Chapter One

Geometry Basics

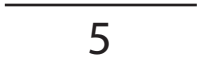
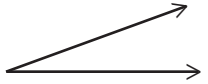
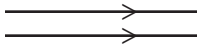
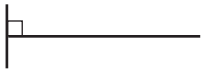
Geometric Notation

Triangles

Bisectors

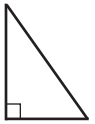
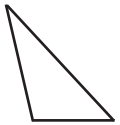
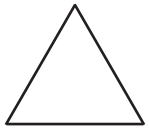
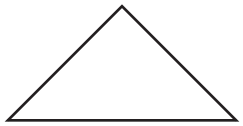
Equations

Basic geometric terminology and notation

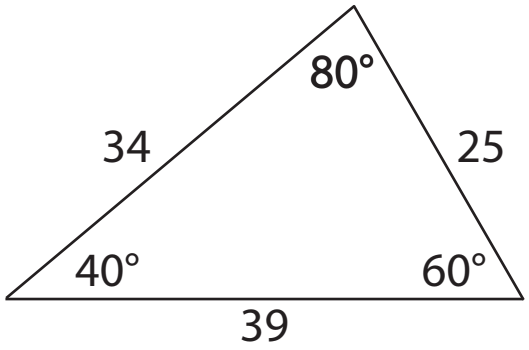
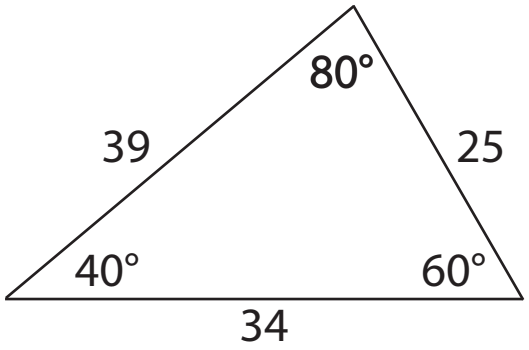
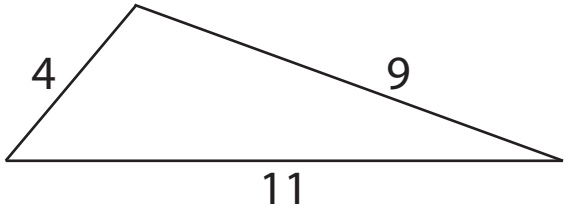
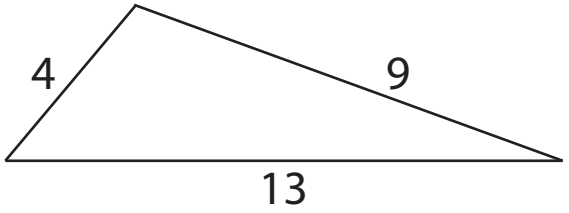
Term	Notation	Definition	Sketch
Point	A	zero-dimensional object	
Line	$\overleftrightarrow{AB}$	one-dimensional object extending infinitely in both directions	
Ray	$\overrightarrow{AB}$	one-dimensional object extending infinitely in one direction	
Line Segment	$\overline{AB}$	finite one-dimensional object	
Length	AB	distance from one point to another	
Angle	$\angle A$	two rays that share a common endpoint, called the vertex	
Congruent	$\cong$	equal in size and shape	
Parallel	$\parallel$	extending forever in a plane but never intersecting	
Perpendicular	$\perp$	intersecting at a 90° angle	

Types of triangles

Every triangle is either acute, right, or obtuse, depending on how big its largest angle is. In addition, every triangle is either equilateral, isosceles, or scalene, depending on how many sides are congruent.

