

CHAPTER THREE: QUADRATICS**Review November 2** ↻ **Test November 9**

The most common functions in math at this level are quadratic functions, whose graphs are parabolas. Important in this chapter and future math is the relationship between x -intercepts, zeros, roots, and solutions. Roughly speaking, all of these refer to values which make y equal zero. For quadratic equations, they can be found by graphing, factoring, completing the square, or using the quadratic formula. Sometimes they will not turn out to be real numbers and instead are what are called imaginary, such as if $x^2 = -1$ which has as solutions the imaginary numbers $x = \pm\sqrt{-1}$. This happens when the parabola is completely above or below the x -axis and never crosses it, and thus there are no x -intercepts.

3-A Graphs of Quadratic Functions**Tuesday • 10/18**

standard form • vertex form • vertex • axis of symmetry • maximum • minimum

- 1 Identify the vertex of a quadratic in vertex form.
- 2 Write a quadratic equation in standard form.
- 3 Find the vertex of a parabola in standard form.
- 4 Find the vertex of a parabola in intercept form.
- 5 Sketch a parabola and its axis of symmetry.

3-B Simplifying Radical Expressions**Thursday • 10/20**

radicand • radical

- 1 Simplify a square root.
- 2 Simplify an n^{th} root.
- 3 Reduce a rational expression.
- 4 Rationalize a denominator with one or two terms.

3-C Complex Numbers**Monday • 10/24**

imaginary number • complex number • complex plane • complex conjugate

- 1 Write a fractional complex number in standard form.
- 2 Plot numbers on the complex plane.
- 3 Solve equations with complex solutions.
- 4 Add, subtract, and multiply complex numbers.
- 5 Divide complex numbers.

3-D Solving Quadratic Equations**Thursday • 10/27**completing the square • quadratic formula • solution • zero • root • x -intercept

- ① Solve a quadratic equation by factoring.
- ② Solve a quadratic equation by isolating a square.
- ③ Solve a quadratic equation by completing the square.
- ④ Solve a quadratic equation with the quadratic formula.
- ⑤ Solve any quadratic equation.
- ⑥ Find the solutions, zeros, roots, or x -intercepts of a quadratic.

3-E Conic Sections**Tuesday • 11/1**

conic section • ellipse • circle • hyperbola • asymptote

- ① Sketch an ellipse given in standard form.
- ② Sketch a hyperbola given in standard form.
- ③ Identify ellipses and hyperbolas.
- ④ Write in standard form the equation of an ellipse or hyperbola written as a polynomial.

3-A Graphs of Quadratic Equations

VERTEX Form of a Quadratic is $y = a(x - h)^2 + k$. The VERTEX (h, k) is the tip of the parabola.

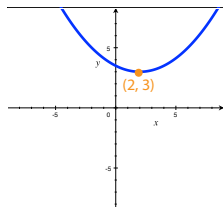
① Identify the vertex of a quadratic in vertex form.

1. Identify h and k .
2. The vertex is (h, k) .

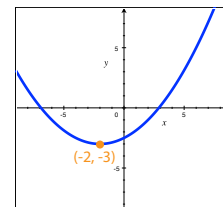
① Identify the vertex.

a) $y = \frac{1}{8}(x - 2)^2 + 3$

1. $h = 2, k = 3$
2. $(2, 3)$



b) $y = \frac{1}{8}(x + 2)^2 - 3$
 $h = -2, k = -3$
 $(-2, -3)$



STANDARD Form of a Quadratic Equation is $y = ax^2 + bx + c$ (or $ax^2 + bx + c = 0$).

② Write a quadratic equation in standard form.

1. Get y or zero by itself on one side.
2. Multiply as needed to remove parentheses.
3. Combine like terms, putting the combined terms in order from highest exponent to lowest.

