify an equation.
❶ Identify whether or not each of the following is an equation.
a) $x+5$ b) $x+5=9+5$

c) $x+5=9+5=14$

❷ Apply an operation to both sides of an equation.
❷ Do the following to each side of the equation $x+5=8$, and simplify.
a) add two

b) multiply by 2

c) put it to the power of 2

❸ Identify notation errors in applying an operation to both sides of an equation.
❸ Identify each error in $\times 4(\frac{1}{4}x = x + 1)$, and rewrite it correctly.

❹ Identify the inverse of an operation by definition.
❹ Identify the inverse of each operation in the equation $175x - 900 = 1600$.

❺ Apply inverses to solve an equation.
❺ $2(x-3)^2+8=40$

❻ Express a numerical answer.
❻ Give an appropriate answer for each question.
a) Solve $7x=9$

b) Solve $7x=9$. Round the answer to the nearest tenth.

c) If a pack of seven pens costs $9, what is the price per pen?

CHAPTER TWO: FUNCTIONS — Due Monday, December 5

Functions are the basic building blocks of algebra and most further math. Functions have a domain, a range, and an inverse, and they can be composed, sketched, and transformed. There are many different families of functions, a few key ones of which are explored in this chapter.

2-A Composition and Inverses

composition • inverse • one-to-one • equation • horizontal line test

❶ Evaluate compositions of functions.

❶ Given $f(x) = 4x - 10$ and $g(x) = x^2 + 2x - 3$, evaluate the following.

a) $f(g(3))$

b) $g(f(3))$

c) $f(g(f(3)))$

❷ Find a simplified expression for a composition of functions.

❷ Using the functions f and g, above, give an expression for the following.

a) $f(g(x))$

b) $g(f(x))$

c) $f(g(f(x)))$

❸ Identify the inverse of a basic function by definition.

❸ Identify the inverse of each of the following functions, and verify that $f^{-1}(f(x)) = x$.

a) $a(x) = x + 5$

b) $b(x) = 5x$

c) $c(x) = x^5$

d) $d(x) = 5^x$

❹ Identify the inverse of a function conceptually.

❹ $f(x) = 7x$ is the number of minutes it takes Miki to run x miles. (Miles are plugged in to calculate minutes.)

❺ Find the inverse of a relation algebraically.

❺ Find the inverse of the following functions.

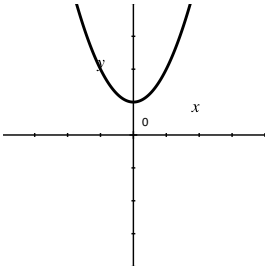
a) $f(x) = x - 8$

b) $g(x) = 2x + 1$

c) $h(x) = x^2$

❻ Find the inverse of a relation graphically.

❻ Sketch the graph of the inverse of the parabola shown below.



❼ Determine whether or not the inverse of a graph is a function.

❼ Why isn't the inverse of the function shown above also a function?

2-C Power and Root Functions

n^{th} root • extraneous solution

❶ Find real n^{th} roots of a number a with a calculator.

❶ Find all real roots, if any.

a) 5^{th} roots of -50

b) 6^{th} roots of 50

c) 6^{th} roots of -50

❷ Evaluate $\sqrt[n]{a^m}$ or $a^{m/n}$ by hand.

❷ $8^{5/3}$

❸ Solve a power equation or root equation.

❸ a) $2(x+5)^4 - 30 = 66$

b) $2(\sqrt[4]{x+5}) + 92 = 80$

2-C Exponential Functions

exponential • growth factor • decay factor • compound interest

❶ Identify a scenario's rate of increase or decrease and its growth or decay factor.

❶ a) 4% less

b) 200% more

c) 3 times as much

d) 3 times more

❷ Write a function for an exponential growth or decay situation, and use it to calculate future values.

❷ An account earns 1.90% annual interest and has $8330 in 2022. Calculate the amount in the account in the following years.

a) 2030

b) 2020

❸ Calculate an account balance with compounded annual interest.

