CHAPTER ONE: FUNDAMENTALS — Test: Thursday, September 5

Thorough understanding and fluency of the concepts and methods of arithmetic and algebra, covered in this chapter and the next, is a cornerstone to success in this course and most future math courses as well. This chapter focuses on the fundamental concepts of numbers (including fractions, decimals, and negatives), exponents, and functions.

1-A Fractions and Decimals — Thursday • 8/8

scientific notation

❶ Reduce a fraction.
❷ Multiply or divide a fraction.
❸ Add or subtract fractions with the same denominator.
❹ Add or subtract fractions with different denominators.
❺ Divide zero or by zero.
❻ Convert a percentage to a decimal.
❼ Convert calculator notation to scientific notation and to standard notation.

1-B Order of Operations and Negatives — Monday • 8/12

sign

❶ Subtract, multiply, or divide by a negative.
❷ Apply order of operations for negatives and parentheses.
❸ Apply order of operations for basic arithmetic and powers.
❹ Use a calculator to evaluate a fraction.

1-C Properties of Exponents — Tuesday • 8/13

❶ Simplify an expression using properties of exponents.
❷ Rewrite an expression without negative exponents.

1-D Functions and their Graphs — Monday • 8/19

x-axis • *y*-axis • origin • coordinates • domain • range • argument • function

❶ Estimate coordinates on a graph.
❷ Use function notation.
❸ Sketch a function by plotting points.
❹ Identify the domain and range of a graphed function.
❺ Read expressions in function notation, and identify the arguments.
❻ Identify the domain of a function in function notation.
❼ Identify whether or not a relation is a function.
❽ Identify whether or not a graph represents a function.

1-E Terms versus Factors — Wednesday • 8/21

conjugate

❶ Identify terms and factors of an expression.
❷ Identify a coefficient.
❸ Add or subtract expressions.
❹ Classify a polynomial in one variable.
❺ Multiply a polynomial by a monomial.
❻ Distribute a monomial
❼ Simplify a fraction with multiple terms in the numerator or denominator.
❽ Take a power or root of an expression.

1-F Polynomial Multiplication Monday • 8/26

term • factor • coefficient • like terms • monomial • degree • constant • linear • quadratic • cubic • polynomial • binomial • trinomial • standard form • leading coefficient • distribute

❶ Multiply two polynomials.
❷ Multiply more than two polynomials.
❸ Multiply a binomial by its conjugate.
❹ Square a binomial.

1-G Solving Equations Tuesday • 8/27

equation • inverse • solution • solve • simplify • round

❶ Identify an equation.
❷ Apply an operation to both sides of an equation.
❸ Identify notation errors in applying an operation to both sides of an equation.
❹ Identify the inverse of an operation by definition.
❺ Apply inverses to solve an equation.
❻ Express a numerical answer.

1-A Fractions and Decimals

❶ Reduce a fraction.

1. Divide the numerator and denominator by a number that divides evenly into both.
2. Repeat step 1 until no number divides evenly into both the numerator and the denominator.

❶ Reduce $\frac{140}{350}$.

1. 10 divides evenly into 140 and into 350. $140 \div 10 = 14$, and $350 \div 10 = 35$. $\frac{140}{350} = \frac{14}{35}$
2. 7 divides evenly into 14 and into 35. $14 \div 7 = 2$, and $35 \div 7 = 5$. $\frac{14}{35} = \frac{2}{5}$

It also would have worked to divide 140 and 350 both by 70 in step 1 instead of doing two separate steps.

❷ Multiply or divide a fraction.

1. If multiplying or dividing by a whole number n, rewrite the number as $\frac{n}{1}$.
2. If dividing, change the divisor (the second number) to its reciprocal. The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$, and the reciprocal of the whole number a is $\frac{1}{a}$.
3. Multiply the numerators together.
4. Multiply the denominators together.

❷ a) $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$ b) $\frac{2}{5} \times 3 = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$ c) $\frac{2}{5} \div \frac{4}{3} = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20}$ d) $\frac{2}{5} \div 3 = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

❸ Add or subtract fractions with the same denominator.

1. Add or subtract the numerators.
2. Keep the same denominator.

❸ $\frac{12}{20} - \frac{5}{20} = \frac{7}{20}$

❹ Add or subtract fractions with different denominators.

1. Multiply the first fraction by $\frac{b}{b}$, where b is the denominator of the second fraction.
2. Multiply the second fraction by $\frac{a}{a}$, where a is the denominator of the first fraction.
3. Add or subtract (see ❹).

❹ $\frac{3}{5} - \frac{1}{4}$

1. $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$
2. $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$
3. $\frac{12}{20} - \frac{5}{20} = \frac{7}{20}$

It is impossible to divide by zero. Attempting to do so yields a result of "undefined". Zero divided by any other number is zero.

❺ Divide zero or by zero.

