CHAPTER ONE: GEOMETRY BASICS

Review August 31 ⇝ Test September 12

Terminology and notation are fundamental for any subject. This chapter sets up basic concepts for describing, sketching, and solving geometric scenarios.

1-A Geometric Notation

Tuesday • 8/22

point • line • ray • line segment • plane • parallel • perpendicular • colinear • coplanar • angle • vertex • congruent • midpoint

❶ Identify and label geometric components.
❷ Refer to angles by name.

1-B Triangles

Thursday • 8/24

acute • right • obtuse • isosceles • equilateral • scalene

❶ Identify types of triangles.
❷ Order the sides or angles of a triangle by size.
❸ Identify impossible lengths of triangle sides.

1-C Bisectors

Monday • 8/28

bisector • midsegment • median • altitude • concurrency

❶ Use a triangle midsegment to find lengths and angles.
❷ Use a perpendicular bisector to find a length.
❸ Sketch the medians of a triangle.
❹ Sketch the altitudes of a triangle.
❺ Find lengths and angles within bisected angles.

1-D Equations

Thursday • 8/31

expression • equation

❶ Show proper notation in solving an equation.
❷ Write the equation of a line given two points.
❸ Write the equation of a perpendicular bisector.
❹ Find the distance between two points.

1-A Geometric Notation

The basic building blocks of geometry are the point, line, and plane.

A zero-dimensional object is a POINT. Points are typically labeled with a capital letter.

A one-dimensional object is a LINE if it extends infinitely in both directions (indicated by arrowheads), a RAY if it extends infinitely in only one direction, and a Line SEGMENT if it does not extend infinitely in either direction.

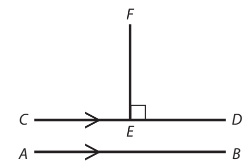
Lines, rays, and line segments can be labeled with two points on them and a bar drawn above. For lines, the bar has arrows on each side, since it extends infinitely in both directions: $\overleftrightarrow{AB}$. For rays, the bar has an arrow on the side that extends infinitely, and the other side is above the point that is the endpoint: $\overrightarrow{AB}$. For line segments, the bar has no arrows, and each point is an endpoint: $\overline{AB}$.

A pair of points without a bar, AB, represents the length of the line segment. Other variations of this notation include $|AB|$ and mAB.

A two-dimensional object is a PLANE if it extends infinitely in all directions.

Lines in a plane that never intersect are PARALLEL, written as $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and indicated graphically with arrowheads in the lines.

Lines that intersect at a 90° angle are PERPENDICULAR, written $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$ and indicated graphically with a box at the intersection.



Points on the same line are COLINEAR. Points and lines on the same plane are COPLANAR.

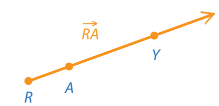
Lines and planes can also be labeled by a single script letter. Use lowercase for lines and uppercase for planes.

Letters used to represent points, lines, and other geometric objects, as well as variables, are italicized when typed.

❶ Identify and label geometric components.

1. Label each point with a capital letter. If they are typed, italicize them.
2. Use commonly accepted notation, such as described above.

❶ Sketch and label a ray that starts at R and passes through A and Y.



An ANGLE is formed by a pair of rays sharing the same endpoint, called the VERTEX.

Angles can be labeled with a number or letter written inside it, or by its vertex, such as A. If labeling it by its vertex, use the angle symbol, such as $\angle A$, to clarify that you are referring to angle A and not point A. If a point is the vertex of more than one angle, use three letters to clarify which angle is intended, with the vertex in the middle, such as $\angle BAC$.

The measure of an angle of an angle can be notated by putting m before the angle, such as $m\angle BAC$.

❷ Refer to angles by name.

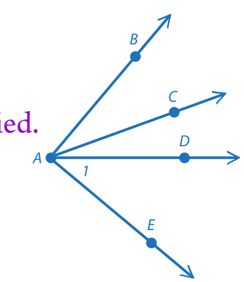
1. If the angle itself is labeled, use this number or letter.
2. If it is not labeled, use the $\angle$ symbol and the vertex.
3. If the vertex is the vertex of other angles in the diagram as well, use additional points to clarify which rays are in the angle specified.

