Name: **PreCalculus Fall Study Guide**

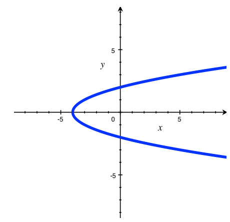
CHAPTER ONE: FUNCTIONS AND THEIR GRAPHS **Due Monday, December 12**

1-A Functions

relation • independent variable • dependent variable • function • vertical line test • argument

❶ Determine whether or not a relation is a function.

❶ Identify which of the following are functions. For those that are not, demonstrate this by giving a single value of x that results in two different values of y.



a) The parabola at right

b) $y = \pm\sqrt{x}$

c) The set of ordered pairs { (3, 5), (5, 2), (3, 9) }

d) $y = \sqrt{x}$

e) y = the team that won the Superbowl in year x

f) y = the year team x won the Superbowl

❷ Identify the argument of a function.

❷ For the equation $\sqrt{2x-1} - 12 = 4\cos(5x - 20) + \tan 6x - 20$, identify the argument of the following functions.

a) the square root function b) the cosine function c) the tangent function

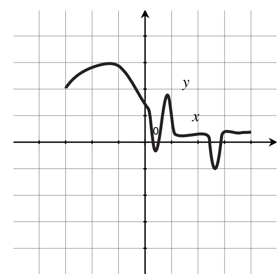
❸ Use function notation.

❸ Write the following in function notation, and then plug in 5 for each independent variable.

a) $y = x^2 + 3x$ b) $A = \pi r^2$ c) Austin makes $10 an hour.

1-B Domain and Range

domain • range



❶ Identify the domain and range of a graphed function.

❶ Identify the domain and range of the function graphed at right.

❷ Identify the domain of a function in function notation.

❷ Identify the domain of the following functions.

a) $a(x) = x$ b) $b(x) = \frac{1}{x}$ c) $c(x) = \sqrt{x}$ d) $d(x) = \log x$

e) $e(x) = \frac{1}{2x-8}$ f) $f(x) = \sqrt{2x-8}$ g) $g(x) = \log(2x - 8)$ h) $h(x) = \frac{\log(2x+7)}{3-\sqrt{20-x}}$

1-C Composition and Inverses

composition • inverse • one-to-one • horizontal line test

❶ Evaluate compositions of functions.

❶ Given $f(x) = 4x - 10$ and $g(x) = x^2 + 2x - 3$, evaluate the following.

a) $f(g(3))$ b) $g(f(3))$

❷ Find a simplified expression for a composition of functions.

❷ Using the functions f and g, above, give an expression for the following.

a) $f(g(x))$ b) $g(f(x))$

❸ Identify the inverse of a basic function by definition.

❸ Identify the inverse of each of the following functions, and verify that $f^{-1}(f(x)) = x$.

a) $a(x) = x + 5$ b) $b(x) = 5x$ c) $c(x) = x^5$ d) $d(x) = 5^x$

❹ Determine whether or not two functions *f* and *g* are inverses of each other.

❹ Are $f(x) = 4x + 8$ and $g(x) = \frac{x}{4} - 8$ inverses of each other?

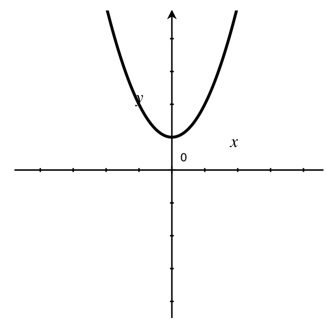
❺ Identify the inverse of a relation conceptually.

❺ $f(x) = 7x$ is the number of minutes it takes Jon to run x miles. (Miles are plugged in to calculate minutes.)

❻ Find the inverse of a relation algebraically.

❻ Find the inverse of the following functions.

a) $f(x) = x - 8$ b) $g(x) = 2x + 1$ c) $h(x) = x^2$



❼ Find the inverse of a relation graphically.

❼ Sketch the graph of the inverse of the parabola shown at right.

❽ Determine whether or not the inverse of a graph is a function.

❽ Why isn't the inverse of the function graphed above also a function?