1. Anything divided by zero is undefined (there is no answer).
2. Zero divided by anything other than itself is equal to zero.

❺ a) $\frac{2}{0}$ is undefined b) $\frac{0}{2} = 0$ c) $\frac{0}{0}$ is undefined

x% means $\frac{x}{100}$. 100% is the same as $\frac{100}{100}$ or 1.

❻ Convert a percentage to a decimal.

1. Move the decimal point two places to the left, and remove the % symbol.

❻ a) 9% = .09 b) .9% = .009

SCIENTIFIC NOTATION is $a \times 10^b$, where $1 \le a < 10$ and b is an integer.

Many calculators use the notation `aEb` instead of $a \times 10^b$ to display scientific notation. Do not write numbers in calculator notation.

❼ Convert calculator notation to scientific notation and to standard notation.

1. To convert `aEb` to scientific notation, change "E" into "× 10", and make b an exponent.
2. To convert $a \times 10^b$ to standard notation, move the decimal point right b spaces (which will be left if b is negative). Fill in 0's as needed.

❼ a) `2.57E3` b) `2.57E-3`

1. 2.57×10^3 2.57×10^{-3}
2. 2570 .00257

1-B Order of Operations and Negatives

The SIGN of a real number is one of three things: positive, negative, or zero.

"Changing the sign" of a number generally refers to making a positive number negative or vice versa.

Multiplying or dividing a nonzero number by a negative number changes the sign of the original number.

❶ Subtract, multiply, or divide by a negative.

1. Multiplying or dividing by a negative number changes the sign.
2. Subtracting a negative number is the same as adding the positive version of the number.

❶ a) $20 \times (-10) = -200$ b) $20 \div (-10) = -2$ c) $-20 \div (-10) = 2$ d) $20 - (-10) = 30$

❷ Apply order of operations for negatives and parentheses.

1. A negative number to an odd power stays negative.
2. A negative number to an even power becomes positive.
3. If a negative sign is not in parentheses, there is only one of them. This means multiply a positive number by negative one: $-5^n = -1(5^n)$, which is always negative. If a negative sign is in parentheses, there are n of them, where n is the exponent: $(-5)^n = (-1)^n(5)^n$. This is positive if n is even.

❷ a) $-2^3 = -8$ b) $(-2)^3 = -8$ c) $-2^4 = -16$ d) $(-2)^4 = 16$

When evaluating an expression with no parentheses, exponents are done first, followed by multiplication and division, and then addition and subtraction.

When evaluating an expression with parentheses in it, each expression within parentheses is evaluated on its own first, following the order above, before the overall expression is evaluated.

Numerators and denominators always have parentheses around them, even if they are not written. For example, $\frac{1+3}{3+5}$ is actually $\frac{(1+3)}{(3+5)}$. These parentheses only have to be shown if the numerator is not written above the denominator. For example, on a calculator, $\frac{1+3}{3+5}$ can be typed as $(1 + 3) / (3 + 5)$, but not as $1 + 3 / 3 + 5$.

❸ Apply order of operations for basic arithmetic and powers.

1. Apply the steps below to everything within parentheses, including hidden parentheses.
2. Calculate exponents.
3. Calculate multiplication and division, including multiplying by -1 for negative signs.
4. Calculate addition and subtraction.

❸ a) $4 + 5 \times 2$
$4 + 10$
14

b) $4 + 5(2)^3$
$4 + 5(8)$
$4 + 40$
44

c) $-5(-2)^3 + 3$
$-5(-8) + 3$
$40 + 3$
43

❹ Use a calculator to evaluate a fraction.

1. Put parentheses around the numerator, and put parentheses around the denominator.
2. Close any parentheses thar are opened, such as for the argument of a square root function.
3. Use the minus button – for subraction and the negative button (-) for a negative number.

❹ $\frac{-8+2\sqrt{100}}{\sqrt{49}-5}$

(-8 + 2(√(100))/(√(49)–5) = 6

1-C Properties of Exponents

The rules below are valid in almost all contexts, including any time a and b are positive or x and y are integers.

Property	Rule	Example
Power of a Product	$(ab)^x = a^x b^x$	$(2x)^3 = 2^3x^3 = 8x^3$
Power of a Quotient	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	$\left(\frac{x}{2}\right)^3 = \frac{x^3}{2^3} = \frac{x^3}{8}$
Power of a Power	$(b^x)^y = b^{xy}$	$(x^5)^3 = x^{15}$
Product of Powers	$b^x b^y = b^{x+y}$	$x^5x^3 = x^8$
Quotient of Powers	$\frac{b^x}{b^y} = b^{x-y}$	$\frac{x^5}{x^3} = x^2$
Zero Exponent	$b^0 = 1$	$2^0 = 1$
Negative Exponent	$b^{-x} = \frac{1}{b^x}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

❶ Simplify an expression using properties of exponents.