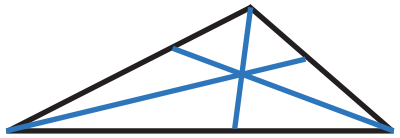
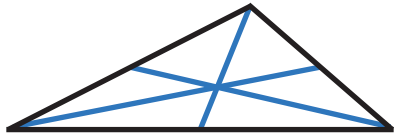
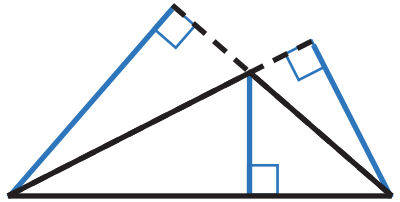
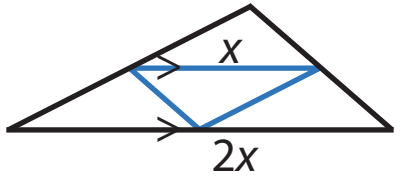
Type	Description	Sketch
Acute	All three angles are under 90°	
Right	One angle is exactly 90°.	
Obtuse	One angle is above 90°.	
Equilateral	All three angles are 60°, making all three sides congruent.	
Isosceles	Two of the sides are congruent.	
Scalene	No sides are congruent.	

Possible Triangles

Restriction	Possible example	Impossible example
The longest side and shortest side must be opposite the largest angle and smallest angle, respectively.		
The longest side must be shorter than the total length of the other two sides.		

Line segments in triangles

Certain line segments are commonly used with triangles. Every triangle has three of each of the following.

Segment	Definition	Sketch	Features
Angle Bisector	cuts angle in half		Every point is equidistant from the sides of the angle. The intersection is within the triangle.
Median	connects midpoint to opposite vertex		The intersection is within the triangle.
Altitude	connects to line containing opposite side at a right angle		If one angle is obtuse, the other two altitudes (and the intersection) are outside the triangle.
Midsegment	connects midpoint of one side to midpoint of another		It is parallel to a side and half as long.

Equation notation

An **expression** is one or more terms added together.

An **equation** is one expression set equal to another.

An **inverse** is the opposite of the original. For example, the inverse of adding 5 is subtracting 5.

Equations are solved by applying one or more inverses equally to each side.

Notation is very important when solving equations. Some common errors are shown below.

Equation	Incorrect	Correct	Error
$x - 5 = 9$	$x - 5 = 9 + 5$	$x - 5 + 5 = 9 + 5$	can't add 5 to one side and not the other
$2x = 12$	$\frac{2x = 12}{2}$	$\frac{2x}{2} = \frac{12}{2}$	can't divide an equals sign
$x^2 = \frac{2}{3}$	$x = \pm\frac{\sqrt{2}}{3}$	$x = \pm\sqrt{\frac{2}{3}}$	root must apply to whole expression
$\sqrt{y} = 3x$	$y = 3x^2$	$y = (3x)^2$	square must apply to whole expression
$-\frac{1}{5}y = x + 3$	$y = (x + 3)\ -5$	$y = (x + 3)\ (-5)$	-5 without parentheses means minus 5
$10^x = 40$	$\log 10^x = 40 \log$	$\log 10^x = \log 40$	function can't be after its argument

The value on one side of an equals sign must equal the value on the other side. For example, if you have 20 grapes, eat half of them, and then given 3 away, $20 \div 2 = 10 - 3 = 7$ gives a correct answer of 7, but the work is incorrect because $20 \div 2 \neq 10 - 3$.

Key geometric equations

Equation	Title	Parameters	Example
$y = mx + b$	line	m = slope b = y-intercept	The equation of the line with slope 4 and y-intercept -6 is $y = 4x - 6$.
$m = \frac{y_2 - y_1}{x_2 - x_1}$	slope	(x_1, y_1) = one point (x_2, y_2) = other point	The slope between the points (-3, 7) and (5, 2) is $m = \frac{2 - 7}{5 - (-3)} = \frac{-5}{8}$.
$M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$	midpoint	(x_1, y_1) = one point (x_2, y_2) = other point	The midpoint between the points (-3, 7) and (5, 2) is $M = (\frac{-3 + 5}{2}, \frac{7 + 2}{2}) = (1, 4.5)$.
$m_2 = -\frac{1}{m_1}$	perpendicular slope	m_1 = original slope	The slope perpendicular to $m_1 = \frac{3}{4}$ is $m_2 = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$.