

CHAPTER FIVE: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Due Thursday, May 25

5-A Exponential Functions

exponential function • exponential growth and decay • compound interest

1 Identify a scenario's rate of increase or decrease and its growth or decay factor.

- 1 a) 4% less b) 200% more c) 3 times as much d) 3 times more

2 Sketch $f(x) = b^x$.

- 2 Compare the graphs of $q(x) = 2^x$, $r(x) = 1.1^x$, and $s(x) = (\frac{3}{4})^x$.

3 Write a function for an exponential growth or decay situation, and use it to calculate future values.

- 3 Annual population growth in California has been 0.87% over the past six years, reaching 39.3 million residents in 2016. Use this growth rate to calculate an estimate of California's population in the following years.

- a) 2025 b) 2011

5-B Logarithmic Functions

logarithm • common log • natural log

1 Simplify the composition of a logarithmic and an exponential function.

- 1 Simplify the following expressions.

- a) $\log_6 6^{2x}$ b) $3^{\log_3 2x}$ c) $\ln e^9$ d) $\log_2 8^x$

2 Evaluate a simple logarithm by hand.

- 2 Evaluate the following logarithms if possible.

- a) $\log_4 16$ b) $\log_4 \frac{1}{16}$ c) $\log_4 2$ d) $\log_4 \frac{1}{2}$
- e) $\log_4 4$ f) $\log_4 1$ g) $\log_4 0$ h) $\log_4 -16$

3 Calculate a common or natural logarithm with a calculator.

- 3 Evaluate the following logarithms if possible.

- a) $\log 1000$ b) $\log .01$ c) $\ln 20$ d) $\log -100$

4 Sketch $f(x) = \log_b x$.

- 4 Compare the graphs of $q(x) = \log_2 x$, $r(x) = \log_{1.1} x$, and $s(x) = \log_{3/4} x$.

5-C Properties of Logarithms

product property • quotient property • power property • negative property • reciprocal property • change of base property

1 Simplify a logarithmic expression.

- 1 Simplify the following expressions. State the property of logarithms used for each step.

- a) $2 \log_3 x + \log_3 10$ b) $\log_3 9^{4x}$ c) $\log_3 (\frac{1}{9})^{4x}$

② Evaluate a logarithm in which the base and the argument are powers of the same number.

② $\log_{32} 8$

③ Use a calculator to evaluate any logarithm.

③ $\log_3 20$

5-D Exponential Equations

① Solve an exponential equation by using log with a base in the equation.

① Solve by using log with base 4.

a) $4^{5x} = 8$

b) $4^{5x} = 8^{x+10}$

② Solve an exponential equation by using common log or natural log.

② Solve by using log.

a) $4^{5x} = 8$

b) $4^{5x} = 8^{x+10}$

③ Solve an exponential equation without a calculator.

③ $4^{5x} = 8^{x+10}$

5-E Exponential Situations

half-life

① Translate a description of an exponential situation into an equation, and solve it.

① A radioactive substance is decaying at a rate of 10% per minute. In how long will only half of the original amount remain?

① It takes 6.6 minutes for half of the mass of a radioactive substance to decay. (That is, it has a half-life of 6.6 minutes.) What is its rate of decay?

5-F Logarithmic Equations

exponentiate

① Solve a logarithmic equation.

① a) $\log_x 400 = 5$

b) $8 \log_9 x - 2 \log_3 4x = 5$

c) $\log(x+1) + \log(x-2) = 1$

6-A Counting Methods

combination • choose • sample space • fundamental counting principle • permutation

1 Count combinations.

- 1 In how many ways can Savannah choose her 3 favorite songs from a playlist of 9 songs?

2 Find the total number of possible outcomes in a series of events.

- a) Choose 3 representatives out of 9 seniors and 2 representatives out of 8 juniors.

- b) Identify the 1st place, 2nd place, 3rd place, and 4th place finisher out of 25 racers.

3 Count permutations.

- 3 In how many ways can Savannah choose her favorite, second favorite, and third favorite song from a playlist of 9 songs?

4 Use a graphing calculator to count combinations and permutations.

- 4 Calculate ${}_{19}^{22}$.

5 Find the size of a sample space by using permutations, if possible.

- 5 Use combinations to express the size of the sample space for each of the following. Then rewrite the solution using permutations if possible, or explain why not.