1. Use the properties above as needed.
2. Reduce fractions if needed.

❶ Simplify.

a) $a^6a^3 = a^9$	b) $(a^6)^3 = a^{18}$	c) $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$	d) $(ab)^6 = a^6b^6$
e) $a^0 = 1$	f) $\frac{a^6}{a^3} = a^3$	g) $\frac{a^3}{a^6} = \frac{1}{a^3}$	h) $a^{-3} = \frac{1}{a^3}$
i) $ab^3(a^2b)^5$ $ab^3(a^{10}b^5)$ $a^{11}b^8$	j) $\left(\frac{5a}{2}\right)^3$ $\frac{5^3a^3}{2^3}$ $\frac{125a^3}{8}$	k) $\left(\frac{5a}{2}\right)^{-3}$ $\left(\frac{2}{5a}\right)^3$ $\frac{8}{125a^3}$	l) $10\left(\frac{5a}{2}\right)^{-3}$ $10\left(\frac{8}{125a^3}\right)$ $\frac{80}{125a^3}$

❶ Simplify $\frac{(2a^4b)^3b^6}{a^{12}bc^2}$, and write it without a fraction. State each property of exponents used.

$(2a^4b)^3 = 2^3(a^4)^3b^3$	Power of a Product	$\frac{8(a^4)^3b^3b^6}{a^{12}bc^2}$
$(a^4)^3 = a^{12}$	Power of a Power	$\frac{8a^{12}b^3b^6}{a^{12}bc^2}$
$b^3b^6 = b^9$	Product of Powers	$\frac{8a^{12}b^9}{a^{12}bc^2}$
$\frac{a^{12}b^9}{a^{12}b} = a^0b^8$	Quotient of Powers	$\frac{8a^0b^8}{c^2}$
$a^0 = 1$	Zero Exponent	$\frac{8(1)b^8}{c^2}$
$\frac{1}{c^2} = c^{-2}$	Negative Exponent	$8b^8c^{-2}$

A factor to the power of -1 is the reciprocal of that factor. This means moving the numerator to the denominator and vice versa.

❷ Rewrite an expression without negative exponents.

1. If a factor in the numerator has a negative exponent, move the factor to the denominator and change the exponent to positive.
2. If a factor in the denominator has a negative exponent, move the factor to the numerator and change the exponent to positive.
3. If a fraction has a negative exponent, take the reciprocal of the fraction (switch the numerator with the denominator), and change the exponent to positive.

Make sure not to move any factors that do not have a negative exponent, such as the 4 in $4x^{-1}$.

❷ Rewrite with no parentheses or negative exponents.

a) $\frac{a^{-2}b}{c^3d}$ $\frac{b}{a^2c^3d}$	b) $\frac{a^2b}{c^{-3}d}$ $\frac{a^2c^3b}{d}$	c) $\frac{ab^{-2}}{c^3d}$ $\frac{a}{b^2c^3d}$	d) $\frac{(ab)^{-2}}{c^3d}$ $\frac{1}{a^2b^2c^3d}$
e) $\left(\frac{a^2b}{c^3d}\right)^{-2}$ $\left(\frac{c^3d}{a^2b}\right)^2$ $\frac{c^6d^2}{a^4b^2}$	f) $\left(\frac{a^{-2}b}{c^3d}\right)^{-2}$ $\left(\frac{c^3d}{a^{-2}b}\right)^2$ $\left(\frac{a^2c^3d}{b}\right)^2$ $\frac{a^4c^6d^2}{b^2}$	g) $(2a)^{-1}$ $\frac{1}{2a}$	h) $2a^{-1}$ $\frac{2}{a}$

1-D Functions and their Graphs **Thursday • 8/18**

The *X*-AXIS is the horizontal line where y is zero. It is the same as the number line.

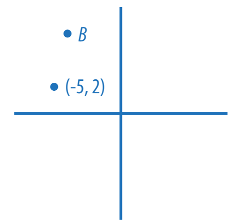
The *Y*-AXIS is the vertical line where x is zero.

The ORIGIN is where the x-axis and y-axis meet, which is the point (0, 0).

The x and y values of a point on a graph are the point's COORDINATES and are listed alphabetically in parentheses: (x, y).

❶ Estimate coordinates on a graph.

1. Negative x values are left of the y-axis, and positive x-values are right of the y-axis.
2. Negative y-values are below the x-axis, and positive y-values are above the x-axis.
3. The closer a coordinate is to zero, the closer the point is to the origin.



❶ Estimate the coordinates of point B at right.

1. It is left of the y-axis, so x is negative.
2. It is above the x-axis, so y is positive.
3. x is approximately -5 but a little closer to zero, and y is much higher than 2.

(-4, 6) is a good estimate.

Functions are written in the form $f(x)$, where x is the independent variable variable (the input) and $f(x)$ is the dependent variable (the output). Letters other than f can be used as well, especially to distinguish between different functions. Variables other than x can be used as well.

