Introduction

These concepts will help your math skills.
These are the tools needed for any algebra-based class.
Understanding the fundamentals will help make math make sense and be easier and less intimidating.

This guide will help your math grade.
A section of every math final you take at SVHS will consist of these 24 problems (but with different numbers or wording each time).
Not all of these are covered in or before Math 1. If you learn the learn the other ones on your own, you can get over 100% on the Math 1 final.
Most high school math problems are built on one or more of these, making it impossible to do them correctly if these are incorrect.

You can use this guide on your own, or with a friend, tutor, or others, by doing the following for each problem.
Make sure you understand each bold concept, including any mathematical terminology.
Make sure you understand the expalanation below it, or an alternative explanation.
Try the sample problem.
Check to see if your answer is correct or if you made one of the common errors shown.
If you are working with others, have them give you additional problems for the concept.

Concepts, Explanations, and Sample Problems

1. Subtract a fraction from a whole number.

Get a common denominator by multiplying the whole number by $\frac{n}{n}$, where *n* is the denominator, and subtract the numerators.

$1 - \frac{3}{11} =$

2. Multiply or divide a fraction by a whole number.

Multiply the fraction by $\frac{n}{1}$ to multiply by *n*, or by $\frac{1}{n}$ to divide by *n*.

$\frac{4}{5} \times 2 =$

3. Divide zero or by zero.

Any number divided by zero is undefined. Zero divided by any other number is zero.

$\frac{9}{0} =$

4. Subtract a decimal percentage from a whle number.

Move the decimal two places left to get rid of the %, then subtract.

$1 - 0.8\% =$

5. Subtract, multiply, or divide by a negative.

Subtracting *-n* is the same as adding *n*. Multiplying or dividing by a negative changes the sign (positive or negative) of the number.

$12 - (-2) =$

6. Apply order of operations for basic arithmetic and powers.

First, do any calculations within parentheses, then apply exponents, then do multiplication and division from left to right, and then do addition and subtraction from left to right.

$1 + 3(2)^2 =$

7. Apply order of operations for negatives and parentheses.

A negative number to an odd power, such as $(-2)^7$, is negative, and a negative number to an even power, such as $(-2)^8$, is positive. But if the negative is not in the parentheses, such as -2^8 or $-(2)^8$, the negative is applied last, so the value is negative no matter what the power is.

Is $(-3)^4$ positive or negative?

8. Use a calculator to evaluate a fraction with multiple terms in the numerator and denominator.

Depending on your calculator, you may need to enter parentheses that are not shown in the written problem. On most calculators, you will need to put parentheses around the entire numerator, around the entire denominator, and around function arguments such as in square roots.

$\frac{9+\sqrt{2}}{\sqrt{6}-1} \approx$

9. Identify and use equations.

An equation has an equals sign. When solving equations, all work should be shown as equations.

Which step(s) below in solving $x^2 + 6x + 9 = 1$ are not equations?

$x^2 + 6x + 8$

$(x + 2)(x + 4)$

$(x + 2) \rightarrow -2, (x + 4) \rightarrow -4$

10. Solve a two-step linear equation exactly.

Use reverse order of operations: Start by adding or subtracting a number from each side, then divide each side by the coefficient.

$13x + 9 = 3$

11. Solve a one-step equation with a fractional coefficient.

Multiply each side of the equation by the denominator to cancel it. If the numerator is not 1, also divide each side of the equation by the numerator.

$-\frac{1}{3}x = -30$

12. Distribute a monomial.

Multiply each term by the monomial.

$5x^2(4x - 2) =$

13. Multiply two polynomials.

Multiply each term of the first binomial by each term of the second binomial, and then combine like terms.

$(4x - 5)(x + 3) =$

14. Square a binomial.

This is multiplying two binomials (above) where the two binomials are the same. Make sure not to leave out the middle term!

$(2a + 5)^2 =$

15. Multiply and divide powers.

Add exponents when multiplying powers, and divide exponents when dividing powers. A variable with no written exponent has an exponent of 1.

Simplify $\frac{x(x^9)}{x^2}$.

16. Apply the negative exponent property.

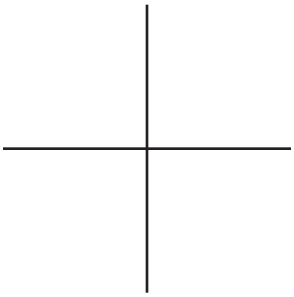
Each factor with a negative exponent can be moved from the numerator to the denominator or vice versa to make the exponent positive. A factor with more than one negative exponent, such as b in $(ab^{-5})^{-2}$, moves once for each negative exponent.

$\left(\frac{2x^{-3}}{7}\right)^{-1} =$

17. Estimate coordinates on a graph.

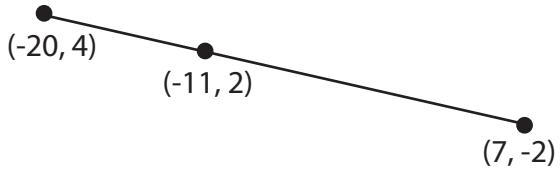
The first coordinate, x, is to the right of the origin if positive or to the left if negative. The second coordinate, y, is above the origin if positive or below if negative.