❸ Amelia has $18,000 in her college fund, invested at 2.40% annual interest compounded monthly. How much will she have in 3½ years?

2-E Logarithmic Functions

logarithm • common log • natural log • change of base property • exponentiate • half-life

❶ Simplify the composition of a logarithmic and an exponential function.

❶ a) $\log_4 4^{3x}$

b) $3^{\log_3 2x}$

c) $\log_2 8^x$

❷ Evaluate a simple logarithm by hand.

❷ Evaluate the following logarithms if possible.

a) $\log_4 16$

b) $\log_4 \frac{1}{16}$

c) $\log_4 2$

d) $\log_4 \frac{1}{2}$

e) $\log_4 4$

f) $\log_4 1$

g) $\log_4 0$

h) $\log_4 -16$

❸ Evaluate a common or natural logarithm with a calculator.

❸ Evaluate the following logarithms if possible.

a) log .01

b) log 2.24

c) ln 25

d) log -100

❹ Simplify a logarithmic expression.

❹ Simplify the following expressions. State the property of logarithms used for each step.

a) $2\log_3 x + \log_3 10$

b) $\log_3 9^{4x}$

c) $\log_3 \left(\frac{1}{9}\right)^{4x}$

❺ Simplify a logarithm in which the base and argument are both powers of the same base.

❺ $\log_{81} 27$

❻ Evaluate any logarithm.

❻ $\log_{16} 33$

❼ Solve an exponential equation.

❼ $3(7^{2x-1}) - 30 = 360$

❽ Solve a logarithmic equation.

❽ a) $\log_x 400 = 5$

b) $8\log_9 x - 2\log_3 4x = 5$

❾ Translate a description of a half-life situation into an equation, and solve it.

❾ Polonium-218 has a half-life of 3.11 minutes. How long will it take 400 grams to decay to 30 grams?

❿ Determine half-life based on decay rate or decay rate based on half-life.

❿ Polonium-218 has a half-life of 3.11 minutes. What is its rate of decay?

❿ Polonium-218 decays at a rate of 20.0% per minute. What is its half-life?

2-E Sketches of Functions

asymptote

❶ Sketch a power function $f(x) = x^n$.

❶ Sketch $f(x) = x^3$.

❷ Sketch a root function $f(x) = \sqrt[n]{x}$.

❶ Sketch $f(x) = \sqrt[3]{x}$.

❸ Sketch an increasing exponential function $f(x) = b^x$.

❹ Sketch a decreasing exponential function $f(x) = b^x$.

❺ Sketch an increasing logarithmic function $f(x) = \log_b x$.

❻ Sketch a decreasing logarithmic function $f(x) = \log_b x$.

2-F Transformations

transformation • pre-image • image • translation • stretch • reflection

❶ Translate a function h units right and k units up.

❶ Translate the pre-image $f(x) = 2x^2 - 5x - 4$ three units left and five units up.

❷ Stretch a function horizontally by a factor of b and vertically by a factor of a.

❷ Stretch the pre-image $f(x) = 2 + \sin x$ by a factor of 2 vertically (twice as tall) and by a factor of $\frac{1}{3}$ horizontally (one third as wide).

❸ Reflect a function across the *y*-axis and/or the across the *x*-axis.

❸ Reflect the pre-image $f(x) = x^2 + 3x - 2$ across the stated axis.

a) the *y*-axis

b) the *x*-axis

❹ Apply multiple transformations to a function.

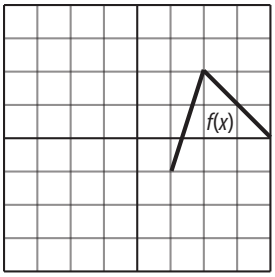
❹ Transform the pre-image $f(x) = x^2 - 6x + 9$ by doing the following.

a) Reflect it across the x-axis, and then translate it up four units.

b) Translate it up four units, and then reflect it across the x-axis.