A formula for $f(x)$ is given in terms of x, such as $f(x) = 2x$. Values of $f(x)$ can be found by using different values of x in the formula.

In addition to numbers, algebraic expressions can be used in functions.

❷ Use function notation.

1. Substitute a value of x (in parentheses) for each x in the formula.
2. Simplify.

❷ Given $f(x) = 5x$ and $g(x) = x^2 + 2x - 10$, find the following.

a) $f(4) = 5(4) = 20$ b) $g(4) = (4)^2 + 2(4) - 10 = 14$ c) $f(x + 2) = 5(x + 2) = 5x + 10$

On a graph, $f(x)$ represents y. A graph can be sketched by calculating the value of $f(x)$ for each of several values of x and connecting the points.

❸ Sketch a function by plotting points.

1. Choose a value of x and calculate the value of $f(x)$ for this x value.
2. Plot the point $(x, f(x))$ on the graph.
3. Repeat steps 1 and 2 until the shape of the graph starts taking place. For best results, make sure to include the highest and lowest points on the graph (if there are any) and points on each side of them.
4. Connect the points in a curve.

❸ $f(x) = x^2 - 2x - 3$

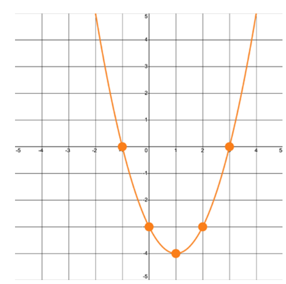
$f(0) = (0)^2 - 2(0) - 3 = -3$

$f(1) = (1)^2 - 2(1) - 3 = -4$

$f(2) = (2)^2 - 2(2) - 3 = -3$

$f(3) = (3)^2 - 2(3) - 3 = 0$

$f(-1) = (-1)^2 - 2(-1) - 3 = 0$



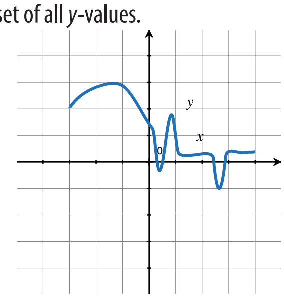
The set of all possible values that can be used as input for a function is called the DOMAIN of the function. On a graph, the domain is the set of all x-values.

The set of all possible values that could be an output of a function is called the RANGE of the function. On a graph, the range is the set of all y-values.

❹ Identify the domain and range of a graphed function.

1. The domain is all x values the graph reaches.
2. The range is all y values the graph reaches.

❹ Identify the domain and range of the function graphed at right.



1. The domain is approximately $-3 \le x \le 4$.
2. The range is approximately $-1 \le y \le 3$.

An ARGUMENT of a Function is an expression input into the function.

Arguments are in parentheses, although in some cases the parentheses do not need to be written, such as in a square root function. These parentheses mean "of", but they do not indicate multiplication.

❺ Read expressions in function notation, and identify the arguments.

1. The arguments are the expressions inside the functions, and are usually in parentheses.
2. Read it as *function "of" argument*, not as multiplication.

❺ Read the notation $f(x) = \sqrt{x+1}$, and identify the arguments.

x is the argument of function f, and $x + 1$ is the argument of the square root function.

f of x equals the square root of $x + 1$.

Many functions can have any real number as their arguments, but some functions result in error for certain inputs. Numbers that would cause an error in a given function are not part of the domain of that function. In Math 2, this only occurs when dividing by zero or taking the square root of a negative, but in more advanced contexts it can happen for other reasons.

Some functions take nonnumerical arguments.

❻ Identify the domain of a function in function notation.

1. If the function takes numerical input, the domain is all real numbers except in specific situations such as:
   - If there is a denominator D, the domain is limited by $D \neq 0$.
   - If there is a radical $\sqrt{R}$, the domain is limited by $R \ge 0$.
   - If there is a logarithm $\log L$, the domain is limited by $L > 0$.
2. If the function is not numerical, the domain is all arguments that make sense in the context of the problem.

❻ a) $a(x) = 6x^2 - x + 4$ b) $b(x) = \frac{x+8}{2x-6}$ c) $c(x) = 4\sqrt{2x-6}$ d) $d(x) = x$'s student ID number

a)	b)	c)	d)
	$2x - 6 \neq 0$	$2x - 6 \ge 0$	
x can be any real number	$x \neq 3$	$x \ge 3$	x can be any student

The definition of a FUNCTION f is a relation such that $f(x)$ is equal to one specific value (if anything) for any single value of x. By definition, it is impossible to get two different outputs for one function input. If a relation has more than one output for any one input, it is not a function.

❼ Identify whether or not a relation is a function.

1. It is a function unless there exists a possible input that results in more than one output. In other words, in a function, there is only one answer to any question.

❼ State whether or not the following relations are functions. For ones that are not functions, write an x value and two different y values for that x value.