❷ Name each angle shown at right.

The top angle is $\angle BAC$.

The middle angle is $\angle BAD$.

The bottom angle is $\angle BAE$, or simply $\angle 1$.



Objects that are exactly the same size and shape (although not necessarily with the same position or orientation) are CONGRUENT, symbolized by "≅".

The MIDPOINT of a Line Segment divides it into two congruent segments.

Tick marks or otherwise identical marks drawn through different objects identify the objects as congruent.

1-B Triangles

An angle between 0° and 90° is ACUTE.
A 90° angle is RIGHT. Right angles are labeled with a small square at the vertex.
An angle between 90° and 180° is OBTUSE.
The largest angle in a triangle determines whether the triangle itself is acute, right, or obtuse.
If two of the angles in a triangle are congruent, the two sides opposite them are congruent as well, and the triangle is ISOSCELES.
If all three angles in a triangle are congruent, the three sides are congruent as well, and the triangle is EQUILATERAL.
Otherwise the triangle is SCALENE.

❶ Identify types of triangles.

1. Identify if any side is congruent to another or if any angle is congruent to another.
   If so, the triangle is isosceles, and if all three sides or angles are equal, it is also equilateral.
   Otherwise the triangle is scalene.
2. Identify whether the largest angle is acute (< 90°), right (90°), or obtuse (> 90°).

❶ Identify the type of triangle shown at right.

1. Two sides are congruent, so it is isosceles.
   The third is not, so it is not equilateral.
2. The largest angle is more than 90°, so it is obtuse.

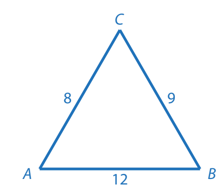
The lengths of the sides of a triangle are limited in two ways:

- The side opposite the largest angle must be the largest side, and the side opposite the smallest angle must be the smallest side.
- Each side must be shorter than the sum of the other two sides.

❷ Order the sides or angles of a triangle by size.

1. The larger an angle is in a triangle, the longer the side opposite it is compared to the other sides, and vice versa.

❷ Rank the angles in the triangle at right (not drawn to scale) from largest to smallest.



1. $C < A < B$

❸ Identify impossible lengths of triangle sides.

1. If all three sides are given, add the two smallest sides. If this total is not larger than the largest side, the triangle does not exist.
2. If two sides AB and BC are given ($AB > BC$), the length of the third side must be longer than $AB - BC$ and shorter than $AB + BC$.

❸ Given $RS = 5$ and $QS = 8$, what are the possible lengths of QR?

2. $QR > 8 - 5 = 3$
   $QR < 8 + 5 = 13$

1-C Bisectors

A BISECTOR of a line segment goes through the line segment's midpoint.

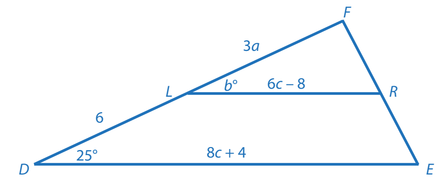
A Triangle MIDSEGMENT connects the midpoints of two sides of a triangle.

The Triangle Midesgment Theorem states that a triangle midsegment is parallel to the third side of the triangle and half as long.

❶ Use a triangle midsegment to find lengths and angles.

1. Each midsegment bisects two sides of the triangle.
2. Each midsegment is parallel to the third side of the triangle.
3. The length of each midsegment is half the length of the third side of the triangle.

❶ Find the values of a, b, and c in the diagram at right, given $\overline{LR}$ is a midsegment of ΔDEF.



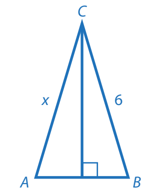
1. $3a = 6$ so $a = 2$
2. $\overleftrightarrow{DE} \parallel \overleftrightarrow{LR}$, so $b = 25°$
3. $6c - 8 = \frac{1}{2}(8c + 4)$, so $c = 5$

The Perpendicular Bisector Theorem states that every point on the perpendicular bisector of a segment is an equal distance from each of the two ends of the segment. The converse is true as well.