1-D Calculator Input and Output

scientific notation • significant figures

❶ Identify calculator errors with parentheses and negatives.

❶ Identify and correct the errors in "24+–8²/√(90/11–(–8)+2" to evaluate $f(8)$, given $f(x) = \frac{24 + x^2}{\sqrt{90/11} - x + 2}$.

❷ Quickly evaluate an expression for multiple values of a variable.

❷ Given the function $f(x) = \frac{24 + x^2}{\sqrt{90/11} - x + 2}$, evaulate $f(8)$ and $f(-8)$.

❸ Convert calculator notation to scientific notation and to standard notation.

❸ a) 2.57 E 3 b) 2.57 E -3

❹ Convert standard notation to scientific notation.

❹ a) 2700 b) 2700 exactly c) .002700

❺ Count significant figures.

❺ a) 304,900 b) .0304900 c) 2.0304900 d) 3.04900×10^{-12}

❻ Use appropriate rounding in a word problem involving decimals.

❻ On a statistics test, a problem asked to find a sample size (number of people) by calculating $(\frac{2}{3})(\frac{1}{3})(\frac{3.98}{.72 - 2/3})^2$.

❼ Graph a function on the calculator.

❼ Display $f(x) = x^2$ over the domain $-8 \le x \le 8$ and range $-5 \le y \le 40$.

❽ Change the viewing area of the screen automatically.

1-E Transformations

transformation • pre-image • image • translation • stretch • reflection

❶ Translate a function *h* units right and *k* units up.

❶ Translate the pre-image $f(x) = 2x^2 - 5x - 4$ three units left and five units up.

❷ Stretch a function horizontally by a factor of *b* and vertically by a factor of *a*.

❷ Stretch the pre-image $f(x) = 2 + \sin x$ by a factor of 2 vertically (twice as tall) and by a factor of $\frac{1}{3}$ horizontally (one third as wide).

❸ Reflect a function across the *y*-axis and/or the across the *x*-axis.

❸ Reflect the pre-image $f(x) = x^2 + 3x - 2$ across the stated axis.

a) the *y*-axis

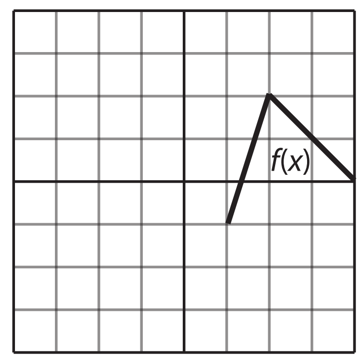
b) the *x*-axis

❹ Apply multiple transformations to a function.

❹ Transform the pre-image $f(x) = x^2 - 6x + 9$ by doing the following.

a) Reflect it across the *x*-axis, and then translate it up four units.

b) Translate it up four units, and then reflect it across the *x*-axis.



❺ Given the graph of $f(x)$, sketch $f(x - h) + k$.

❺ Given $f(x)$, graph $f(x + 3) + 1$.

❻ Given the graph of $f(x)$, sketch $a \cdot f(^x/_b)$.

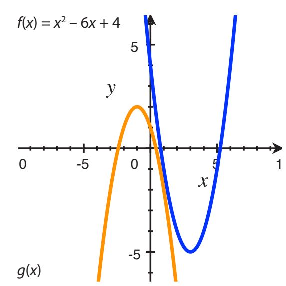
❻ Given $f(x)$, graph $2f(2x)$.

❼ Given the graph of $f(x)$, sketch $f(-x)$ or $-f(x)$.

❼ Given $f(x)$, graph $f(-x)$ and $-f(x)$.

❽ Use the equation of a pre-image to find the equation of a graph.

❽ Write the equation for $g(x)$ graphed at right.



1-F Quadratics

quadratic equation • vertex • minimum • maximum • quadratic formula • *x*-intercept • discriminant

❶ Identify the vertex of a quadratic in vertex form.

❶ a) $y = \frac{1}{8}(x - 2)^2 + 3$

b) $y = \frac{1}{8}(x + 2)^2 - 3$

❷ Find the vertex of a parabola in standard form.

❷ $f(x) = 2x^2 - 10x + 7$

❸ Find the vertex of a parabola in intercept form.