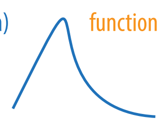
a) $y = 5x^2 + 9x - 10$	b) $y = \pm\sqrt{x}$	c) $y =$ age of person x	d) $y =$ brother of person x
function	not a function: $y = 3$ or -3 for $x = 9$	function	not a function: $y =$ Jon or Ed for $x =$ Mia

The graph of a function cannot have two different points with the same y value, since that would mean there are two different values of $f(x)$ for that value of x.

❽ Identify whether or not a graph represents a function.

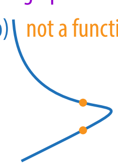
1. It is a function unless there is a point on the graph that is directly above another point on the graph.

❽ a)



function

b)



not a function

1-E Terms versus Factors

A TERM is the product of one or more factors. The terms of an expression are the components of the expression that equal the whole expression when added together.

A FACTOR is the sum of one or more terms. The factors of an expression are the components of the expression that equal the whole expression when multiplied together.

❶ Identify terms and factors of an expression.

1. Components of an expression separated by addition are terms. Addition can be notated by "+" or by "–" (because "–" means add the opposite).
2. Components of an expression separated by multiplication are factors. Multiplication can be shown in many ways, including ×, •, parentheses, or no symbol.
3. If an expression can be rewritten as an expression multiplied by one or more other expressions, each of these expressions are factors of the original expression (See chapter 5).
4. A term can consist of more than one factor, and a factor can consist of more than one term.

❶ List the factors of the expression $5x^2(3x+4)(x^2-9x+2)$, and state how many terms each factor has.

2. The factors are $5x^2$, $(3x+4)$, and (x^2-9x+2).
4. $5x^2$ is one term, $(3x+4)$ is the two terms $3x$ and 4, and (x^2-9x+2) is the the three terms x^2, $-9x$, and 2.

❶ State the terms and the factors of the expression $5a-5b$.

1. The terms are $5a$ and $-5b$, because $5a+-5b$ is equal to $5a-5b$.
3. The factors are 5 and $(a-b)$, because $5\times(a-b)$ is equal to $5a-5b$.

The COEFFICIENT of a term is the constant factor of the term. In other words, it is a number and not a variable. It is normally written at the beginning of the term.

❷ Identify a coefficient.

1. The coefficient of each term is the number that the rest of the term is multiplied by.
2. If the term is subtracted, the coefficient is negative.
3. If no number is written, then the coefficient is 1 (or -1).
4. If the term is a fraction, the coefficient is the numerical part of the fraction.
5. If the term is a constant, then it is its own coefficient.

❷ Identify the coefficients of the expression $5x^6+x^3+\frac{8x^2}{7}-9x+2$.

1. The coefficent of $5x^6$ is 5, because 5 times x^6 is $5x^6$.
3. The coefficient of x^3 is 1, because 1 times x^3 is x^3.
4. The coefficient of $\frac{8x^2}{7}$ is $\frac{8}{7}$, because $\frac{8}{7}$ times x^2 is $\frac{8x^2}{7}$.
2. The coefficient of $-9x$ is -9, because -9 times x is $-9x$.
5. The coefficient of 2 is 2, because there is no variable.

LIKE Terms are terms that are identical to each other, not including the coefficient.

❸ Add or subtract expressions.

1. Group each term with any other term that is identical, not including the coefficient.
2. Add or subtract the coefficients in each group, leaving the rest of the term the same.

❸ $(4x^2+10x-5)+(x^2+2)$

1. $(4x^2+x^2)+(10x)+(-5+2)$
2. $5x^2+10x-3$

A MONOMIAL is a nonzero coefficient multiplied by a variable to a power. The power, which can be any nonnegative whole number, is the DEGREE of the monomial.

Degree	Name	Example
0	CONSTANT	5 (that is, $5x^0$)
1	LINEAR	$5x$ (that is, $5x^1$)
2	QUADRATIC	$5x^2$
3	CUBIC	$5x^3$
n	n^{th} degree	$5x^{12}$

A POLYNOMIAL is an expression of the sum of one or more monomials, after like terms are combined. The degree of a polynomial in one variable is the highest exponent.

A BINOMIAL is a polynomial with two terms.

A TRINOMIAL is a polynomial with three terms.

A polynomial in one variable with its terms written in order from highest exponent to lowest is in STANDARD Form.

The LEADING Coefficient of a Polynomial is the coefficient of the term with the highest exponent.

❹ Classify a polynomial in one variable.

1. A single term is a monomial, two terms is a binomial, and three terms is a trinomial.
2. Consider the term with the highest exponent:

   If there is no variable at all, the polynomial is a constant.

   If there is a variable but no exponent (that is, there is an unwritten exponent of 1), the polynomial is linear.

   If the variable is to the second power, the polynomial is quadratic.

   If the variable is to the third power, the polynomial is cubic.

