CHAPTER FIVE: CIRCLES

Review January 17 ⟿ Test January 23

Circles are the most basic of shapes, and they are fundamental to many fields of mathematics.

5-A Congruent and Supplementary Angles — Monday • 1/9

complementary • supplementary • transversal • linear angles • vertical angles • corresponding angles • alternate exterior angles • alternate exterior angles

❶ Use properties of angles to determine angle measures.

5-B Components of Circles — Wednesday • 1/11

circle • radius • arc • semicircle • minor arc • major arc • sector • chord • diameter • central angle • inscribed angle • circumscribed angle • secant • tangent • normal

❶ Find the measure of a circle's circumference and area based on its radius.
❷ Find the measure of an arc and sector of a circle based on the radius and angle.
❸ Solve for the measure of an arc or angle in a circle.

5-C Circle Theorems — Friday • 1/13

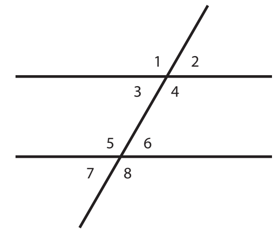
❶ Sketch and label a diagram to illustrate a circle theorem.
❷ Prove a circle theorem.
❸ Use circle theorems to calculate measures of angles or lengths.

5-A Congruent and Supplementary Angles

Two angles are COMPLEMENTARY if they total 90°.

Two angles are SUPPLEMENTARY if they total 180°.

A TRANSVERSAL is a line that cuts through two other lines at two different points. A transversal through parallel lines creates eight angles, as shown below, each of which is supplementary with or congruent to each of the others.



There are four pairs of LINEAR Angles, such as ∠1 and ∠2. Linear angles are supplementary.

There are four pairs of VERTICAL Angles, such as ∠1 and ∠4. Vertical angles are congruent.

There are four pairs of CORRESPONDING Angles, such as ∠1 and ∠5. Corresponding angles are congruent.

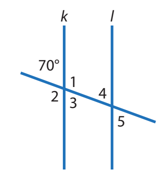
There are four pairs of ALTERNATE EXTERIOR Angles, such as ∠1 and ∠8. Alternate exterior angles are congruent.

There are four pairs of ALTERNATE INTERIOR Angles, such as ∠3 and ∠6. Alternate interior angles are congruent.

❶ Use properties of angles to determine angle measures.

1. Identify any pair of angles that are linear (together they make a line). The measure of one can be found by subtracting the other from 180°.
2. Identify any pair of angles that are vertical (they are opposite each other but made from the same lines). These angles are congruent.
3. Identify any pair of angles made by a transversal through parallel lines. The angles are congruent if they are corresponding (they are positioned the same on each of the two parallel lines) or if they are alternate interior or alternate exterior (they would be on top of each other if the diagram were rotated 180° about its center).

❶ Given *k* and *l* are parallel in the diagram at right, state the relationship between the 70° angle and each of the other labeled angles, and use this to determine the measure of each labeled angle.



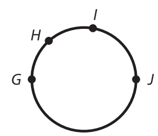
1. ∠1 and the 70° angle are linear, so they are supplementary and ∠1 = 180° – 70° = 110°. ∠2 = 110° for the same reason.
2. ∠3 and the 70° angle are vertical, so they are congruent and ∠3 = 70°.
3. ∠4 and the 70° angle are corresponding, so they are congruent and ∠4 = 70°.
   ∠5 and the 70° angle are alternate exterior angles, so they are congruent and ∠5 = 70°.

5-B Components of Circles

A CIRCLE with RADIUS r is the set of points exactly r units from a given center.

An ARC of a circle is a partial circle. A SEMICIRCLE is exactly half a circle. A MINOR Arc is a semicircle or smaller. A MAJOR Arc is a semicircle or larger.

Arcs are labeled with an arc above the two endpoints, such as $\overset{\frown}{HJ}$ at right. For major arcs, use a middle point as well, such as $\overset{\frown}{HGJ}$.



$\pi \approx 3.1416$ is an irrational number, meaning that it goes on forever with no pattern, and thus cannot be written as an exact fraction.

