Chapter One

Functions and their Graphs

Functions

Domain and Range

Composition and Inverses

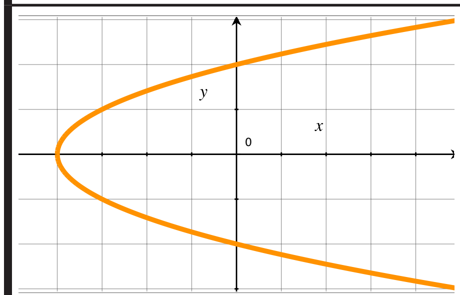
Calculator Input and Output

Transformations

Quadratics

Functions

A **function** yields a specific output (value of the dependent variable) for every possible input (value of the independent variable, or **argument**). Even a single example of a single input having two different outputs means a relation is not a function.

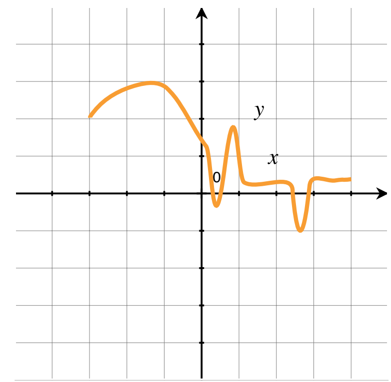
Example	Independent Variable	Dependent Variable	Function?
$A = \pi r^2$	r	A	yes
$d(w) = 7w$	w	$d(w)$	yes
$\{ (1, 4), (2, 5), (2, 9) \}$	x	y	no, because if $x = 2$, y could be either 5 or 9
y = holiday in month x	month	holiday	no, because if x = November, y could be either Thanksgiving or Veterans' Day
	x	y	no, because if $x = 0$, y could be either 2 or -2

Domain and Range

The **domain** of a function is every value that could be plugged in, and the **range** is every value that could result.

The domain of the graph at right is limited to x values between -3 and 4. $f(5)$, for example, has no value.

The range is limited to y values between -1 and 3. Nowhere on the graph does $f(x)$ equal 4, for example.



Type of Function	Restrictions	Example	Domain of Example
Polynomial	none	$a(x) = 5x^2 + 2x$	all real numbers
Rational	denominator must not be zero	$b(x) = \frac{x+5}{2x-6}$	$2x - 6 \neq 0$, so $\mathbf{x \neq 3}$
Even Roots	argument must be greater than or equal to zero	$c(x) = \sqrt{2x-6}$	$2x - 6 \geq 0$, so $\mathbf{x \geq 3}$
Logarithms	argument must be greater than zero	$d(x) = \log(2x-6)$	$2x - 6 > 0$, so $\mathbf{x > 3}$
Real-World	argument must make sense	$e(x)$ = number of high school students in grade x	High school only has grades 9 through 12, so **x = 9, 10, 11, or 12**

Composition

Compostion of functions is plugging an entire function or its value into another function.

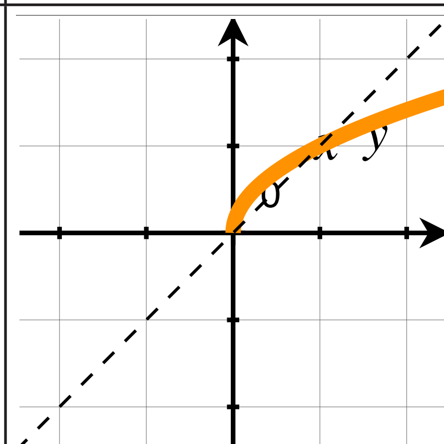
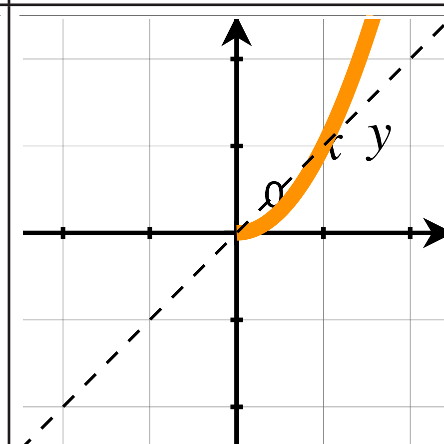
The examples below use the functions $a(x) = 4x^2 + 2x - 5$ and $b(x) = x + 3$.

Function Argument	Procedure	Example	Result
function value	Find the inner function value, and plug it into the outer function.	$a(b(2))$	$b(2) = 5$ $a(5) = \mathbf{105}$
function expression	Replace the independent variable (each time) with the expression being plugged in.	$a(b(x))$	$b(x) = x + 3$ $a(x + 3) =$ $4(x + 3)^2 + 2(x + 3) - 5$ $= \mathbf{4x^2 + 26x + 37}$
composition	Do as above multiple times, working from inside to outside.	$b(b(a(b(2))))$	$a(b(2)) = 105$ (above) $b(105) = 108$ $b(108) = \mathbf{111}$

$a(b(x))$ can also be written $(a \circ b)\ (x)$.

Inverses

If a function's independent and dependent variables are switched, the new relation is the **inverse** of the original. If it too is a function, it undoes the original function, and is labeled by $^{-1}$, such as $a^{-1}(x)$.

Representation	Way to find inverse	Example	Inverse of example
single operation	Identify the opposite of the operation.	$a(x) = x + 5$	$a^{-1}(x) = x - 5$
algebraic	Switch x and y, and solve for the new y.	$b(x) = 2x + 5$	$b^{-1}(x) = \frac{x-5}{2}$
words	Switch the variables.	$c(x)$ = the number of minutes in x hours	$c^{-1}(x)$ = the number of hours in x minutes
points	For each point, switch x with y.	{ (1, 4), (0, 9), (2, -5) }	{ (4, 1), (9, 0), (-5, 2) }
graph	Rotate the graph across $y = x$ diagonal.		