② $2x^2 = 3(x + 5)^2 + 100$

1. $3(x + 5)^2 + 100 - 2x^2 = 0$

2. $3(x^2 + 10x + 25) + 100 - 2x^2 = 0$

$3x^2 + 30x + 75 + 100 - 2x^2 = 0$

3. $x^2 + 30x + 175 = 0$

For a quadratic equation in standard form $f(x) = ax^2 + bx + c$, $h = \frac{-b}{2a}$ and $k = f(h)$.

③ Find the vertex of a parabola in standard form.

1. Identify a and b .
2. Calculate $h = -b \div 2a$.
3. Plug h into the equation to find k .
4. The vertex is (h, k) .

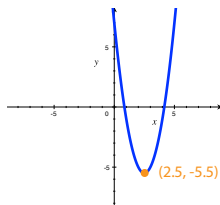
③ $f(x) = 2x^2 - 10x + 7$

1. $a = 2, b = -10$

2. $h = \frac{-(-10)}{2(2)} = 2.5$

3. $k = f(2.5) = 2(2.5)^2 - 10(2.5) + 7 = -5.5$

4. The vertex is $(2.5, -5.5)$.



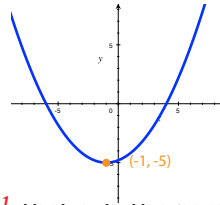
INTERCEPT Form of a Quadratic is $y = a(x - p)(x - q)$. The x -intercepts are $(p, 0)$ and $(q, 0)$, and the x -coordinate of the vertex is midway between these:
 $h = \frac{p+q}{2}$. As always, $k = f(h)$.

④ Find the vertex of a parabola in intercept form.

1. Identify p and q .
2. Calculate $h = (p + q) \div 2$.
3. Plug h into the equation to find k .
4. The vertex is (h, k) .

④ $f(x) = \frac{1}{5}(x - 4)(x + 6)$

1. $p = 4, q = -6$
2. $h = \frac{4-6}{2} = -1$
3. $k = \frac{1}{5}(-1 - 4)(-1 + 6) = -5$
4. The vertex is $(-1, -5)$.



The AXIS OF SYMMETRY of a Parabola is the line $x = h$ that cuts the parabola exactly in half.

If a is positive, the parabola opens upward and k is the MINIMUM of the quadratic.

If a is negative, the parabola opens downward and k is the MAXIMUM of the quadratic.

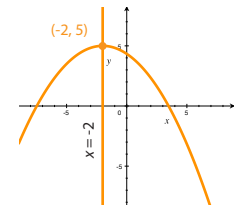
The closer a is to 0, the wider the parabola is.

⑤ Sketch a parabola and its axis of symmetry.

1. Find and plot the vertex (h, k) (see ①, ③, or ④).
2. Draw the axis of symmetry, which is the vertical line $x = h$ through the vertex.
3. Use the value of a to determine the direction and approximate shape of the parabola.

⑤ $y = \frac{1}{6}(x + 2)^2 + 5$

1. The vertex is $(-2, 5)$
2. The axis of symmetry is $x = -2$.
3. $a = \frac{1}{6}$ is positive and close to zero, so the parabola opens down and is fairly wide.



3-B Simplifying Radical Expressions

An expression under a root sign is a RADICAND. Together with the root sign it is called a RADICAL.

$a^{1/n}$ means $\sqrt[n]{a}$. Therefore, the properties of powers listed in section 5-1 can be applied to roots as well. In particular, by the product of powers property

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}.$$

1 Simplify a square root.

1. Use the product property to separate the radical into two radicals, the first of which is the square root of a square. Keep in mind that a variable with an even exponent can be written as a square: $x^{2a} = (x^a)^2$.
2. Take the square root of the first radical.
3. Repeat steps 1 and 2 with the remaining radical, if possible.
4. Simplify if needed.

