CHAPTER FIVE: DISCRETE PROBABILITY DISTRIBUTIONS — Review November 30 ✎ Test December 7

A probability distribution shows the likelihood of each possible outcome. This chapter deals with discrete probability distributions, in which there are a set number of possible outcomes in any given range (e.g., no cars, 1 car, 2 cars), as opposed to continuous probability distributions, in which there are infinitely many possible outcomes which can be sorted into discrete categories (e.g., 0 to 10 cm, 10 to 20 cm, 20 to 30 cm). A particularly important discrete probability distribution is the binomial probability distribution, in which a trial such as predicting a coin flip is attempted n times and P(r) represents the probability of exactly r of those n trials being successful.

5-A Introduction to Probability Distributions — Monday • 11/21

discrete variable • continuous variable • probability distribution

❶ Classify a variable as discrete or continuous.
❷ Determine whether or not a discrete variable will be treated as continuous.
❸ Give the probability distribution of an event.
❹ Calculate the mean and standard deviation of a discrete probability distribution.

5-B Geometric and Binomial Probabilities — Monday • 11/28

geometric probability distribution • binomial probability distribution • binomial experiment

❶ Calculate the probability that the first success will be on the n^{th} trial.
❷ Calculate the probability that the first success will be after the n^{th} trial.
❸ Calculate the probability of getting exactly r successes in a binomial experiment.
❹ Explain the components of a binomial experiment calculation.
❺ Calculate the probability of getting at most or at least r successes in a binomial experiment.
❻ Calculate binomial probabilities with the calculator.

5-C Binomial Distributions — Wednesday • 11/30

❶ Make a histogram for a binomial probability distribution.
❷ Calculate the mean and standard deviation for number of successes in a binomial distribution.

5-A Introduction to Probability Distributions

A CONTINUOUS Variable has infinitely many possible values within any given range: There is always a possible value between any two other values. Continuous data must be rounded.

A DISCRETE Variable has a specific number of possible values within any given range: Either it is nonnumerical or there exist values that it could not be that are between values that it could be. Discrete data are exact.

❶ Classify a variable as discrete or continuous.

1. The variable is continuous unless the data are nonnumerical or there exists an impossible value between two possible values.
2. In general, data that are counted are discrete and data that are measured are continuous.

❶

a) Shoe size is discrete because, for example, 9 and 9.5 are possible but 9.1 is not.

b) Foot length is continuous because it is measured and there are infinitely many possible values between any two given values, such as decimal values between 6 inches and 7 inches.

Continuous variables are sorted into ranges. Some discrete variables—those with many possible values—are also sorted into ranges, and thus treated as continuous variables.

❷ Determine whether or not a discrete variable will be treated as continuous.

1. The variable is truly discrete if it is useful to identify the probility of each individual possible value.
   The variable is treated as continuous if it is more useful to identify the probability of ranges of possible values.
2. In a histogram, labeling the boundaries of the bars as category boundaries (as in chapter 3) is for data treated as continuous, and labeling each bar itself with a single value (as in this chapter) is for truly discrete data.

❷

a) Number of people in a car is truly discrete data. There would be a bar for 1 person, a bar for 2 people, etc.

b) Number of people in a school would be treated as continuous data. There might be a bar for 1 to 500 people, a bar for 501 to 1000 people, etc.

A PROBABILITY DISTRIBUTION shows all the possible outcomes or ranges of outcomes of an event and how likely each one is. The sum of the probabilities in a probability distribution is 1 (that is, 100%) because it includes all possibilities.

❸ Give the probability distribution of an event.

1. List each possible outcome.
2. State the probability of each.

❸ Show the probability distribution for a coin flip.

1. heads, tails
2. 50% chance of heads, 50% chance of tails

The mean of a probability distribution is the same as its expected value: $\mu = \Sigma x P(x)$.
The standard deviation is of a probability distribution is $\sigma = \sqrt{\Sigma(P(x)(x-\mu)^2)}$.
These formulas are the same as those in 3-C, except they use $P(x)$ instead of f. Calculations are slightly simpler, because $\Sigma P(x) = 1$ rather than $\Sigma f = n$.

❹ Calculate the mean and standard deviation of a discrete probability distribution.

1. Identify the value x of each possible outcome.
2. Identify the probability $p(x)$ of each possible outcome. (If $\Sigma P(x) \neq 1$, fix step 1.)
3. Calculate $xP(x)$ for each possible outcome.
4. To get the mean, calculate $\mu = \Sigma x P(x)$.
5. Subtract μ from each x value.
6. Square each difference in step 5.
7. Multiply each square in step 6 by $P(x)$ for that value.
8. To get the variance, add the products in step 7.
9. To get the standard deviation, take the square root of the variance in step 8.