Parentheses

All numerators, denominators, negatives, and arguments have parentheses around them, whether or not they are written.

Reason	Example	Correct	Incorrect
Numerator	$\frac{4+8}{2}$	$(4+8)/2 = \frac{12}{2} =$ **6**	$4+8/2 = 4+\frac{8}{2} = 8$
Denominator	$\frac{24}{8+4}$	$24/(8+4) = \frac{24}{12} =$ **2**	$24/8+4 = \frac{24}{8}+4 = 7$
Negative	$f(-5)$, given $f(x) = x^2$	$(-5)^2 =$ **25**	$-5^2 = -25$
Argument	$\sqrt{3+1}+5$	$\sqrt{}(3+1)+5 = 2+5 =$ **7**	$\sqrt{}(3+1+5) = \sqrt{9} = 3$

Significant Figures

The number of **significant figures** in a value is how many digits were measured or calculated. In normal contexts, nonzeros are always significant, but zeros are not sigifnicant if they are at the end and there is no decimal point or if they are at the beginning and there is a decimal point.

Be careful with calculator scientific notation. In the last example, 3.82×10^{-4} is the same as .000382, but it is very different from 3.82!

Value	Number of significant figures	Rounded to 3 SF
32,490	4	32,500
.032490	5	.0325
3.817007260E-4	10	.000382

In special contexts, significant figures should be counted differently.

Value and Context	Number of significant figures	Reason
"exactly 3000 meters"	4	The distance was measured exactly, down to the meter.
growth factor of 1.025	2	The growth rate is .025.
decay factor of .998	1	The decay rate is .002.

Rounding

In addition to rounding up ending digits of 5, 6, 7, 8, and 9, keep the following guidelines in mind.

Use exact values when possible.	When exact values are known, do not round them.	Given $f(x) = x^{20}$, evaluate $f(4/3)$.	Good: $(4/3)^{20} \approx$ **315** Bad: $1.33^{20} \approx 300$
Use consistent decimal places appropriate for the context.	Use an appropriate, consistent number of decimal places for each answer within a specific context with the same units.	Add 8% tax to a \$6 item and to a \$10 item.	Good: **\$6.48 and \$10.80** Bad: \$6.48 and \$10.8
Use enough significant figures.	Three significant figures are usually good for a final answer, but use more for the work. Avoid rounding to a single digit.	Convert 17 inches to feet.	Good: **1.4 feet** Bad: 1 foot
Don't use digits that aren't significant.	Don't use more significant figures in the answer than were originally given.	Convert 1.5 pounds to grams.	Good: **680 grams** Bad: 680.389 grams
Be careful with growth factors and decay factors.	When multiplying by $1 + r$, count the significant figures in r, not in $1 + r$.	What growth factor takes 5 years to yield an increase of 7.32%?	Good: **1.0142** Bad: 1.01

Transformations

Transformation of a function changes its graph and its corresponding equation. In all contexts, anything done directly to x makes a horizontal change, and anything done directly to y makes a vertical change. See PreCalculus Chart 1 for detailed examples.

Transformation	Vertical	Horizontal	Change in graph
Translation	$f(x) + k$	$f(x - h)$	Each point is moved k units up or h units to the right.
Stretch	$a \cdot f(x)$	$f(\frac{x}{b})$	Each point is a times as far from the x-axis or b times as far from the y-axis.
Reflection	$-f(x)$	$f(-x)$	Each point is reflected to the other side of the x-axis or y-axis.

Quadratics

A **quadratic** equation is one in which the highest power is 2. Its graph is a parabola, the tip of which is called the **vertex**.

Form	Equation	To find the vertex	To find the x-intercepts
vertex	$y = a(x - h)^2 + k$	Identify h and k in the equation.	Plug in $y = 0$ and solve for x by isolating the square or by another method.
standard	$y = ax^2 + bx + c$	Calculate $h = \frac{-b}{2a}$, and then plug in $x = h$ to find k.	Plug in $y = 0$ and solve for x using the quadratic formula or another method.
intercept	$y = a(x - p)(x - q)$	Calculate $h = \frac{p+q}{2}$, and then plug in $x = h$ to find k.	Identify p and q in the equation.

Solving Quadratic Equations

Quadratic equations can be solved in multiple ways. The quadratic formula will work in all situations, but in some cases factoring or completing the square is simpler.

Method	Procedure	$x^2 - 4x + 3 = 15$	Most common error
factor	Set one side equal to zero, factor the other side, and set each factor equal to zero.	$x^2 - 4x - 12 = 0$ $(x - 6)(x + 2) = 0$ $x - 6 = 0$, so $x = 6$ $x + 2 = 0$, so $x = -2$	not setting one side equal to zero before factoring: $(x - 1)(x - 3) = 15$
complete the square	Make one side a perfect square and make the other side a number, so that the square root can be taken of each side.	$x^2 - 4x + 4 = 16$ $(x - 2)^2 = 16$ $x - 2 = \pm 4$ $x - 2 = 4$, so $x = 6$ $x - 2 = -4$, so $x = -2$	not using $\pm$ to show both square roots: $x + 2 = 4$
quadratic formula	Put the equation in standard form equal to zero, and plug *a*, *b*, and *c* into the quadratic formula.	$x^2 - 4x - 12 = 0$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$ $x = \frac{4 \pm \sqrt{64}}{2}$ $x = \frac{4+8}{2} = 6$ $x = \frac{4-8}{2} = -2$	not using parentheses for *b* if it is negative: $x = \frac{-(-4) \pm \sqrt{-4^2 - 4(1)(-12)}}{2(1)}$