   If the variable is to the fourth power or higher, look up what it is called, or simply refer to the polynomial as "n^{th} degree", where n is the exponent.

❹ Classify the following polynomials, write them in standard form, and identify the leading coefficient.

a) $x + 4x^3$
cubic binomial
$4x^3 + x$
4

b) $-15x$
linear monomial
$-15x$
-15

c) $8x^2 - 2x^9 + 3$
9^{th} degree trinomial
$-2x^9 + 8x^2 + 3$
-2

d) $2 - \frac{7x^3}{5} + 6x^2 + x$
cubic polynomial
$-\frac{7x^3}{5} + 6x^2 + x + 2$
$-\frac{7}{5}$

To DISTRIBUTE a monomial is to multiply an expression by it. To do so, multiply each term in the expression by the monomial.

❺ Multiply a polynomial by a monomial.

1. Multiply each term in the expression by the monomial.

❺ $2(5x^3 - 4x + 10)$

$10x^3 - 8x + 20$

When distributing a monomial, each term is only multiplied once. This means if a term consists of multiple factors, such as $(x + 3)(2x - 1)$, only the terms in one of the factors are multiplied.

When multiplying a fraction by a monomial, only multiply the terms in the numerator (see 1-A ❷).

A function and its argument make up a single factor. The argument itself is not a factor.

❻ Distribute a monomial.

1. Identify one factor of each term of the expression. Ignore the denominator if there is one.
2. Multiply each term of these factors by the monomial.

❻ Multiply the expression $((x + 3)(2x - 1) + 4x(9x^2 + 3) + \sqrt{5})$ by 10.

1. $(x + 3)$, $4x$, and $\sqrt{5}$
2. $(10x + 30)(2x - 1) + 40x(9x^2 + 3) + 10\sqrt{5}$

❻ Identify the error, if any, in each of the following attempts to multiply $\frac{11(4x) - \sqrt{10x}}{5}$ by 2.

a) $\frac{22(4x) - \sqrt{20x}}{5}$ 10x is an argument and should not be multiplied.

b) $\frac{22(4x) - 2\sqrt{10x}}{10}$ The denominator should not be multiplied.

c) $\frac{22(8x) - 2\sqrt{10x}}{5}$ The term $11(4x)$ should be multiplied by 2 one time, but it was multiplied by 2 once on the 11 and again on the $4x$.

d $\frac{22(4x) - 2\sqrt{10x}}{5}$ This is correct, because each term in the numerator was multiplied by 2 one time.

❼ Simplify a fraction with multiple terms in the numerator or denominator.

1. Identify the terms of the numerator and the denominator. An argument by itself is not a term.
2. Identify a factor that divides evenly into all of the terms.
3. Rewrite the fraction with the factor divided out of each term. If it is variable, specify that it cannot be zero.

❼ $\frac{6x^2 - 9x\sqrt{30x}}{6x^2 - 9x}$

1. The terms are $6x^2$ and $-9x\sqrt{30x}$ in the numerator, and $6x^2$ and $-9x$ in the denominator.
2. $3x$ divides evenly into each of these terms:

   $6x^2 \div 3x = 2x$

   $-9x\sqrt{30x} \div 3x = -3\sqrt{30x}$ (not $-3\sqrt{10}$)

   $6x^2 \div 3x = 2x$

   $-9x \div 3x = -3$
3. $\frac{11(4x) - 3\sqrt{10x}}{5}$, $x \neq 0$ (We divided by $3x$, so our answer doesn't work if $3x$ is zero.)

Note that $6x^2$ in the numerator and $6x^2$ in the denominator do not cancel each other out, because there are other terms.

Unlike multiplying or dividing an expression, which results in each individual *term* being multiplied or divided, taking a power or root of an expression results in the power or root of each individual *factor*.

❽ Take a power or root of an expression.

1. Identify the factors of the expression.
2. Take the power or root of each factor. If the root is even, such as a square root, put $\pm$.

❽Simplify.

a) $10xy$ to the third power

1. The factors are 10, x, and y.
2. $1000x^3y^3$

b) the square root of $49x^8y^2$

The factors are 49, x^8, and y^2.

$\pm 7x^4y$

1-F Polynomial Multiplication

❶ Multiply two polynomials.

1. Multiply each term in one polynomial by each term in the other polynomial.
2. Combine like terms.

❶ $(4x - 3)(x + 5)$

1. $4x(x) + 4x(5) + \text{-}3(x) + \text{-}3(5)$

   $4x^2 + 20x - 3x - 15$

2. $4x^2 + 17x - 15$

❷ Multiply more than two polynomials.

1. Multiply two of the polynomials together (see ❶).
2. Multiply the result by the next polynomial.
3. Combine like terms.
4. Go back to step 2 if there is another polynomial to multiply.