❹ A spinner has 50 spaces. The gold one is worth 50 points, the 3 red ones are each worth 20 points, and the 9 brown ones are each worth 10 points.

x	$P(x)$	$xP(x)$	$x-\mu$	$(x-\mu)^2$	$P(x)\,(x-\mu)^2$
50	.02	1.00	46	2116	42.32
20	.06	1.20	16	256	15.36
10	.18	1.80	6	36	6.48
		$\mu = 4.00$			$\sigma^2 = 64.16$
					$\sigma \approx 8.01$

5-B Geometric and Binomial Probabilities

In the probability distributions below, n represents the number of trials, r represents the number of successes, p represents the probability of success on each trial (which must be the same for each trial), and q represents the probability of failure on each trial. p and q are complements.

The GEOMETRIC Probability Distribution, $P(n) = q^{n-1}p$, gives the probability that the first success will be the n^{th} trial.

Similarly, $P(n) = q^n$ gives the probability that the first success will be after the n^{th} trial.

❶ Calculate the probability that the first success will be after the n^{th} trial.

1. Identify n, the number of trials.
2. Identify q, the probability of failure on each individual trial.
3. Calculate $P(n) = q^n$.

❶ Natalie is predicting rolls on 8-sided dice. Find the probability that her first successful prediction will be after her fourth roll.

1. There are $n = 4$ rolls.
2. The probability of not getting an 8 is $q = \frac{7}{8}$ on each roll.
3. $P(4) = \left(\frac{7}{8}\right)^4 = \frac{2401}{4096}$

❷ Calculate the probability that the first success will be on the n^{th} trial.

1. Identify n and q (see steps 1 and 2, above).
2. Identify p, the probability of success on each individual trial.
3. Calculate $P(n) = q^{n-1}p$.

❷ Natalie is predicting rolls on 8-sided dice. Find the probability that her first successful prediction will be on her fourth roll.

1. $n = 4$, $q = \frac{7}{8}$
2. The probability of a getting an 8 is $p = \frac{1}{8}$ on each roll.
3. $P(4) = \left(\frac{7}{8}\right)^3\left(\frac{1}{8}\right) = \frac{343}{4096}$

The BINOMIAL Probability Distribution, $P(r) = \binom{n}{r}p^r q^{n-r}$, gives the probability that exactly r out of n trials of the same independent event will be successes. This type of problem is called a BINOMIAL EXPERIMENT, and is a special case of 4-C ❹: $\binom{n}{r}$ is the number of possible orders, and $p^r q^{n-r}$ is the probability of each order.

❸ Calculate the probability of getting exactly r successes in a binomial experiment.

1. Identify n, the number of trials.
2. Identify r, the number of successes.
3. Identify p, the probability of success on each individual trial.
4. Identify q, the probability of failure on each individual trial.
5. Calculate $P(r) = \binom{n}{r}p^r q^{n-r}$.

❸ Out of five 6-sided dice, exactly three roll a 6.

1. There are $n = 5$ rolls.
2. There are $r = 3$ 6's.
3. The probability of a getting a 6 is $p = \frac{1}{6}$ on each roll.
4. The probability of not getting a 6 is $q = \frac{5}{6}$ on each roll.
5. $P(3) = \binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 = 10\left(\frac{1}{216}\right)\left(\frac{25}{36}\right) = \frac{250}{7776}$

❹ Explain the components of a binomial experiment calculation.

1. p^r is the probability of r successful trials, each with probability p of success (see 4-C ❷).
2. q^{n-r} is the probability of $n - r$ unsuccessful trials, each with probability q of failure (see 4-C ❷).
3. $\binom{n}{r}$ is the number of possible orders for r of the n trials to be successes and the rest to be failures (see 4-A ❶). Each order has probability $p^r q^{n-r}$.
4. $\binom{n}{r}p^r q^{n-r}$ is the probability that one of the $\binom{n}{r}$ orders will occur, that is, that exactly r of the n trials will be successes (see 4-C ❹).

❹ Out of five 6-sided dice, exactly three roll a 6.

1. $\left(\frac{1}{6}\right)^3$ is the probability of three dice all rolling a 6.
2. $\left(\frac{5}{6}\right)^2$ is the probability of two dice not rolling a 6.
3. $\binom{5}{3} = 10$ is the number of possible orders of 3 6's and 2 non-6's.
4. $\binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 = \frac{250}{7776}$ is the probability that exactly three out of five 6-sided dice roll a 6.