The circumference of a circle is 2π times its radius: $C = 2\pi r$.

The area of a circle is π times the square of its radius: $A = \pi r^2$.

Given that an arc with measure m is $\frac{m}{360°}$ of a circle, the length of an arc can be found by multiplying the circle's circumference by this fraction: $L = \frac{m}{360°} \cdot 2\pi r$.

Likewise, the area of a SECTOR of a Circle defined by an arc can be found by multiplying the circle's area by this fraction: $A = \frac{m}{360°} \cdot \pi r^2$.

❶ Find the measure of a circle's circumference and area based on its radius.

1. Calculate $C = 2\pi r$ for the circumference.
2. Calculate $A = \pi r^2$ for the area.

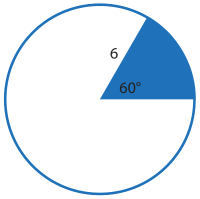
❶ Find the circumference and area of a circle with radius 10.

1. $C = 2\pi(10) = 20\pi$
2. $A = \pi(10)^2 = 100\pi$

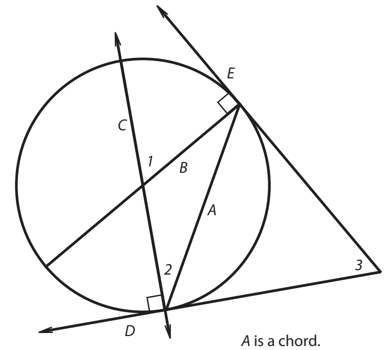
❷ Find the measure of an arc and sector of a circle based on the radius and angle.

1. Calculate the circumference and area of the whole circle (see ❶).
2. Multiply these values by the fraction of the circle included by the $m°$ arc, which is $\frac{m}{360°}$.

❷ Find the arc length and sector area for the 60° angle shown at right.



1. $C = 2\pi(6) = 12\pi$
   $A = \pi(6)^2 = 36\pi$
2. $L = \frac{m}{360°} \cdot 12\pi = 2\pi$
   $A = \frac{m}{360°} \cdot 36\pi = 6\pi$



1 is a central angle.
2 is an inscribed angle.
3 is a circumscribed angle.
A is a chord.
B is a diameter.
C is a secant.
D and *E* are tangents.

A CHORD is a line segment from one point on a curve to another. A DIAMETER is a chord of a circle that passes through the center.

Arcs formed by congruent chords are congruent.

A CENTRAL Angle is composed of two radii. It is the same size, in degrees, as the arc it determines.

An INSCRIBED Angle is composed of two chords that intersect the circle at the same point. It is half the size, in degrees, as the arc it determines.

The arcs determined by two congruent chords in a circle are congruent.

A CIRCUMSCRIBED Angle is composed of two tangents. It is supplementary to the central angle opposite it.

The arcs determined between two parallel lines are congruent.

A SECANT is a line that cuts through a circle.

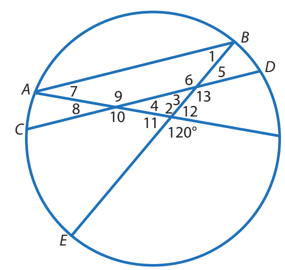
A TANGENT to a Curve is a line that touches the curve at a single point without crossing it.

The NORMAL to a Curve at a given point is the line passing through it that is perpendicular to the tangent. If the curve is a circle, the normal passes through the center.

❸ Solve for the measure of an arc or angle in a circle.

1. Identify any central angles. Their measures are the same as that of their intercepted arcs.
2. Identify any inscribed angles. Their measures are half that of their intercepted arcs.
3. Use identified congruencies to label unknown values.
4. Use congruencies based on vertical angles, alternate angles, etc. to label unknown values.
5. Mark as perpendicular the intersection of any tangent line and diameter.
6. Subtract known values of angles in triangles from 180°, known values of angles in quadrilaterals from 360°, known values of arcs or central angles in circles from 360°, etc., to find the remaining values.

❸ Given $\overline{AB}$ and $\overline{CD}$ are parallel, and given $\overset{\frown}{AE} = 72°$, find the measure of each angles 1 through 7.