❷ Use a perpendicular bisector to find a length.

1. Sketch a triangle by connecting the ends of the line segment with a point on the perpendicular bisector. This triangle is isosceles.

❷ Find the value of x in the diagram at right.



1. $\overline{AC} \cong \overline{BC}$, so $x = 6$

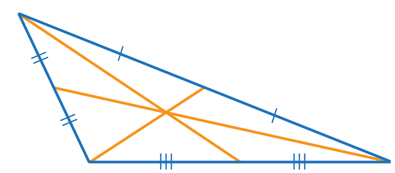
A MEDIAN goes from a triangle vertex to the midpoint of the opposite side. The medians of a triangle all meet at a point called the

An ALTITUDE goes from a triangle vertex to the line containing the opposite side and is perpendicular to this line. In an obtuse triangle, two of the altitudes are outside the triangle.

❸ Sketch the medians of a triangle.

1. Identify the midpoint of each side. Label each half as congruent to the other half.
2. Sketch each median by connecting each midpoint with the opposite vertex.

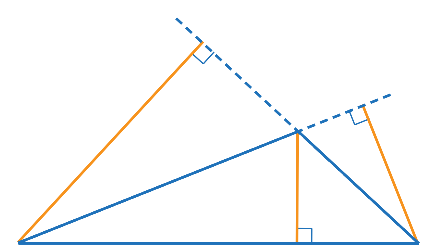
❸ Sketch the medians of the triangle at right.



❹ Sketch the altitudes of a triangle.

1. If the triangle is obtuse, extend the sides past their opposite vertices.
2. Sketch each altitude by drawing a perpendicular from each side to the opposite vertex. Label each angle as right.

❹ Sketch the altitudes of the triangle at right.

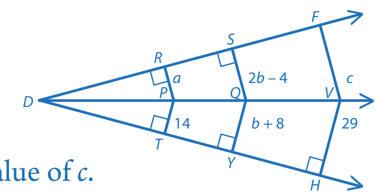


A bisector of an angle cuts the angle in half. The Angle Bisector Theorem states that every point on the bisector of an angle is an equal distance from each of the two rays forming the angle. (This distance is a segment perpendicular to the ray.) The converse is true as well.

❺ Find lengths and angles within bisected angles.

1. Identify any known line segment that is perpendicular to a side of the angle and that connects the side of the angle to the angle bisector.
2. The length of this segment is equal to the length of the segment that is perpendicular to the other side of the angle and that connects that side to the angle bisector.

❺ Find the values of a, b, and c in the diagram at right.



a) $\overline{PR} \perp \overrightarrow{DR}$ and $\overline{PT} \perp \overrightarrow{DT}$. By the perpendicular bisector theorem, $\overline{PR} \cong \overline{PT}$, so $a = 14$.

b) $\overline{QS} \perp \overrightarrow{DS}$ and $\overline{QY} \perp \overrightarrow{DY}$. By the perpendicular bisector theorem, $\overline{QS} \cong \overline{QY}$, so $2b - 4 = b + 8$, making $b = 12$.

c) Because it is unknown whether or not $\overline{VF}$ is perpendicular to $\overrightarrow{DF}$, we do not have enough information to find the value of c.

A point where three or more lines meet is a Point of CONCURRENCY.

The point of concurrency for a triangle's angle bisectors is called the *incenter*, because it is the center of a circle inscribed within the triangle.

The point of concurrency for a triangle's perpendicular bisectors is called the *circumcenter*, because it is the center of a circle circumscribed about the triangle.

The point of concurrency for a triangle's altitudes is called the *orthocenter*, because the altitudes are orthogonal (perpendicular) with the sides.

The point of concurrency for the medians is called the *centroid*, because this would be the center of mass if the triangle were a physical object.