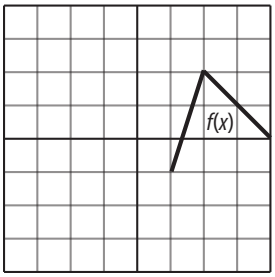
❺ Given the graph of $f(x)$, sketch $f(x - h) + k$.

❺ Given $f(x)$ below, graph $f(x + 3) + 1$.



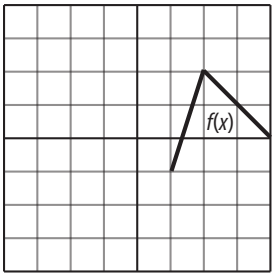
❻ Given the graph of $f(x)$, sketch $a \cdot f\left(\frac{x}{b}\right)$.

❻ Given $f(x)$ below, graph $2f(2x)$.



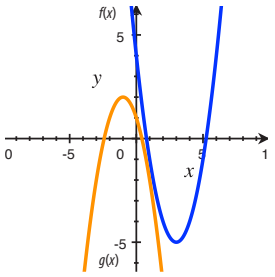
❼ Given the graph of $f(x)$, sketch $f(-x)$ or $-f(x)$.

❼ Given $f(x)$ below, graph $f(-x)$ and $-f(x)$.



❽ Use the equation of a pre-image to find the equation of a graph.

❽ Given $f(x) = x^2 - 6x + 4$, write an equation for $g(x)$ below.



CHAPTER THREE: QUADRATICS — Due Thursday, December 8

The most common functions in math at this level are quadratic functions, whose graphs are parabolas. Important in this chapter and in future math is the relationship between x-intercepts, zeros, roots, and solutions. Roughly speaking, all of these refer to values which make y equal zero. For quadratic equations, they can be found by graphing, factoring, completing the square, or using the quadratic formula. Sometimes they are imaginary rather than real, such as in the equation $x^2 = -1$ which has as solutions the imaginary numbers $x = \pm\sqrt{-1}$. This happens when the parabola is completely above or below the x-axis and never crosses it, resulting in no x-intercepts.

3-A Graphs of Quadratic Functions

standard form • vertex form • vertex • axis of symmetry • maximum • minimum

❶ Identify the vertex of a quadratic in vertex form.

❶ a) $y = \frac{1}{8}(x-2)^2 + 3$ b) $y = \frac{1}{8}(x+2)^2 - 3$

❷ Write a quadratic equation in standard form.

❷ $2x^2 = 3(x+5)^2 + 100$

❸ Identify the vertex of a parabola in standard form.

❸ $f(x) = 2x^2 - 10x + 7$

❹ Identify the vertex of a parabola in intercept form.

❹ $f(x) = \frac{1}{5}(x-4)(x+6)$

❺ Sketch a parabola and its axis of symmetry.

❺ $y = -\frac{1}{6}(x+2)^2 + 5$

3-B Simplifying Radical Expressions

radicand • radical

❶ Simplify a square root.

❶ $\sqrt{75x^2y^6z^7}$

❷ Simplify an n^{th} root.

❷ $\sqrt[3]{800x^3y^6z^8}$

❸ Rationalize a denominator with one or two terms.

❸ a) $\frac{12}{\sqrt{20}}$

b) $\frac{12}{8-\sqrt{20}}$

3-C Complex Numbers

imaginary number • complex number • complex plane • complex conjugate

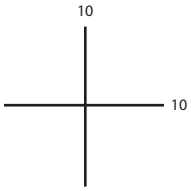
❶ Write a fractional complex number in standard form.

❶ Write $\frac{-10+14i}{9}$ in standard form.

❷ Plot numbers on the complex plane.

❷ a) $5 - 2i$ b) -2

c) $4i$ d) $\frac{-15+11i}{2}$



❸ Solve equations with complex solutions.

❸ $2x^2 + 91 = 3$

❹ Add, subtract, and multiply complex numbers.

❹ a) $(5 + 2i) - (8 - 3i)$ b) $(5 + 2i)(8 - 3i)$

❺ Divide complex numbers.

