# CHAPTER FOUR: RIGHT TRIANGLE TRIGONOMETRY — Test: Thursday • 11/21

*Other than circles, triangles are the most fundamental shape. Many aspects of advanced abstract mathematics and practical applications are based on properties of triangles. In particular, the field of trigonometry is founded on relationships between side lengths of right triangles.*

## 4-A Special Right Triangles — Monday • 11/4

❶ Find unknown lengths in a 45° right triangle.

❷ Find unknown lengths in a 30° right triangle.

❸ Solve problems by identifying 30° right triangles or 45° right triangles.

## 4-B Trigonometric Functions in Right Triangles — Wednesday • 11/6

trigonometric function • sine • cosine • tangent • cosecant • secant • cotangent

❶ Find the values of each of the six trigonometric functions of a nonright angle in a right triangle with two known sides.

❷ Find values of cotangent, secant, and cosecant on the calculator.

❸ Calculate a side length in a right triangle based on a known angle and known side length.

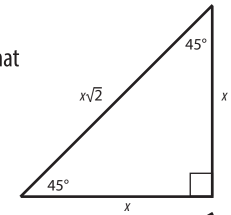
## 4-C Inverse Trigonometric Functions in Right Triangles — Tuesday • 11/12

inverse trigonometric function • $\sin^{-1}$ (arcsin) • $\cos^{-1}$ (arccos) • $\tan^{-1}$ (arctan)

❶ Calculate an angle measure in a right triangle based on two known side lengths.

❷ Solve a right triangle.

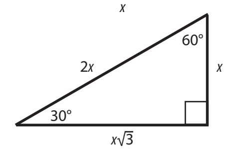
## 4-A Special Right Triangles




By the Pythagorean theorem, in an isosceles right triangle with leg length $a$ and hypotenuse length $c$, $a^2 + a^2 = c^2$. Therefore, $c = \sqrt{2a^2}$, that is, $c = a\sqrt{2}$. In other words, in a 45° right triangle, the hypotenuse must be $\sqrt{2}$ times the length of either leg.

❶ Find unknown lengths in a 45° right triangle.

1. If a leg length is known, multiply it by $\sqrt{2}$ to find the hypotenuse
   If the hypotenuse is known, divide it by $\sqrt{2}$ to find the leg lengths.

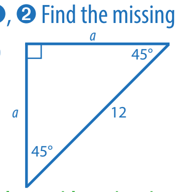



If an equilateral triangle with side length is cut in half by its altitude, making a pair of 30° right triangles, the hypotenuse is twice as long as the shorter leg: $c = 2a$. Thus, by the Pythagorean theorem, $a^2 + b^2 = (2a)^2$, making the longer leg $b = \sqrt{3a^2}$, that is, $b = a\sqrt{3}$.

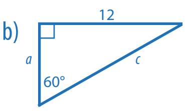
❷ Find unknown lengths in a 30° right triangle.

1. If the length of the shorter leg is known, multiply it by $\sqrt{3}$ to find the length of the longer leg, and multiply it by 2 to find the hypotenuse.
   If the length of the longer leg is known, divide it by $\sqrt{3}$ to find the length of the shorter leg, and then multiply the length of this leg by 2 to find the hypotenuse.
   If the hypotenuse is known, divide it by 2 to find the length of the shorter leg, and then multiply the length of this leg by $\sqrt{3}$ to find the length of the longer leg.

❶, ❷ Find the missing lengths.

a) 


1. $a = \frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$

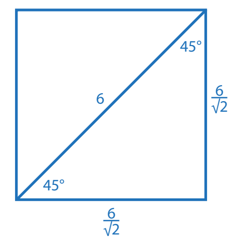
b) 


1. $a = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$
   $c = 2 \cdot 4\sqrt{3} = 8\sqrt{3}$

❸ Solve problems by identifying 30° right triangles or 45° right triangles.

1. Sketch the situation, drawing additional sides if necessary.
2. Label angles that are 30°, 45°, 60°, or 90°.
3. Use the ratios above to label the side lengths.
4. Use the side lengths to calculate the answer.

❸ What is the area of a square with a diagonal of 6?




4. area $= \frac{6}{\sqrt{2}} \cdot \frac{6}{\sqrt{2}} = \frac{36}{\sqrt{4}} = 18$

## 4-B Trigonometric Functions in Right Triangles

The three primary TRIGONOMETRIC Functions are SINE (sin), COSINE (cos), and TANGENT (tan). They show the ratio of the length of one side to another in a right triangle.

Both of the acute angles in any right triangle are comprised of the *hypotenuse* and one leg, called the *adjacent* leg. The leg that is not part of the angle is called the *opposite* leg.

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

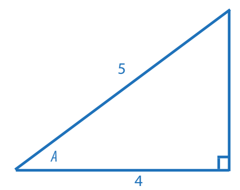
The three reciprocal trigonometric functions are COSECANT (csc), SECANT (sec), and COTANGENT (cot). They are the reciprocals of sine, cosine, and tangent, respectively.

$\frac{1}{\sin A} = \csc A = \frac{\text{hypotenuse}}{\text{opposite}}$ $\frac{1}{\cos A} = \sec A = \frac{\text{hypotenuse}}{\text{adjacent}}$ $\frac{1}{\tan A} = \cot A = \frac{\text{adjacent}}{\text{opposite}}$

❶ Find the values of each of the six trigonometric functions of a nonright angle in a right triangle with two known sides.