Angle	Measure	Justification
1	72° ÷ 2 = 36°	∠1 is half the measure of the inscribed arc *AE*.
2	120°	∠2 and the labeled 120° angle are vertical angles.
3	36°	∠3 and ∠1 are corresponding angles.
4	180° – 120° – 36° = 24°	∠1, ∠2, and ∠3 make a triangle, which totals 180°.
5	36°	∠5 and ∠1 are alternate interior angles.
6	180° – 36° = 144°	∠6 and ∠5 are linear angles.
7	24°	∠7 and ∠4 are corresponding angles.

Note that the other labeled angles can be found by subtracting a linear angle from 180°.

5-C Circle Theorems

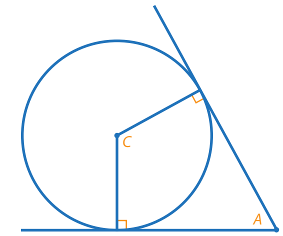
Many theorems can be proved about circles and the lines that intersect them. Below are some key theorems from *Big Ideas Math*, including those informally stated in 5-B.

Tangent Line to Circle	In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.
Congruent Corresponding Chords	In the same circle, or in congruent circles, two minor arcs are congruent if and only if their correspoinding chords are congruent.
Perpendicular Chord Bisector	If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.
Perpendicular Chord Bisector converse	If a chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.
Equidistant Chords	In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.
Measure of an Inscribed Angle	The measure of an inscribed angle is one-half the measure of its intercepted arc.
Inscribed Quadrilateral	A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.
Tangent and Intersected Chord	If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.
Angles Inside the Circle	If two chords intersect inside a circle, then the measure of each angle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
Angles Outside the Circle	If a tangent and a secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.
Circumscribed Angle	The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercetps the same arc.
Segments of Chords	If two chords intersect in the interior of a circle, the the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.
Segments of Secants	If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.
Segments of Secants and Tangents	If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

❶ Sketch and label a diagram to illustrate a circle theorem.

1. Sketch the circle.
2. Sketch the other components mentioned in the theorem.
3. Label congruent components, right angles, etc., as needed to establish the theorem.

❶ Circumscribed angle theorem



$\angle A = 180° - \angle C$

❷ Use a two-column proof to prove a circle theorem.

1. In the left column, state what is given. In the right column, write “given.”
2. In the left column, state what can be concluded based on what is written above. In the left column, state the property, postulate, theorem, etc., that establishes the validity of the statement in the left column, or explain it if no such concept is available.
3. Repeat step 2 until the statement in the left column is that which the theorem claims.

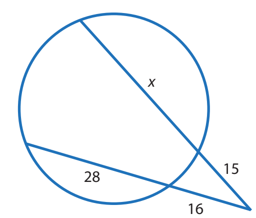
❷ Circumscribed angle theorem

1. A given angle is circumscribed. Call this angle A.	given
2. $\angle A$ is formed by two tangents to a circle.	definition of circumscribed angle
3. The tangents are perpendicular with the radii they intersect. Call these points B and D.	tangent line to circle theorem
4. $\angle B$ and $\angle D$ are each 90°.	Perpendicular lines make 90° angles.
5. The radii through points B and D make a central angle. Call this angle C.	definition of central angle
6. $ABCD$ is a quadrilateral.	Quadrilaterals have four vertices.
7. The sum of the interior angles of $ABCD$ is 360°	The sum of the interior angles of an n-gon is $180°(n-2)$.
8. $\angle A + 90° + 90° + \angle C = 360°$	The interior angles of $ABCD$ are $\angle A$, $\angle B$, $\angle C$, and $\angle D$.
9. $\angle A = 180° - \angle C$	subtraction property of equality

❸ Use circle theorems to calculate measures of angles or lengths.

1. Identify a theorem that can provide additional information based on the given information.
2. Apply the theorem.
3. Repeat steps 1 and 2 as needed.

❸ Find the value of x in the diagram.



1. segments of secants theorem
2. $16(16+28) = 15(15+x)$

$704 = 225 + 15x$

$x \approx 32$