❸ $f(x) = \frac{1}{5}(x - 4)(x + 6)$

❹ Solve a quadratic equation by factoring.

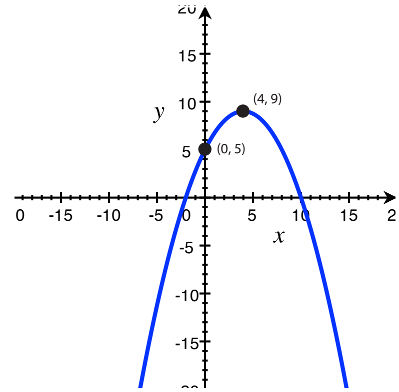
❹ $2x^2 + x = 15$

❺ Solve a quadratic equation with the quadratic formula, and simplify.

❺ $12x^2 + 17x = 7$

❻ Use the discriminant to determine the number of x-intercepts of a parabola.

❻ $y = 9x^2 - 6x + 1$



❼ Sketch the graph of a parabola from an equation.

❼ $y = x^2 - 12x + 20$

❽ Write the equation of a parabola from a graph.

❽ Write the equation of the parabola at right.

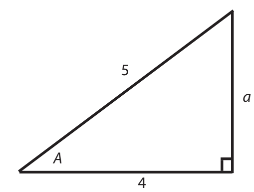
CHAPTER TWO: TRIGONOMETRIC FUNCTIONS **Due Tuesday, December 13**

2-A Angles in Right Triangles

trigonometric function • sine • cosine • tangent • cotangent • secant • cosecant • inverse trigonometric function • $\sin^{-1}$ (arcsin) • $\cos^{-1}$ (arccos) • $\tan^{-1}$ (arctan)

❶ Find the values of each of the six trigonometric functions of an angle in a right triangle with two known sides.

❶ Find the sine, cosine, tangent, cosecant, secant, and cotangent of angle *A* shown at right.

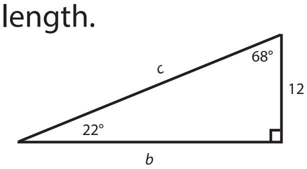


❷ Find values of cotangent, secant, and cosecant on the calculator.

❷ Evaluate sec 25°.

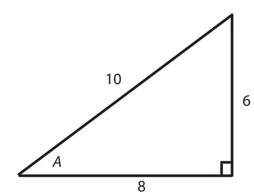
❸ Calculate a side length in a right triangle based on a known angle and known side length.

❸ Solve for *c* in the triangle at right.



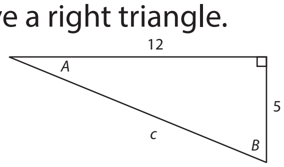
❹ Calculate an angle measure in a right triangle based on two known side lengths.

❹ Solve for *A* in the triangle at right.

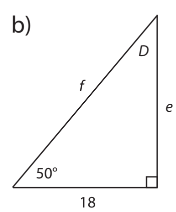


❺ Solve a right triangle.

❺ a)



b)



2-B Angles in Circles

standard position • coterminal • initial side • terminal side • quadrant • reference angle

❶ Find the values of the trigonometric functions for an angle in standard position that passes through a specific point (x, y).

❶ Find the cosine of an angle θ in standard position that passes through the point (3, -2).

❷ Find angles coterminal to a given angle.

❷ Find three angles coterminal to 240°.

❸ Sketch an angle in standard position.

❸ Sketch 840° in standard position.

❹ Find an angle's reference angle.

❹ Find the reference angle for a 300° angle.

❺ Use a reference angle to find the values of the trigonometric functions for an angle θ that is a multiple of 30° or 45°.

❺ Find all six trigonometric ratios for 300°

2-C Radians

radian

❶ Convert between degree and radian measure.

❶ a) Convert $\frac{11\pi}{6}$ to degrees. b) Convert 135° to radians.

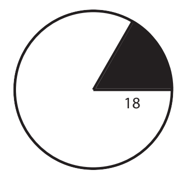
❷ For angles measured in radians, draw sketches, find coterminal angles and reference angles, and find trig ratios.