- a) Claire chooses her 3 favorite months.

- b) Claire chooses her favorite, second favorite, and third favorite month.

- c) Use a 6-sided die to choose a color for each of three teams.

- d) Put 6 colors in a hat and draw three of them to choose a different color for each of three teams.

6-B Set Notation and Venn Diagrams

set • element • subset • cardinality • intersection • union • complement • empty set • universal set • Venn diagram

1 Read set notation.

- 2 State the following in words, given A is the set of aces and B is the set of black cards.

a) A

b) U

c) \emptyset

d) $|A|$

e) $|U|$

f) $A \cup B$

g) $A \cap B$

h) $|A \cap B|$

i) A'

j) $(A \cap B)'$

k) $(A \cup B)'$

2 Identify a region in a Venn diagram from set notation.

- 2 Shade the following regions in a Venn diagram.

a) $A \cap B$

b) $A \cup B$

c) $(A \cup B)'$

d) $C \cup (A \cap B)$

3 Use a Venn diagram to find cardinalities of sets.

- 3 In a class of 30 students, 16 are athletes, 6 are actors, and 9 are neither.

6-C Probability of a Single Event

mutually exclusive • given • conditional probability

① Use the size of the sample space to find the probability of an event.

① Eve draws two cards. Find the probability that...

a) the first card is a ace

b) both cards are aces

② Find the probability that either of two specific events will occur.

② Find the probability of a card being as stated.

a) red or an ace

b) a 9 or an ace

③ Find probabilities based on given information.

③ Find the following probabilities for Eve's two cards.

a) The second card is an ace.

b) The second card is an ace, given the first card is an ace.

c) The first card is an ace, given the second card will be an ace.

6-D Probability of Specific Multiple Events

dependent events • independent events

① Identify whether events are independent or dependent.

① Ashley rolls three dice, trying to roll 6's, and she draws two cards, trying to draw kings.

② Find the probability of multiple events all occurring.

② James draws five cards. Calculate the probability that the first two cards are aces and the fourth card is not an ace.

② Janaya rolls three 6-sided dice. Calculate the probability that they all roll the same number.

6-E Probability of General Multiple Events

binomial experiment

① Calculate the probability of an event that can happen in different ways.

① Hanna grabs 3 random pens from a drawer with 6 black pens, 4 red pens, and 1 purple pen. What is the probability that they are all the same color?

② Use a complement to calculate a probability.

② What is the probability that at least one of the three pens Hanna grabs, above, is red?

③ Calculate the probability of an event that can occur in different orders.

③ David rolls five 6-sided dice. What is the probability that exactly two of them roll '6'?

④ Explain the components of a binomial experiment calculation.

⑤ Calculate the probability of getting at most or at least r successes in a binomial experiment.

⑤ Find the probability that out of five 6-sided dice, fewer than four will roll '6'.

6-F Probability Distributions

probability distribution • expected value

① Give the probability distribution of a simple event.

① Show the probability distribution for a coin flip.

② Make a histogram showing the probability distribution for a binomial experiment.

② Predict four rolls of a 6-sided die.

③ Calculate the expected value of a probability distribution.

③ A 10-space spinner has 6 blue spaces worth 10 points each, 3 red spaces worth 25 points each, and one black space worth -100 points.

CHAPTER SEVEN: LINEAR CORRELATION

due Tuesday, May 30

7-A The Line of Best Fit

model • linear regression • line of best fit • interpolation • extrapolation • residual

① Find the equation of the line of best fit for a data set.

| | | | | | |
|--------------------------------|-----|-----|-----|-----|-----|
| ① daily high temperature (°F): | 76 | 79 | 86 | 84 | 80 |
| ice cream Annika sells (\$): | 224 | 289 | 330 | 360 | 299 |

② Use a line of best fit equation to predict a value of the dependent variable.

② Predict Annika's ice cream sales for days with the following high temperatures.

a) 78° b) 86° c) 50°

- ③ Calculate a residual for a point on a regression line.
③ Calculate Annika's residual for 86° .