$$\textcircled{1} \sqrt{75x^2y^6z^7}$$

$$1. \sqrt{25x^2y^6z^6} \sqrt{3z}$$

$$2. 5xy^3z^3 \sqrt{3z}$$

2 Simplify an n^{th} root.

1. Do steps 1-4 above, except use n^{th} roots instead of square roots. Keep in mind that a variable with an exponent divisible by n can be written as an n^{th} power: $x^{na} = (x^a)^n$.

$$\textcircled{2} \sqrt[3]{800x^3y^6z^8}$$

$$2. \sqrt[3]{8x^3y^6z^6} \sqrt[3]{100z^2}$$

$$3. 2xy^2z^2 \sqrt[3]{100z^2}$$

A fraction can only be reduced if every term in the numerator and the denominator can be divided by the same factor.

3 Reduce a rational expression.

1. Identify the terms that add together to equal the numerator.
2. Repeat step 2 for the denominator.
3. Divide each term by any factor that can divide into every term in both the numerator and denominator. This factor must not equal zero since you are dividing by it, so specify what value(s) of the variable, if any, would not work.
4. Repeat step 3 until there are no more factors that divide into every term.

$$\textcircled{3} \frac{6x^2 - 14x\sqrt{6x}}{8x^4 + 24x^3}$$

1. The terms in the numerator are $6x^2$ and $-14x\sqrt{6x}$. Note that $-14x\sqrt{6x}$ is a single term, rather than $-14x$ and $\sqrt{6x}$ being two separate terms.

2. The terms in the denominator are $8x^4$ and $24x^3$.

3. $2x$ can be divided out from each of the four terms, assuming $2x$ does not equal zero.

$$\frac{3x - 7\sqrt{6x}}{4x^3 + 12x^2}, x \neq 0$$

The CONJUGATE of a number $a + \sqrt{b}$ is $a - \sqrt{b}$.

Multiplying a number by its conjugate results in a rational (nonradical) number: $(a + \sqrt{b})(a - \sqrt{b}) = a^2 + a\sqrt{b} - a\sqrt{b} - (\sqrt{b})^2 = a^2 - b$.

A denominator with a radical is not considered simplified. To RATIONALIZE a Denominator is to rewrite a fraction so that there is no radical in the denominator.

④ Rationalize a denominator with one or two terms.

1. Multiply the numerator and denominator by the denominator if it is a single term, or by the conjugate of the denominator if it has two terms.
2. Simplify the radical.
3. Reduce.

④ Simplify.

a) $\frac{12}{\sqrt{20}}$

1. $\frac{12}{\sqrt{20}} \cdot \frac{\sqrt{20}}{\sqrt{20}} = \frac{12\sqrt{20}}{20}$
2. $\frac{12(2\sqrt{5})}{20} = \frac{24\sqrt{5}}{20}$
3. $\frac{6\sqrt{5}}{5}$

b) $\frac{12}{8 - \sqrt{20}}$

$$\frac{12}{8 - \sqrt{20}} \cdot \frac{8 + \sqrt{20}}{8 + \sqrt{20}} = \frac{96 + 12\sqrt{20}}{8^2 - 20}$$

$$\frac{96 + 12(2\sqrt{5})}{44} = \frac{96 + 24\sqrt{5}}{44}$$

$$\frac{48 + 12\sqrt{5}}{22}$$

3-C Complex Numbers

A number involving the square root of a negative is IMAGINARY (Nonreal). The Imaginary Unit i is defined as $i = \sqrt{-1}$.

A COMPLEX Number is a number that can be written in the form $a + bi$, where a and b are real numbers. Since b can be zero, all numbers can be considered complex.

Standard form of a complex number is $a + bi$.

① Write a fractional complex number in standard form.

1. Split it into its real part and its imaginary part, each with the same denominator.

① Write $\frac{-10+14i}{9}$ in standard form.

$$\frac{-10}{9} + \frac{14}{9}i$$

A complex number $a + bi$ can be represented on the Complex PLANE by the coordinates (a, b) .

② Plot numbers on the complex plane.