❷ Do the following for the angle $\theta = \frac{14\pi}{3}$.

a) Find a coterminal angle. b) Sketch it. c) Find its reference angle. d) Find the cosine.

❸ Find the area of a sector.

❸ Find the area of the sector at right.



2-D The Unit Circle

unit circle

❶ Use a unit circle to find the values of the trigonometric functions for an angle θ that is a multiple of $^{\pi}/_{6}$ (30°) or $^{\pi}/_{4}$ (45°).

❶ a) cos 120° b) tan 150° c) sec $\frac{3\pi}{2}$ d) sin $\frac{10\pi}{3}$

❷ Use a unit circle to find the value of an inverse trigonometric function for an angle shown in the unit circle.

❷ Find the following in degrees (if possible).

a) $\sin^{-1} \frac{1}{2}$ b) $\cos^{-1} \frac{1}{2}$ c) $\sin^{-1} -\frac{1}{2}$ d) $\tan^{-1} -1$

e) $\tan^{-1} \frac{\sqrt{3}}{3}$ f) $\cos^{-1} 2$

❷ Find the following in radians (if possible).

a) $\sin^{-1} \frac{\sqrt{2}}{2}$ b) $\tan^{-1} -1$

2-E Trigonometric Identities

reciprocal identity • quotient identity • Pythagorean identity • double angle identity • conjugate • verify

❶ Rewrite a trigonometric expression without fractions.

❶ Simplify $\frac{\sin x}{\cos^2 x}$.

❷ Rewrite a trigonometric expression using only sine and cosine.

❷ Rewrite $\csc^2 x \cot x$.

❸ Rewrite a trigonometric expression using a Pythagorean identity.

❸ Rewrite using a Pythagorean identity.

a) $6 \cot^2 x + 2$

b) $5 \cos x - 3 \sin^2 x$

❹ Split a fraction with multiple terms in the numerator into separate fractions.

❹ Write $\frac{1 - \sin x}{\cos x}$ as two separate fractions.

❺ Add or subtract terms when one or both are fractions.

❺ Subtract $\frac{1}{\sin x} - \frac{\sin x}{\tan x}$.

❻ Use a conjugate to simplify a trigonometric fraction.

❻ Simplify $\frac{\cos x}{1 - \sin x}$.

❼ Factor trigonometric expressions.

❼ Factor.

a) $x^2 + 8x + 15$

b) $\tan^2 x + 8 \tan x + 15$

c) $2x^3 + 16x + 30$

d) $2 \tan^3 x + 16 \tan^2 x + 30 \tan x$

❽ Verify a trigonometric identity.

❽ Verify the following identities, and label each step with one of the methods ❶ through ❼ shown in the notes.

a) $\cot x + \tan x = \csc x \sec x$

b) $\frac{1}{\sin x - \sin x \cos x} = \csc^3 x + \cot x \csc^2 x$

CHAPTER THREE: TRIGONOMETRIC EQUATIONS **Due Thursday, December 15**

3-A Solving Simple Trigonometric Equations Algebraically

general solution

❶ Use the unit circle to find two solutions to a simple trigonometric equation.

❶ a) $\sin \theta = \frac{1}{2}$ b) $\cos \theta = \frac{\sqrt{3}}{2}$ c) $\tan \theta = \frac{\sqrt{3}}{3}$

❷ Use an inverse function to find two solutions to a simple trigonometric equation.

❷ a) $\sin \theta = \frac{1}{2}$ b) $\cos \theta = \frac{\sqrt{3}}{2}$ c) $\tan \theta = \frac{\sqrt{3}}{2}$

❸ Find all solutions to a simple trigonometric equation within a given range.

❸ Find all solutions to $\sin \theta = \frac{1}{2}$ in the range $-360° < \theta < 810°$.

❹ Find the general solution to a simple trigonometric equation.

❹ a) $\sin \theta = \frac{1}{2}$ b) $\tan \theta = 1.38$

3-B Solving Complicated Trigonometric Equations Algebraically

❶ Use algebra and basic trig identities to simplify an equation, and solve it.

❶ Find all solutions to $5 \sec^2 3\theta = 10$ in the range $0° < \theta < 360°$.