7-B Statistically Significant Correlations

data • sample • correlation coefficient • population • p value • statistically significant • data snooping

- ① Calculate r for a data set, and interpret the value.
① Calculate the correlation coefficient for Annika's data, and explain what it means.
- ② Determine whether or not the correlation found in a sample is statistically significant.
② Are Annika's data statistically significant?

7-C Causal Relationships

affect • effect • causal relationship • confounding variable

- ① Distinguish between *affect* and *effect* meaning *influence*.
① Alcohol ___ffects fine motor control. Nausea, amnesia, and unconsciousness are also possible ___ffects. Smaller people are particularly ___ffected, although anyone can experience the ___ffects of alcohol.
- ② Discuss possible reasons for correlation in a sample.
② Identify possible reasons for the correlation between exercise and health.
- ③ Critically evaluate a causal claim from a sample correlation.
③ From 1999 to 2010, the number of lawyers in Georgia has been positively correlated with the annual number of Americans who die by becoming tangled in their bed sheets: $r = .961, p = .0000003$.

CHAPTER EIGHT: VECTORS

due Thursday, June 1

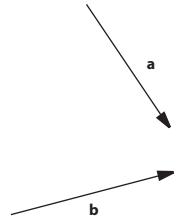
8-A Two-Dimensional Vectors

vector • tail • head • component • magnitude • scalar

- ① Identify the vector from one point to another.
① Sketch the following vectors, and express them in component form.
a) \mathbf{a} goes from $(2, 5)$ to $(8, 0)$ b) \mathbf{b} goes from $(3, 7)$ to $(9, 2)$ c) \mathbf{c} goes from $(9, 2)$ to $(3, 7)$
- ② Find the magnitude of a vector.
② $\mathbf{h} = \langle 6, -5 \rangle$
- ③ Multiply a vector by a scalar.
③ Find $-10\mathbf{a}$, given $\mathbf{a} = \langle 2, -3 \rangle$.
- ④ Find sums and differences of vectors algebraically.
④ Find $2\mathbf{a} - 10\mathbf{b}$, given $\mathbf{a} = \langle 8, 6 \rangle$ and $\mathbf{b} = \langle 2, -1 \rangle$.

5 Find sums and differences of vectors geometrically.

5 Find $2\mathbf{b} - 3\mathbf{a}$, given \mathbf{a} and \mathbf{b} shown at right.



6 Find the angle a vector makes with a horizontal line.

6 $\mathbf{a} = \langle -6, 5 \rangle$

8-B Vector Equations

position vector

1 Write a vector equation of a line through a given point and parallel to a given vector.

1 Write the equation of the line through the point $(3, 4)$ in the direction $\langle 5, -2 \rangle$.

2 Write a vector equation of a line through two points.

2 Write the equation through the points $(9, 5)$ and $(-3, 1)$.

3 Determine whether or not a line passes through a given point.

3 Does the line $\mathbf{r} = t\langle 10, 4 \rangle + \langle -9, 5 \rangle$ pass through the point $(21, 17)$?

4 Find the point of intersection of two lines written as vector equations.

4 Find the point of intersection of the lines $\mathbf{r} = t\langle 2, 3 \rangle + \langle 5, -10 \rangle$ and $\mathbf{q} = s\langle 8, -4 \rangle + \langle -7, 12 \rangle$.

8-C Three-Dimensional Vectors

1 Apply techniques for two-dimensional vectors to three-dimensional vectors.

1 Find the magnitude of $\mathbf{a} = \langle 5, -4, 2 \rangle$.

1 Is the point $(12, 9, 4)$ on the line $\mathbf{a} = t\langle 4, 6, -2 \rangle + \langle 0, -9, 12 \rangle$?

8-D Unit Vectors

unit vector $\cdot \mathbf{i} \cdot \mathbf{j} \cdot \mathbf{k}$

1 Express a vector as a linear combination of unit vectors.

1 $\langle 10, 5, -3 \rangle$

- ② Find a unit vector parallel to a given vector.

② $\mathbf{a} = 10\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$

8-E Angles Between Vectors

dot product • orthogonal

- ① Find the dot product of two vectors.
 - ① Find $\mathbf{a} \cdot \mathbf{b}$, given $\mathbf{a} = \langle 10, 2, 5 \rangle$ and $\mathbf{b} = \langle -3, 7, 1 \rangle$.