1-D Equations

An Algebraic EXPRESSION is one or more terms added together.

An Algebraic EQUATION is one expression set equal to another.

An equation is solved by applying one or more inverse operations to each side of the equation.

❶ Show proper notation in solving an equation.

1. Identify the units used in word problems.
2. Apply the necessary operation to each side of the equation (not to the equation as a whole). Be sure each symbol is written in an appropriate place. Show every step except those that are both simple enough to recognize easily and not new to the current chapter.
3. Write the new equation. Be sure the expressions on either side of the equals sign are equivalent.
4. Repeat steps 2 and 3 as needed.

❶ Collin has \$900 this year and plans to save \$175 per year. In what year will he have \$1600?

Incorrect	Reason	Correct
		x = years after 2017
$900 + 175x = 1600$		$900 + 175x = 1600$
$\div\, 175\ (175x = 700)$	An equals sign cannot be divided,	$175x = 700$
	and the ÷ symbol can't be before the dividend	$175x \div 175 = 700 \div 175$
$x = 700 \div 175 = 4 + 2017 = 2021$	$700 \div 175 \neq 4 + 2017$	$x = 4$
		He will have \$1600 in the year $4 + 2017 = 2021$.

$y = mx + b$ is the equation of a line, where m is the slope, b is the y-intercept, and x and y are variables.
The slope between two points can be found by dividing the vertical distance between them by the horizontal distance between them: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
If the slope is known, the y-intercept of a line can be calculated by plugging in the x value and the y value of any point on the line and solving for b.

❷ Write the equation of a line given two points.

1. Use the slope formula above to calculate m. Make sure to subtract the x values in the same direction as you subtracted the y values.
2. In the equation $y = mx + b$, plug in the given values of x and y from one of the two points and the calculated value of m.
3. Solve for b.
4. Write the equation using the calculated value of m and of b.

❷ Write an equation of the line passing through the points (-1, 6) and (3, 4).

1. $m = \frac{4-6}{3-(-1)} = \frac{-2}{4} = -\frac{1}{2}$
2. $4 = -\frac{1}{2}(3) + b$
3. $\frac{5}{2} = b$
4. $y = -\frac{1}{2}x + \frac{5}{2}$

The midpoint of the line segment between two points is the point that averages the two x-coordinates and averages the two y-coordinates: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
The slope of a perpendicular is the negative reciprocal of the original slope: $m_2 = -\frac{1}{m_1}$.

❸ Write the equation of a perpendicular bisector.

1. Use the slope formula to calculate the slope of the original line segment.
2. Take the negative reciprocal of this slope to find the slope m of the perpendicular bisector.
3. Use the midpoint formula to calculate the midpoint of the original line segment.
4. In the equation $y = mx + b$, plug in the x and y values of the midpoint and the calculated value of m.
5. Solve for b.
6. Write the equation using the calculated value of m and of b.

❸ Write an equation of the perpendicular bisector of the line segment with endpoints (-1, 6) and (3, 4).

1. $m_1 = \frac{4-6}{3-(-1)} = \frac{-2}{4} = -\frac{1}{2}$
2. $m = -\left(\frac{1}{-\frac{1}{2}}\right) = 2$
3. $(x, y) = \left(\frac{-1+3}{2}, \frac{6+4}{2}\right) = (1, 5)$
4. $5 = 2(1) + b$
5. $3 = b$
6. $y = 2x + 3$

The distance between two points can be found using the Pythagorean theorem $c^2 = a^2 + b^2$: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

❹ Find the distance between two points.

1. Find the horizontal distance between the points, $a = x_2 - x_1$.
2. Find the vertical distance between the points, $b = y_2 - y_1$.
3. Use the Pythagorean theorem. Be sure to put any negative number in parentheses before squaring it.

❹ Find the distance between the points (-1, 6) and (3, 4).

1. $a = 3 - (-1) = 4$
2. $b = 4 - 6 = -2$
3. $AB = \sqrt{4^2 + (-2)^2} = \sqrt{20} \approx 4.5$