❷ Solve a trigonometric equation by factoring.

❷ Find all solutions to $\tan^3 3\theta + \tan^2 3\theta = 20 \tan 3\theta$ in the range $0° \le \theta \le 90°$.

3-C Graphs of Sine and Cosine Functions

amplitude • period • phase shift

❶ Stretch $y = \sin x$ to have a specified amplitude and period.

❶ Write a sine equation that has an amplitude of 4 and a period of $\frac{\pi}{3}$.

❷ Translate $y = a \sin bx$ with a phase shift of c to the right and a vertical shift of d upward.

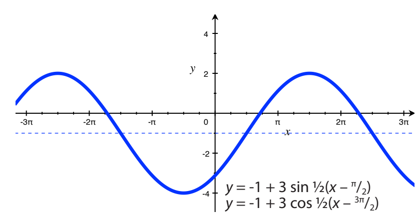
❷ Translate $y = 3 \sin 2x$ left by $\frac{\pi}{4}$ and up by 2.

❸ Identify the amplitude, period, phase shift, and vertical shift of an equation of the form $y = d + a \sin b(x - c)$.

❸ $y = 4 - 5 \sin 6(x + \frac{\pi}{8})$

❹ Write the equation of a graphed sine or cosine function.

❹ Write a sine equation and a cosine equation for the graph at right.



❺ Sketch $y = d + a \sin b(x - c)$ or $y = d + a \cos b(x - c)$.

❺ Graph $y = 1 - 3 \sin \frac{3}{2}(x - \frac{2\pi}{3})$ and $y = 1 - 3 \cos \frac{3}{2}(x - \frac{2\pi}{3})$.

❻ Sketch $y = d + a \sin (bx - bc)$ or $y = d + a \cos (bx - bc)$.

❻ Graph $y = 1 - 3 \sin (\frac{3x}{2} - \pi)$.

3-D Solving Trigonometric Equations Graphically

❶ Solve a system of two equations by graphically finding the points of intersection.

❶ Find the points of intersection of $f(x) = 2 \sin 3x$ and $g(x) = x + 1$.

❷ Solve an equation by finding the points of intersections of two graphs.

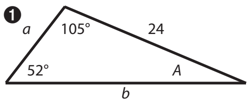
❷ $2 \sin 3x = x + 1$

4-A The Law of Sines

law of sines • altitude • ambiguous case

❶ Solve an AAS or ASA triangle.

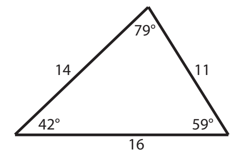
❶



❷ Calculate an altitude of a triangle from a specific base.

❷ For the triangle at right, calculate the altitude from each base.

a) from base 16 b) from base 14 c) from base 11



❸ Given SSA, identify whether zero, one, or two triangles exist.

❸ State the number of triangles that exist for the given information.

a) $A = 30°$, $a = 5$, $c = 20$ b) $A = 30°$, $a = 25$, $c = 20$ c) $A = 30°$, $a = 15$, $c = 20$

❹ Solve an SSA triangle.

❹ Solve and sketch the following triangles, if they exist.

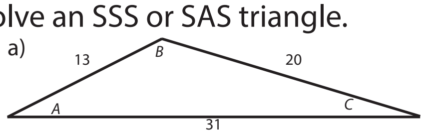
a) $b = 20$, $C = 50°$, $c = 16$ b) $b = 20$, $C = 50°$, $c = 25$ c) $b = 20$, $C = 50°$, $c = 10$

4-B The Law of Cosines

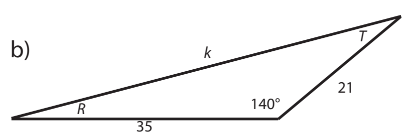
law of cosines

❶ Solve an SSS or SAS triangle.

❶ a)



b)



4-C Areas of Triangles

❶ Find the area of a nonright triangle.

❶ Find the area of a triangle with $m = 8$, $n = 13$, and $p = 15$.

❶ Find the area of a triangle with $m = 20$, $n = 14$, and $P = 116°$.

❶ Find the area of a triangle with $m = 24$, $M = 99°$, $N = 31°$.