❷ $(x + 2)(x + 5)(x - 10)$

1. $(x^2 + 5x + 2x + 10)(x - 10)$

   $(x^2 + 7x + 10)(x - 10)$

2. $x(x^2 + 7x + 10) - 10(x^2 + 7x + 10)$

   $x^3 + 7x^2 + 10x - 10x^2 - 70x - 100$

3. $x^3 - 3x^2 - 60x - 100$

The CONJUGATE of a Binomial is the original binomial except with the sign of one of the terms switched: The conjugate of $a + b$ is $a - b$.

A binomial can be multiplied by itself or by its conjugate the same as multiplying any two binomials (see ❶), or you can follow these patterns:

Binomial times itself (squaring a binomial): $(a + b)^2 = a^2 + 2ab + b^2$

Binomial times its conjugate: $(a + b)(a - b) = a^2 - b^2$

❸ Multiply a binomial by its conjugate.

1. Square each term of the binomial.
2. Subtract the second square from the first.

❸ $(3x - 10)(3x + 10)$

1. $a^2 = (3x)^2 = 9x^2$

   $b^2 = 10^2 = 100$

2. $(3x - 10)(3x + 10) = 9x^2 - 100$

❹ Square a binomial.

1. Square each term of the binomial.
2. Multiply the two terms together, and double this product.
3. Add the three results of the steps above.

❹ $(3x - 10)^2$

1. $a^2 = (3x)^2 = 9x^2$

   $b^2 = (\text{-}10)^2 = 100$

2. $2ab = 2(3x)(\text{-}10) = \text{-}60x$

3. $(3x - 10)^2 = 9x^2 - 60x + 100$

1-G Solving Equations

An EQUATION is one expression set equal to another expression.

An expression on one side of an equal sign must be equal to the expression on the other side.

❶ Identify an equation.

1. If there is no equal sign, there is no equation.
2. If there is one equal sign, what is written is an equation.
3. If there are multiple equal signs, each one is in the middle of an equation that includes everything up to but not including the next equal sign.

❶ a) $x + 5$ b) $x + 5 = 9 + 5$ c) $x + 5 = 9 + 5 = 14$

1. This is not an equation. 2. This is an equation. 3. $x + 5 = 9 + 5$ is an equation, and $9 + 5 = 14$ is an equation.

Any operation can be done to the expression on one side of an equation as long as it is done to the whole expression and also to the whole expression on the other side as well.

❷ Apply an operation to both sides of an equation.

1. For addition or subtraction, apply it at the end of both expressions.
2. For multiplication or division, apply it to each term of both expressions. Use parentheses around each expression if it has more than one term
3. For powers or roots, apply it to each factor of both expressions. Use parentheses around each expression if it has more than one factor.

Never include an equal sign within parentheses.

❷ Do the following to each side of the equation $x + 5 = 8$, and simplify.

a) add two
$x + 5 + 2 = 8 + 2$
$x + 7 = 10$

b) multiply by 2
$2(x + 5) = 2(8)$
$2x + 10 = 16$

c) put it to the power of 2
$(x + 5)^2 = 8^2$
$x^2 + 10x + 25 = 64$

When applying an operation to both sides of an equation, it is not necessary to show this step. However, if you do show it, the following are all incorrect: applying it only on one side, applying only to part of one or both sides, applying it to the equation as a whole, or writing it in an illogical place such as $+2\ 8$ instead of $8 + 2$.

❸ Identify notation errors in applying an operation to both sides of an equation.

1. Operations must be applied to each side, not written on only one side.
2. Symbols for addition, subtraction, multiplication, and division must be between two expressions, not before both of them.
3. An equal sign cannot be operated on. Make sure it is not within parentheses or part of a numerator, denominator, or argument.
4. Multiplying must apply to every term. Use parentheses when multiplying expressions with more than one term.
5. Division must apply to every term. Use appropriately sized fraction bars when dividing expressions with more than one term.
6. Powers must apply to every factor. Use parentheses when applying powers to expressions with more than one factor.
7. Roots must apply to every factor. Use appropriately sized square-root symbols when taking roots of expressions with more than one factor.
8. "x" should not be a variable and a multiplication sign in the same equation.

❸ Identify each error in $x4(\frac{1}{4}x = x + 1)$, and rewrite it correctly.

2. The multiplication sign should not be at the beginning.
3. The equal sign cannot be multiplied. It should not be in parentheses.
4. The whole expression $x + 1$ needs to be in parentheses.
8. The multiplication sign should not be the same as the variable.

$4(\frac{1}{4}x) = 4(x + 1)$

The INVERSE of an operation is the operation that cancels the original operation.

❹ Identify the inverse of an operation by definition.

1. The inverse of addition is subtraction.
2. The inverse of multiplication is division.
3. The inverse of a power is a root.

❹ Identify the inverse of each operation in the equation $5x - 9 = 16$.

The inverse of subtracting 9 is adding 9.

The inverse of multiplying by 5 is dividing by 5.