1. Write the number as $a + bi$. For real numbers, $b = 0$. For fractions, see ①.

2. Plot the point (a, b) .

② Plot the following on the complex plane.

a) $5 - 2i$

b) -2

c) $4i$

d) $\frac{-15+11i}{2}$

1. $5 - 2i$

$-2 + 0i$

$0 + 4i$

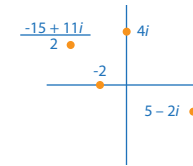
$\frac{-15}{2} + \frac{11}{2}i$

2. $(5, -2)$

$(-2, 0)$

$(0, 4)$

$(\frac{-15}{2}, \frac{11}{2})$



③ Solve equations with complex solutions.

1. Solve the equation normally, but change any number $\sqrt{-c}$ to $i\sqrt{c}$.

③ $2x^2 + 91 = 3$

$$x^2 = -44$$

$$x = \pm\sqrt{-44}$$

$$x = \pm i\sqrt{44}$$

$$x = \pm 2i\sqrt{11}$$

④ Add, subtract, and multiply complex numbers.

1. Treat i like a variable, but change i^2 to -1 .

④ Calculate.

a) $(5 + 2i) - (8 - 3i)$

$$5 + 2i - 8 + 3i$$

$$-3 + 5i$$

b) $(5 + 2i)(8 - 3i)$

$$40 + 16i - 15i - 6i^2$$

$$40 + i - 6(-1) = 46 + i$$

The Complex Conjugate of a number $a + bi$ is $a - bi$.

Multiplying a number by its complex conjugate results in a real (nonimaginary) number: $(a + bi)(a - bi) = a^2 + abi - abi - (bi)^2 = a^2 + b^2$.

5 Divide complex numbers.

1. Multiply the numerator and denominator by the complex conjugate of the denominator.
2. Convert i^2 to -1 , and combine like terms.
3. Put into $a + bi$ form, and reduce.

5 $\frac{3 + 4i}{2 - 10i}$

1. $\frac{3 + 4i}{2 - 10i} \cdot \frac{2 + 10i}{2 + 10i} = \frac{6 + 30i + 8i + 40i^2}{2^2 + 10^2}$

2. $\frac{6 + 38i - 40}{2^2 + 10^2} = \frac{-34 + 38i}{104}$

3. $\frac{-34}{104} + \frac{38}{104}i = \frac{-17}{52} + \frac{19}{52}i$

3-D Solving Quadratic Equations

If $ab = 0$, then $a = 0$ or $b = 0$.

① Solve a quadratic equation by factoring.

1. Isolate zero on one side of the equation.
2. Factor.
3. Set each factor equal to zero.
4. Solve.

① $18x^3 = 50x$

1. $18x^3 - 50x = 0$

2. $2x(9x^2 - 25) = 0$

$2x(3x + 5)(3x - 5) = 0$

3. $2x = 0$

$3x + 5 = 0$

$3x - 5 = 0$

4. $x = 0$

$x = -\frac{5}{3}$

$x = \frac{5}{3}$

If one side of an equation is a square and the other side is a constant, the equation can be solved by taking the square root of each side.

② Solve a quadratic equation by isolating a square.

1. Rewrite the equation so that there is a squared polynomial alone on one side and a constant alone on the other.
2. Square root each side.
3. Solve the + equation.
4. Solve the - equation.

② $8(2x + 3)^2 - 80 = 120$

1. $8(2x + 3)^2 = 200$

$(2x + 3)^2 = 25$

2. $2x + 3 = \pm 5$

3. $2x + 3 = 5$

$x = 1$

4. $2x + 3 = -5$

$x = -4$

If there isn't already a squared polynomial in a quadratic equation, you can make one by COMPLETING THE SQUARE.

③ Solve a quadratic equation by completing the square.

1. Subtract c from each side.
2. Divide each term by a .
3. Complete the square by adding $(\frac{b}{2})^2$ to each side.
4. Rewrite the square side as $(x + \frac{b}{2})^2$.
5. Square root each side.
6. Solve the + equation.
7. Solve the - equation.