Plot the points (-3, 3), (-3, -6), and (6, -3).



18. Find the slope of a graphed line.

Slope $= \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are any two points on the line.



19. Use function notation.

Substitute the value or expression in parentheses for each instance of the variable represented. On a graph, the input is an x value and the output is the y value for that x value, making the point (x, y).

Given $f(x) = x + 5$ and $g(x) = 2x^2 - x$, evaluate $f(3)$ and $g(3)$ and state what these values mean about the graphs of f and g.

20. Identify whether or not a graph represents a function.

A function cannot have more than one y-value for a single x-value, such as if a vertical line intersects a graph at more than one point.

Is the wave graphed below a function?

21. Distinguish between terms and factors.

Factors are multiplied together, and terms are added together. A factor can consist of more than one term, such as $(x + 3)$. A term must be a monomial but can consist of more than one factor, such as $8x$.

In the expression $10x^2 + 2x$, is $2x$ a term or a factor, or both or neither?

22. Multiply a factored expression by a monomial.

Multiply all of the terms of one factor by the monomial.

Is $(4x)(x - 3)$ times 5 equal to $(4x)(5x - 15)$, $(20x)(x - 3)$, or $(20x)(5x - 15)$?

23. Simplify a rational expression.

Divide each term by the largest factor that divides into each term evenly. Make sure to divide every term one time, regardless of how many factors it has.

Is $\frac{12x + (6y)(10z)}{20x + 4y + 4z + 2}$ equivalent to $\frac{3 + (6y)(5z)}{5 + 2y + 2z}$?

24. Apply inverse operations to each side of an equation.

You can add or subtract at the end of each side of an equation. You can multiply or divide each term in one factor on each side of an equation, but if a side has more than one factor, ignore the other factors on that side. You can take a power or root of each factor in one term on each side of an equation, but if a side has more than one term, do not use this method.

Identify the error below in solving $x^2 + 9 = 36$.

$\sqrt{x^2 + 9} = \sqrt{36}$

$x + 3 = \pm 6$

$x = 3, x = -9$

1. Correct Answer: $\frac{8}{11}$
If you get $\frac{-2}{11}$, you subtracted $\frac{1}{11}$ instead of $\frac{11}{11}$.
If you get $\frac{2}{11}$, you made the above mistake and also subtracted in the wrong order.

2. Correct Answer: $\frac{8}{5}$
If you get $\frac{8}{10}$, you multiplied by $\frac{2}{2}$ (which equals 1) instead of by $\frac{2}{1}$ (which equals 2).

3. Correct Answer: undefined
If you get 0, you did 0 divided by 9 instead of 9 divided by 0.

4. Correct Answer: 0.992
If you get -0.2, you subtracted 0.8 instead of 0.8%.
If you get 0.92, you subtracted 8% instead of 0.8%.

5. Correct Answer: 14
If you get 10, you subtracted 2 instead of -2.

6. Correct Answer: 13
If you get 16, you added $1 + 3$ before multiplying.
If you get 37, you did 3×2 and then squared it (which is $3 \times 3 \times 2 \times 2$) instead of 3×2^2 (which is $3 \times 2 \times 2$).
If you get 64, you made both mistakes above.

7. Correct Answer: positive
If you get -81 instead of 81, you did -3^4 instead of $(-3)^4$.

8. Correct Answer: 7.18
If you get 9.95, you didn't use any parentheses.
If you get 8.58, you used parentheses around the 2 and around the 6 but not around the numerator or the denominator.
If you get 9.98, you put a closing parenthesis for the 2 or for the numerator, but not both.

9. Correct Answer: None of these are equations.
None of these steps have equals signs.
$x^2 + 6x + 8$ is an expression. It should be the equation $x^2 + 6x + 8 = 0$.
$(x + 2)(x + 4)$ is an expression. It should be the equation $(x + 2)(x + 4) = 0$.
$(x + 2) \rightarrow -2$ and $(x + 4) \rightarrow -4$ have no algebraic meaning. They should be the equations $x + 2 = 0$ and $x + 4 = 0$ with solution $x = -2$ and $x = -4$.

10. Correct Answer: $x = \frac{-6}{13}$
If you get $x = \frac{12}{13}$, you subtracted 7 from one side but added 7 to the other side.
If you get $x = \frac{13}{6}$, you tried to divide each side by 6 instead of by 13.
If you get a decimal answer such as $x = -0.46$, you have rounded, resulting in an approximate solution but not the exact solution.

11. Correct Answer: $x = 90$
If you get $x = 10$, you divided one side by $-\frac{1}{3}$ but multiplied the other side by $-\frac{1}{3}$.
If you get a negative solution, you incorrectly multiplied or divided two negatives.