❺ Calculate the probability of getting at most or at least r successes in a binomial experiment.

1. Identify n, p, and q (see ❸).
2. Calculate the probability (see 4-C ❺).

❺ Out of seven 4-sided dice, at least three roll a 4.

1. $n = 7, p = \frac{1}{4}, q = \frac{3}{4}$
2. The event as stated has five possibilities: $P(3) + P(4) + P(5) + P(6) + P(7)$. The complement has only three: $P(0) + P(1) + P(2)$.

$P(0) = \binom{7}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^7 = 1(1)\left(\frac{2187}{16384}\right) = \frac{2187}{16384}$

$P(1) = \binom{7}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^6 = 7\left(\frac{1}{4}\right)\left(\frac{729}{4096}\right) = \frac{5103}{16384}$

$P(2) = \binom{7}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^5 = 21\left(\frac{1}{16}\right)\left(\frac{243}{1024}\right) = \frac{5103}{16384}$

$P(0, 1, \text{or } 2) = \frac{2187}{16384} + \frac{5103}{16384} + \frac{5103}{16384} = \frac{12393}{16384}$

$P(\geq 3) = \frac{16384}{16384} - \frac{12393}{16384} = \frac{3991}{16384}$

Binomial probabilities can be calculated directly on the calculator.

`binompdf(n,p,r)` is the probability of exactly r successes.

`binomcdf(n,p,r)` is the probability of at most r successes.

`1–binomcdf(n,p,r–1)` is the probability of at least r successes, which is the complement of at most $r - 1$ successes.

❻ Calculate binomial probabilities with the calculator.

1. If it is not already, write the problem in terms of yielding *exactly*, *at most*, or *at least* r successes.
2. Identify n, r, and p.
3. Plug the variables into the appropriate function above.

❻ Out of five 6-sided dice, more than two roll a 6.

1. *More than* 2 means *at least* 3.
2. $n = 5, r = 3, p = \frac{1}{6}$
3. $P(3, 4, \text{or } 5)$ = `1–binomcdf(5,1/6,2)` ≈ 3.55%

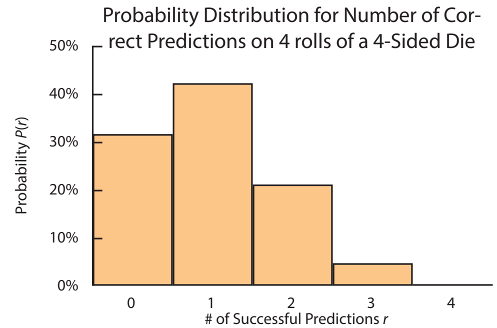
5-C Binomial Distributions

A binomial probability distribution can be shown in a histogram, in which each possible value of r has a bar showing its probability.

❶ Make a histogram for a binomial probability distribution.

1. Identify n, p, and q.
2. Calculate the height of each bar by `binompdf(n,p,r)`. n and p are the same each time, but a different r is used for each bar (0 through n).
3. Label the x-axis "number of successes" from 0 to n, and label the y-axis "$P(r)$", starting at 0%.
4. Graph each bar.
5. Title the graph.

❶ Give the probability distribution for predicting 4 rolls on a 4-sided die.



1. $n = 4$, $p = \frac{1}{4}$, $q = \frac{3}{4}$
2. $P(0) =$ `binompdf(4,1/4,0)` $= .316$
   $P(1) =$ `binompdf(4,1/4,1)` $= .422$
   $P(2) =$ `binompdf(4,1/4,2)` $= .211$
   $P(3) =$ `binompdf(4,1/4,3)` $= .047$
   $P(4) =$ `binompdf(4,1/4,4)` $= .003$

The mean and expected value of a binomial distribution is $\mu = np$. If rounded, it is also the most likely result.
The standard deviation of a binomial distribution is $\sigma = \sqrt{npq}$.

❷ Calculate the mean and standard deviation for number of successes in a binomial distribution.

1. Identify n, p, and q.
2. Calculate $\mu = np$.
3. Calculate $\sigma = \sqrt{npq}$.

❷ Find the mean and standard deviation for number of correct predictions out of 4 rolls on a 4-sided die.

1. $n = 4$, $p = \frac{1}{4}$, $q = \frac{3}{4}$
2. $\mu = 4\left(\frac{1}{4}\right) = 1$
   $\sigma = \sqrt{4\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)} \approx .866$