③ $2x^2 + 20x + 34 = 16$

1. $2x^2 + 20x = -18$
2. $x^2 + 10x = -9$
3. $x^2 + 10x + 25 = -9 + 25$
4. $(x + 5)^2 = 16$
5. $x + 5 = \pm 4$
6. $x + 5 = 4$
 $x = -1$
 $x + 5 = -4$
 $x = -9$

Any quadratic equation in standard form can be solved by plugging a , b , and c into the QUADRATIC FORMULA: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

④ Solve a quadratic equation with the quadratic formula.

1. Put the equation in standard form.
2. Identify a , b , and c .
3. Plug a , b , and c into the quadratic formula.
4. Simplify.

④ $12x^2 + 17x = 7$

1. $12x^2 + 17x - 7 = 0$
2. $a = 12, b = 17, c = -7$
3. $x = \frac{-17 \pm \sqrt{17^2 - 4(12)(-7)}}{2(12)}$
4. $x = \frac{-17 \pm 25}{24} = \frac{1}{3}$ OR $-\frac{7}{4}$

⑤ Solve any quadratic equation.

1. If you can get a square by itself on one side and a constant by itself on the other (such as by completing the square), square root each side. See ② and ③.

2. Otherwise put the equation in standard form and do one of the following:

a) Factor the expression, set each factor equal to zero, and solve (see ①).

b) Use the quadratic formula (see ④).

$$\textcircled{5} \quad x^2 + 10x + 30 = 9$$

$$1. \quad x^2 + 10x + 25 = 4$$

$$(x + 5)^2 = 4$$

$$x + 5 = \pm 2$$

$$x = -7 \text{ or } -3$$

$$2. \quad x^2 + 10x + 21 = 0$$

$$a) \quad (x + 7)(x + 3) = 0$$

$$x + 7 = 0 \text{ or } x + 3 = 0$$

$$x = -7 \text{ or } x = -3$$

$$b) \quad \frac{-10 \pm \sqrt{10^2 - 4(1)(21)}}{2(1)}$$

$$x = \frac{-10 \pm 4}{2} = \frac{-14}{2} \text{ or } \frac{-6}{2} = -7 \text{ or } -3$$

A SOLUTION is a value that makes an equation true.

A ZERO or ROOT is a value that makes an expression or function equal to zero.

An X-INTERCEPT is a point $(x, 0)$ on a graph, where x is a real (nonimaginary) zero.

The DISCRIMINANT of a Quadratic Equation is the radicand $b^2 - 4ac$. If this is positive, the quadratic equation has two real solutions and thus two x -intercepts. If it is zero, it has one, and if it is negative, it has none.

⑥ Find the solutions, zeros, roots, or x -intercepts of a quadratic.

1. If the expression has no equals sign or is equal to y , make it equal to zero.

2. Solve the equation to find the solutions, zeros, or roots.

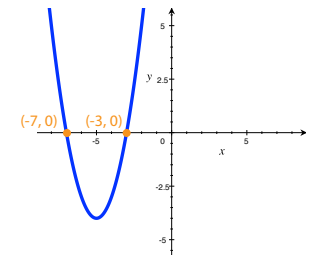
3. The x -intercepts are the same values as the zeros, unless the zeros are imaginary, in which case there are no x -intercepts.

⑥

The solutions to $x^2 + 10x + 21 = 0$ are $x = -7$ and -3 .

The zeros or roots of $f(x) = x^2 + 10x + 21$ are $x = -7$ and -3 .

The x -intercepts of $f(x) = x^2 + 10x + 21$ are $(-7, 0)$ and $(-3, 0)$.



3-E Conic Sections

Parabolas, hyperbolas, ellipses, and circles are called CONIC SECTIONS.