12. Correct Answer: $20x^3 - 10x^2$
If you get $20x^3 - 2$, you only multiplied the first term of the polynomial.

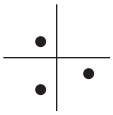
13. Correct Answer: $4x^2 + 7x - 15$
If you have a coefficient other than 7 for your linear term, you did not find $12x - 5x$.

14. Correct Answer: $4a^2 + 20a + 25$
If you get $4a^2 + 25$, you did $2a \times 2a$ and 5×5 but you left out $2a \times 5$ and $5 \times 2a$.

15. Correct Answer: x^8
If you get x^7, you left out the first x (that is, x^1).
If you get x^5, you divided the exponents instead of subtracting them.

16. Correct Answer: $\frac{7x^3}{2}$
If you get $14x^3$, you used $(2x)^{-3}$ for the numerator instead of $2x^{-3}$.
If you get $\frac{7}{2x^3}$, you only applied one of the two negative exponents to the x.

17. Correct Answer: *as shown at right, or something similar*
If your points don't match this pattern, you probably mixed up positive versus negative or ignored the fact that 6 is twice as big as 3.



18. Correct Answer: $\frac{-2}{9}$
If you get $\frac{-9}{2}$, you divided the change in x by the change in y instead of the change in y by the change in x.
The answer cannot be positive because the line goes downhill. If your answer is positive, you made an error in subtracting negatives, or you used one point first for x but the other point first for y.

19. Correct Answer: $f(3) = 8$ means the point (3, 8) is on the graph of f, and $g(3) = 15$ means the point (3, 15) is on the graph of g.
If you get $f(3) = 3x + 15$ or $g(10) = 6x^2 - 3x$, you multiplied the expressions by 3 instead of changing each x to 3.

20. Correct Answer: yes
You might have thought it was not a function because of the repeated y values, but only repeated x values make it not a function.

21. Correct Answer: both
It is a term because adding one or more other terms—in this case adding $10x^2$—results in the whole expression.
It is a factor because multiplying by one or more other factors—in this case multiplying by $(5x + 1)$—results in the whole expression.

22. Correct Answer: $(4x)(5x - 15)$ and $(20x)(x - 3)$ are both correct.
If you get $(20x)(5x - 15)$, you multiplied the expression by 5 twice because you multiplied each term in every factor instead of each term within one factor. This would be the same mistake as $2(3) \times 5 = (2 \times 5)\ (3 \times 5) = 10 \times 15 = 150$.

23. Correct Answer: no
If you changed $12x$ to 3 and $20x$ to 5, you divided these terms by $4x$ but divided the other terms by 2.
If you only have three terms in the denominator instead of four, you calculated $2 \div 2$ as 0 instead of 1.
If you divided $(6y)$ and $(10z)$ both by 2 to get $(3y)(5z)$, you divided the $(6y)(10z)$ term by 4 instead of by 2.

24. Correct Answer: You cannot take the square root of x^2 and 9 separately.
If you did, you are treating the left side of the equation as $9x^2$ (one term with multiple factors) instead of $x^2 + 9$ (one factor with multiple terms).
Instead, you can subtract 9 from each side so that there is only one term on each side of the equation, and then take the square root on each side.

1. $2 - \frac{1}{15} =$

2. $\frac{8}{3} \times 10 =$

3. $\frac{1}{0} =$

4. $1 - 2.2\% =$

5. $50 \div (-10) =$

6. $15 - 5(8)^4 =$

7. For what values of x are $(-9)^x$ and -9^x both negative?

8. Round $\frac{\sqrt{7}+\sqrt{3}}{\sqrt{8}-2}$ to the nearest hundredth.

9. The solution to the equation $x + 20 = 4(x - 2)$ is $x = \frac{28}{3}$. Using equations only, show all steps to find this solution.

10. Find the exact solution to $9 - 7x = 21$.

11. Solve $\frac{3}{4}x = 8$.

12. $2x(x^8 - 5x + 3) =$

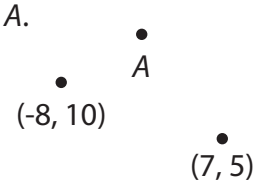
13. $(2x + 9)(3x - 1) =$

14. $(x - 10)^2 =$

15. Simplify $\frac{x^{20}(x^{10})}{x^2(x)}$.

16. Write $9abc^{-4}$ without a negative exponent.

17. Estimate the coordinates of point A.



18. Find the slope of the line through (-5, 9) and (3, -3).

19. Use function notation to find two points on the graph of $b(x) = x^2 + 2x - 1$.

20. Sketch a graph that is not a function but that would be a function if turned sideways.

21. In the expression $3xy$, is x a term, a factor, both, or neither?

22. Identify the error in rewriting $2(x + 4)(3x - 1)$ as $(2x + 8)(6x - 2)$.

23. Simplify $\frac{8x+4}{8x+12}$.

24. Do the first step in solving $4(x + 1)^2 = 9$.