The SOLUTIONS of an Equation are values of the variable that make the equation true. SOLVING an Equation means finding the solutions.

Equations that only have one instance of the variable once like terms are combined (such as $8x^2 + 9 = 20$ or $8x^2 + 9 = 3x^2 + 20$) can be solved by applying one or more inverse operations to each side of the equation. These are done in reverse order of operations.

An equation can have any number of solutions. In particular, since every positive number has two square roots (the square roots of x are $\pm\sqrt{x}$), most quadratic equations have two solutions.

❺ Apply inverses to solve an equation.

1. Identify each operation that took place on the variable, in order by order of operations.
2. On each side of the equation, apply the inverse of each operation, in reverse order.

❺ $2(x-3)^2 + 8 = 40$

1. The following operations took place, in order:
   a) 3 was subtracted from x.
   b) The difference was squared.
   c) The square was multiplied by 2.
   d) 8 was added to to the product.
2. d) Subtract 8 from each side: $2(x-3)^2 = 32$
   c) Divide each side by 2: $(x-3)^2 = 16$
   b) Take the square root of each side: $x - 3 = \pm 4$
   a) Add 3 to each side: $x = 3 \pm 4$

The solutions are $x = -1$ and $x = 7$.

To SIMPLIFY an answer is to express it in its simplest form. For rational answers, this generally means writing it with no fractions or decimals in the numerator or denominator, and reducing. Simplifying irrational answers is explained in chapter 7.

To ROUND an answer is to get a decimal approximation of the answer such as by evaluating it in a calculator.

Simplified answers are the actual answers. Rounded answers are not the actual answers, but are instead approximations of them. Do not round when directions say to simplify.

❻ Express a numerical answer.

1. If the instructions are to round, then round to the indicated place, or choose an appropriate place to round to if none is specified. Write "≈" instead of "=".
2. If the instructions are to simplify, then eliminate any fractions or decimals within fractions, reduce fractions, and simplify square roots (see 7-B).
3. If the instructions do not specify whether to round or simplify, then do either one unless there is a reason in the context of the problem to do one or the other.

❻ Give an appropriate answer for each question.

a) Solve $7x = 9$. $x = \frac{9}{7}$

b) Solve $7x = 9$. Round the answer to the nearest tenth. $x \approx 1.3$

c) If a pack of seven pens costs \$9, what is the price per pen? Each pen costs \$1.29.

CHAPTER TWO: FUNCTIONS — Test: Thursday, October 5

Functions are the basic building blocks of algebra and most further math. Functions have a domain, a range, and an inverse, and they can be composed, sketched, and transformed. There are many different families of functions, a few key ones of which are explored in this chapter.

2-A Composition and Inverses — Monday • 9/11

composition • inverse • one-to-one • equation

❶ Evaluate compositions of functions.
❷ Find a simplified expression for a composition of functions.
❸ Identify the inverse of a basic function by definition.
❹ Identify the inverse of a function conceptually.
❺ Find the inverse of a relation algebraically.
❻ Find the inverse of a relation graphically.
❼ Determine whether or not the inverse of a graph is a function.

2-B Power and Root Functions — Wednesday • 9/13

n^{th} root • extraneous solution

❶ Find real n^{th} roots of a number a with a calculator.
❷ Evaluate $\sqrt[n]{a^m}$ or $a^{m/n}$ by hand.
❸ Solve a power equation or root equation.

2-C Exponential Functions — Friday • 9/15

exponential • growth factor • decay factor • compound interest

❶ Identify a scenario's rate of increase or decrease and its growth or decay factor.
❷ Write a function for an exponential growth or decay situation, and use it to calculate future values.
❸ Calculate an account balance with compounded annual interest.

2-D Logarithmic Functions — Tuesday • 9/19

logarithm • common log • natural log • change of base property • exponentiate • half-life

❶ Simplify the composition of a logarithmic and an exponential function.
❷ Evaluate a simple logarithm by hand.
❸ Evaluate a common or natural logarithm with a calculator.
❹ Simplify a logarithmic expression.
❺ Simplify a logarithm in which the base and argument are both powers of the same base.
❻ Evaluate any logarithm.
❼ Solve a basic exponential equation.
❽ Solve an equation in which one exponential expression is equal to another.
❾ Solve a logarithmic equation.
❿ Translate a description of a half-life situation into an equation, and solve it.
⓫ Determine half-life based on decay rate or decay rate based on half-life.

2-E Sketches of Functions — Monday • 9/25

asymptote

❶ Sketch a power function $f(x) = x^n$.
❷ Sketch a root function $f(x) = \sqrt[n]{x}$.
❸ Sketch an increasing exponential function $f(x) = b^x$.
❹ Sketch a decreasing exponential function $f(x) = b^x$.
❺ Sketch an increasing logarithmic function $f(x) = \log_b x$.
❻ Sketch a decreasing logarithmic function $f(x) = \log_b x$.