1. Use the Pythagorean theorem $a^2 + b^2 = c^2$ to find the length of the third side.
2. Identify which length is opposite the given angle, which is adjacent to it, and which is the hypotenuse.
3. For each trig function, use the two appropriate lengths in a fraction based on the equations above.

❶ Find the sine, cosine, tangent, cosecant, secant, and cotangent of angle $A$ shown at right.




1. $a^2 + 4^2 = 5^2$
   $a = \sqrt{9} = 3$
2. opposite = 3, adjacent = 4, hypotenuse = 5
3. $\sin A = \frac{3}{5}$ $\cos A = \frac{4}{5}$ $\tan A = \frac{3}{4}$
   $\csc A = \frac{5}{3}$ $\sec A = \frac{5}{4}$ $\cot A = \frac{4}{3}$

Most calculators do not have buttons for the reciprocal trigonometric functions.

❷ Find values of cotangent, secant, and cosecant on the calculator.

1. Type `1/` and then the reciprocal function.

❷ Evaluate sec 25°.

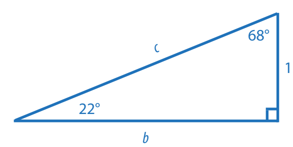
1. $\sec 25° =$ `1/cos(25)` $\approx 1.10$

A trigonometric function can be used to calculate an unknown side length from a known angle and known side length in a right triangle.

❸ Calculate a side length in a right triangle based on a known angle and known side length.

1. Write a sine, cosine, or tangent ratio involving the known side, the unknown side, and either nonright angle.
2. Solve for the unknown side.

❸ Solve for $c$ in the triangle at right.




1. $\sin 22° = \frac{12}{c}$
2. $c \sin 22° = 12$
3. $c = \frac{12}{\sin 22°} \approx 32.0$

We could have also started with $\cos 68° = \frac{12}{c}$, $\tan 68° = \frac{b}{12}$, or $\tan 22° = \frac{12}{b}$, but $\sin 68° = \frac{b}{c}$ and $\cos 22° = \frac{b}{c}$ cannot be solved because these equations have two unknowns. The simplest of these to solve is $\tan 68° = \frac{b}{12}$, because the variable is not in the denominator.

## 4-C Inverse Trigonometric Functions in Right Triangles

Whereas sin, cos, and tan take an angle and find a ratio for it, the INVERSE Trigonometric Functions $SIN^{-1}$, $COS^{-1}$, and $TAN^{-1}$ take a ratio and find an angle for it. For example, $\sin(30°) = ½$ and $\sin^{-1}(½) = 30°$.

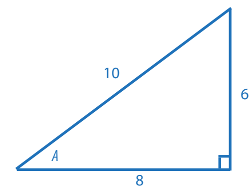
Given an acute angle $A$ in a right triangle, $\sin^{-1}(\sin A) = A$, $\cos^{-1}(\cos A) = A$, and $\tan^{-1}(\tan A) = A$.

$\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ are also called ARCSIN, ARCCOS, and ARCTAN.

❶ Calculate an angle measure in a right triangle based on two known side lengths.

1. Write a sine, cosine, or tangent ratio involving the angle and the two known sides.
2. Solve for the angle by applying $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ to each side of the equation.

❶ Solve for $A$ in the triangle at right.




1. $\sin A = \frac{6}{10}$
2. $\sin^{-1}(\sin A) = \sin^{-1}\left(\frac{6}{10}\right)$
3. $A \approx 36.9°$

We could have also started with $\cos A = \frac{8}{10}$ or $\tan A = \frac{6}{8}$.

To solve a triangle is to find every unknown side and angle.

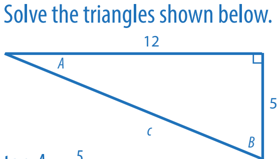
Angles are labeled with capital letters. Each side is labeled with the same letter as the angle opposite it, but lowercase.

❷ Solve a right triangle.

1. If both angles are unknown, solve for one of them using $\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$ (see ❶).
2. Find the second angle by subtracting the first from 90°.
3. If two sides are unknown, solve for one of them using sin, cos, or tan (see 4-B).
4. Find the last side by using sin, cos, or tan (see 4-B), or by using the Pythagorean theorem.

❷ Solve the triangles shown below.

a)




1. $\tan A = \frac{5}{12}$

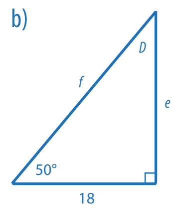
   $\tan^{-1}(\tan A) = \tan^{-1}\left(\frac{5}{12}\right)$

   $A \approx 22.6°$
2. $B \approx 90° - 22.6° = 67.4°$
3. 
4. $\sin 22.6° = \frac{5}{c}$

   $c \sin 22.6° = 5$

   $c = \frac{5}{\sin 22.6°} = 13.0$

b)




$D = 90° - 50° = 40°$

$\tan 50° = \frac{e}{18}$

$18 \tan 50° = e \approx 21.5$

$\cos 50° = \frac{e}{18}$

$f \cos 50° = 18$

$f = \frac{18}{\cos 50°} \approx 28.0$