❺ $\frac{3+4i}{2-10i}$

3-D Factoring

factoring • common monomial

❶ Factor a common monomial out of each term of a polynomial.

❶ $40x^5 - 8x^3 + 20x^2$

❷ Factor a polynomial by grouping.

❷ $2x^3 - 8x^2 + 5x - 20$

❸ Factor $ax^2 + bx + c$ by grouping.

❸ $12x^2 - 4x - 5$

❹ Factor a perfect square trinomial, a difference of two squares, a difference of two cubes, or a sum of two cubes.

❹ a) $x^2 - 100y^2$ b) $x^{10} - 100y^4$

c) $x^3 - 1000y^3$ d) $27m^3 + 8p^3$

e) $4x^2 - 20x + 25$

❺ Factor any polynomial.

❺ $80x^7 - 180x^5$

3-E Solving Quadratic Equations

completing the square • quadratic formula • solution • zero • root • x-intercept

❶ Solve a quadratic equation by factoring.

❶ $18x^3 = 50x$

❷ Solve a quadratic equation by isolating a square.

❷ $8(2x + 3)^2 - 80 = 120$

❸ Solve a quadratic equation by completing the square.

❸ $2x^2 + 20x + 34 = 16$

❹ Solve a quadratic equation with the quadratic formula.

❹ $12x^2 + 17x = 7$

❺ Solve any quadratic equation.

❺ $x^2 + 10x + 30 = 9$

❻ Find the solutions, zeros, roots, or x-intercepts of a quadratic.

❻ Fill in the blanks.

a) The solutions to $x^2 + 10x + 21 = 0$ are _____.

b) The zeros or roots of $f(x) = x^2 + 10x + 21$ are _____.

c) The x-intercepts of $f(x) = x^2 + 10x + 21$ are _____.

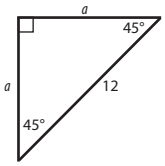
CHAPTER FOUR: RIGHT TRIANGLES — Due Friday, December 9

Other than circles, triangles are the most fundamental shape. Many aspects of advanced abstract mathematics and practical applications are based on properties of triangles. In particular, the field of trigonometry is founded on relationships between side lengths of right triangles.

4-A Special Right Triangles

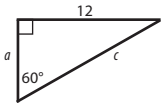
❶ Find unknown lengths in a 45° right triangle.

❶



❷ Find unknown lengths in a 30° right triangle.

❷



❸ Solve problems by identifying 30° right triangles or 45° right triangles.

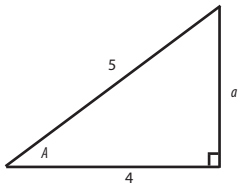
❸ What is the area of a square with a diagonal of 6?

4-B Trigonometric Functions

trigonometric function • sine • cosine • tangent • cosecant • secant • cotangent

❶ Find the values of each of the six trigonometric functions of a nonright angle in a right triangle with two known sides.

❶ Find the sine, cosine, tangent, cosecant, secant, and cotangent of angle *A* shown below.

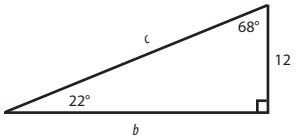


❷ Find values of cotangent, secant, and cosecant on the calculator.

❷ Evaluate sec 25°.

❸ Calculate a side length in a right triangle based on a known angle and known side length.

❸ Solve for *c* in the triangle at right.

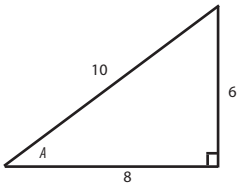


4-C Inverse Trigonometric Functions

inverse trigonometric function • $\sin^{-1}$ (arcsin) • $\cos^{-1}$ (arccos) • $\tan^{-1}$ (arctan)

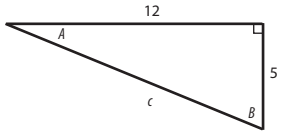
❶ Calculate an angle measure in a right triangle based on two known side lengths.

❶ Solve for *A* in the triangle at right.



❷ Solve a right triangle.

❷ a)



b)