- ② Find the angle between two vectors.
 - ② Find the angle between $\mathbf{a} = \langle 4, 5, -10 \rangle$ and $\mathbf{b} = \langle 8, 0, -1 \rangle$.

- ③ Determine whether two vectors are orthogonal, parallel, or neither.
 - ③ $\mathbf{a} = \langle -3, -20, 12 \rangle$ and $\mathbf{b} = \langle -2, 5, 8 \rangle$

CHAPTER NINE: DERIVATIVES

due Thursday, June 1

9-A Sequences and Series

sequence • arithmetic • geometric • series • summation notation

- ① Identify arithmetic and geometric sequences.
 - ① What type of sequence is 5, -10, 20, -40, ...?

- ② Find a formula for the n^{th} term of an arithmetic or geometric sequence.
 - ② Find an equation for the following sequences.
 - a) 49, 40, 31, 22, ...
 - b) 192, 144, 108, 81, ...

- ③ Find the sum of a series by adding.
 - ③ $\sum_{x=5}^8 (2x + 11)$

- ④ Write an arithmetic or geometric series in summation notation.
 - ④ Write the following series in summation notation.
 - a) the first 40 terms of $14 + 17 + 20 + 23 + \dots$
 - b) $-10 + 20 - 40 + 80 - 160 + 320$

5 Count the number of terms in a series written in summation notation.

5 $\sum_{x=20}^{80} (3x - 16)$

6 Find the sum of an arithmetic or geometric series.

6 Calculate the following sums.

a) the first 85 terms of $-5 - 2 + 1 + 4 + \dots$

b) $\sum_{x=8}^{100} (5x - 1)$

c) $\sum_{x=1}^{30} 40\left(\frac{10}{9}\right)^x$

9-B Limits

limit • discontinuous • rational function

1 Find the limit of a simple function at a given value.

1 a) $\lim_{x \rightarrow 3} 2x$

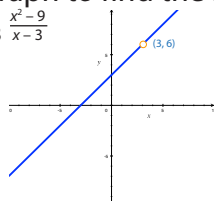
b) $\lim_{x \rightarrow 0} \frac{2}{|x|}$

2 Use an example and the concept of limits to derive the formula for the sum of an infinite geometric series.

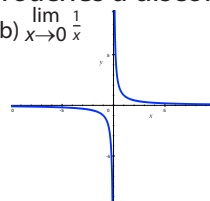
2 $120 + 60 + 30 + 15 + \dots$

3 Use a graph to find the limit of a function f as it approaches a discontinuity.

3 a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$



b) $\lim_{x \rightarrow 0} \frac{1}{x}$



4 Calculate values to estimate the limit of a function f as x approaches a number a .

4 $\lim_{x \rightarrow 3} \frac{x^2 - 9}{5x - 15}$

5 Algebraically find the limit of a rational function f as x approaches a number a .

5 $\lim_{x \rightarrow 3} \frac{x^2 - 9}{5x - 15}$

9-C Slopes of Curves

1 Calculate an estimate of the slope of a function at a given value of x .

1 Estimate the slope of $f(x) = x^3$ when $x = 2$.

2 Algebraically find the slope function of a power function, and use it to calculate the exact slope at a given value of x .

2 Find the slope of the given function when $x = 2$.

a) $f(x) = x^2$

b) $f(x) = x^3$

3 Calculate the exact slope of a polynomial function at a given value of x .

3 Find the slope of $f(x) = x^3 + 10x^2$ when $x = 4$.

9-D Derivatives

derivative • differentiation • power rule • tangent to a curve

1 State the derivative function of a real-world function, using correct units.

1 $B(t)$ is the position of a ball, in meters from the start, after it has rolled for t seconds.

2 Find the derivative of a polynomial function.

2 Find the derivative function for $f(x) = 6x^{10} + x^3 - 4x^2 + 7x - 9$.

3 Use derivatives to find the slope of a function at a given point.

3 Find the slope of the following curves at the given points.

a) $f(x) = 5x^4 - 2x$, at $x = 3$

b) $g(x) = \sin x$, at $x = 60^\circ$

4 Find the equation of a line tangent to a curve at a given x -value a .

4 Find the equation of the line tangent to $f(x) = x^2 - 8x + 5$ at $x = 2$.