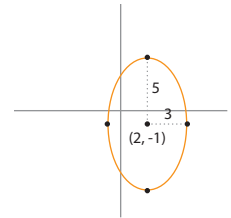
The standard form of the equation of an ELLIPSE is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where (h, k) is the center, a is the horizontal radius, and b is the vertical radius. A CIRCLE is an ellipse in which a and b are equal.

1 Sketch an ellipse given in standard form.

1. Identify $h, k, a,$ and b .
1. Mark the point (h, k) as the center of the ellipse.
2. Measure a distance of a to the right and to the left of the center, and plot these points.
3. Measure a distance of b above and below center, and plot these points.
4. Connect the four plotted points from #2 and #3 in an ellipse.

$$1 \quad \frac{(x-2)^2}{9} + \frac{(y+1)^2}{25} = 1$$

$$1. \quad h = 2, k = -1, a = 3, b = 5$$



The standard form of the equation of a Horizontal HYPERBOLA is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, where (h, k) is the center, a is the distance from the center to the branches of the hyperbola, and $y = \pm \frac{b}{a}(x-h) + k$ are the asymptotes.

The equation of Vertical Hyperbola is the same except the y term is positive instead of the x term.

2 Sketch a hyperbola given in standard form.

1. Sketch it as if it were an ellipse.
2. Sketch a rectangle around the ellipse.
3. Sketch the diagonals of the rectangle and extend them past the rectangle.
4. Using the diagonals as asymptotes, draw the hyperbola starting at the right and left points of the ellipse if the x term is positive, or the top and bottom points of the ellipse if the y term is positive.

$$2 \quad \frac{(x-2)^2}{9} - \frac{(y+1)^2}{25} = 1$$

3 Identify ellipses and hyperbolas.

1. Solve the equation for a positive constant if it is not already.
2. Identify the coefficient of the x^2 term and the coefficient of the y^2 term.
3. If both coefficients are positive, the conic is an ellipse. The ellipse is a circle if the coefficients are the same. Otherwise, the conic is a hyperbola.

3 Identify the following conic sections.

$$a) \quad 5x^2 + 5y^2 + 8x - 3y = 12$$

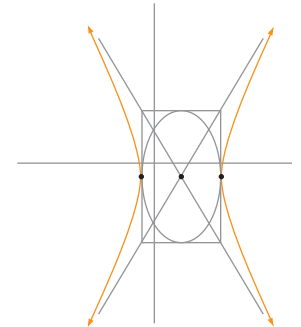
$$2. \quad 5x^2, 5y^2$$

3. It is an ellipse, and specifically a **circle**.

$$b) \quad \frac{(y+8)^2}{4} - \frac{3(x+1)^2}{10} = 1$$

$$\frac{1}{4}y^2, -\frac{3}{10}x^2$$

It is a **hyperbola**.



The equation of an ellipse or hyperbola can be written as a polynomial in the form $ax^2 + by^2 + cx + dy + e = 0$.

- ④ Write in standard form the equation of an ellipse or hyperbola written as a polynomial, and describe it.
1. Complete the square for x .
 2. Complete the square for y .
 3. Simplify.
 4. Divide each term by the constant term so that the equation is set equal to 1.
 5. If both terms are positive, it is an ellipse with center (h, k) , horizontal radius a , and vertical radius b . If only one of the terms is positive, it is a hyperbola in the direction of the positive term, with center (h, k) and asymptotes with slope $\frac{b}{a}$.
- ④ $x^2 + 4y^2 + 10x - 24y + 45 = 0$
1. $(x + 5)^2 + 4y^2 - 24y + 45 = 5^2$
 2. $(x + 5)^2 + 4(y - 3)^2 + 45 = 5^2 + 4(-3)^2$
 3. $(x + 5)^2 + 4(y - 3)^2 = 16$
 4. $\frac{(x+5)^2}{16} + \frac{(y-3)^2}{4} = 1$ is an ellipse centered at $(-5, 3)$ with a horizontal radius of 4 and a vertical radius of 2.